FRICTION MODELING OF ROBOT MANIPULATOR JOINTS

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Keywords: DC Motor, Direct and Inverse Kinematics, Dynamics of Robot Manipulator, LuGre Friction Model.

Abstract: In this paper the idea is to investigate friction in robot manipulator joints. First, the differential equations that describes DC Servo motor in each robot manipulator joints must be obtained. Second, the kinematic and dynamics equations must be obtained for two joints robot manipulator. The obtained equation are used for developing controller system considering dynamic type of friction (LuGre friction model) in robot manipulator joints. The results showed that dynamic friction drastically changes the trajectory of robot manipulator.

Introduction

Researchers have been studying friction phenomena for decades and discovered that different friction phenomena by experimental studies. Based on experimental investigation the mathematical formulation of different friction phenomena has been obtained. Friction is generally divided into two categories and these are static and dynamic friction. Static models are simple mathematical expressions that can be easily implemented and today they are widely used. However, the dynamic models include more phenomena and are more exact, but they are difficult to implement. When system is moving at slow velocity, friction between system elements becomes considerably significant and causes severe errors in control system if it’s not handled properly. One of the major difficulties in friction compensation is parameter uncertainties since friction forces varies with time, temperature and material properties. In this paper the dynamic model of planar two robot manipulator with LuGre friction which is dynamic type of friction in robot manipulator joints were considered. First the dynamic and kinematic equations of robot manipulator are developed and then the mathematical expressions for LuGre friction are derived.

DC Servo Motor

The differential equations that describe DC Servo motor [1] shown in Fig.1 can be written in the following form:

\[ L_\omega \frac{di_a}{dt} = -R_\omega i_a - K_\omega \omega_m + u_\omega J_m \frac{d\omega_m}{dt} = K_F i_a - T_L - T_f, \frac{d\theta_m}{dt} = \omega_m \]  

(1)

where \( i_a \) is the armature current, \( R_\omega \)is armature resistance, \( L_\omega \) is the armature inductance, \( u_\omega \)is the armature voltage, \( \omega_m \)is the angular velocity of motor shaft, \( J_m \) is the inertia of the motor shaft, \( T_L \) is the torque that acts on the motor shaft from the load and \( \theta_m \) is the motor angle. \( K_F \) and \( K_\omega \) are the torque and field constants, where \( K_F = K_\omega T_f \) is the friction torques of the system.

Kinematics and dynamics of two joint robot manipulator

In Fig.1 the planar two joint robot manipulator is shown. In order to obtain equations of joints first direct kinematics must be performed using Denavit Hartenberg procedure [3,4]. The parameters obtained using D-H procedure are given in Tab.1.

![Schematic view of the robot manipulator](image)

Tab. 1 Schematic view: a) Setup of DC-Motor and the load of friction torque [1], b) Two joints robot manipulator [2]

Kinematics parameters obtained using D-H method and are shown in Tab.1.

Table 1. D-H parameters of two joint planar robot manipulator

<p>| | | | | |</p>
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</thead>
<tbody>
<tr>
<td>q_1</td>
<td>0</td>
<td>a_1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>q_2</td>
<td>0</td>
<td>a_2</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
The result of direct kinematics procedure using Denavit Hartenberg procedure can be written in the following form:

\[
T_2^2 = \begin{pmatrix}
\cos(q_1 + q_2) & -\sin(q_1 + q_2) & 0 & a_1 \cos(q_1) + a_2 \cos(q_1 + q_2) \\
\sin(q_1 + q_2) & \cos(q_1 + q_2) & 0 & a_1 \sin(q_1) + a_2 \sin(q_1 + q_2) \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1
\end{pmatrix}
\]  

(2)

Joint equations obtained using inverse kinematics procedure can be written in the following form:

\[
q_i = \arctan(2(\sin(q_{i1}), \cos(q_{i2}), \sin(q_{i3})) = \frac{1 + \cos(q_{i2}) \cos(q_{i3})}{p_x^2 + p_y^2}, \quad \cos(q_{i4}) = \frac{1 + \cos(q_{i2}) \cos(q_{i3})}{p_x^2 + p_y^2}
\]

(3)

LuGre Friction model

Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. There are two types of friction models and these are static and dynamic friction models. LuGre friction model is dynamic friction model. The name “LuGre” is an abbreviation of the Lund Institute of Technology and INPG Grenoble [3]. Here the \(z(t)\) variable represents the average bristle deflection. For small displacement the model behaves like spring. Linearization around zero velocity and zero state gives:

\[
\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z, \quad F = \sigma_0 z + \sigma_1(v) \frac{dz}{dt} + f(v)
\]

(6)

where \(z\) denotes the average bristle deflection. For small displacement the model behaves like spring. Linearization around zero velocity and zero state gives:

\[
\frac{d(\delta z)}{dt} = \delta v, \quad \delta F = \alpha_0 \delta z + (\sigma_1(0) + f'(0)) \delta v,
\]

(7)

where parameter \(\sigma_0\) and \(\sigma_1(v)\) are the bristle stiffness and damping, respectively. In case of constant velocity and constant friction:

\[
F(v) = g(v) \delta g(v) + f(v)
\]

(8)

where \(g(v)\) and \(f(v)\) represents Stibbeck effect, and viscous friction respectively. The formula which gives good approximation of Stibbeck effect can be written in the following form:

\[
g(v) = \alpha_0 + \alpha_1 e^{-(v/k_0)z^2},
\]

(9)

\(\alpha_0 + \alpha_1\) represents stiction, \(\alpha_0\) represents Coulomb friction.

Control System Using Dynamic Compensation Scheme (LuGre Friction Model)

Consider the problem of tracking an operational space trajectory using the dynamic LuGre friction model. The differential equation for robot manipulator can be written in the following form:

\[
D(q) \ddot{q} + C(q, \dot{q}) \dot{q} = \tau - F,
\]

(10)

where \(F\) represents real friction.

\[
\tau = D(q) \ddot{q} + \sigma_0 \dot{q} + \sigma_1 \dot{q} + \sigma_2 \dot{q} - K_p \dot{q} - K_d \dot{q}
\]

(11)

where the terms \(\sigma_i, (i = 0, 1, 2)\) are coefficients of LuGre friction. The problem of control scheme is that the bristle state can’t be measured so observer must be added in the form:

\[
\dot{z} = \dot{\dot{q}} - \frac{|\dot{q}|}{g(\dot{q})} z
\]

(12)

Controller design

Now that dynamic equations are obtained, a feedback linearization will be applied in controlling manipulator. This type of linearization cancels nonlinearities in a nonlinear system so that the closed-loop dynamics is in a linear form. Since LuGre friction model have a non-measurable internal state \(z\), the observer must be implemented in order to perform estimation of parameter. The schematic view of controller system is shown in Fig. 2.
Results

The system was modeled using Matlab Simulink and it is shown in Fig. 3. The time of simulations was set to 10 s. The robot manipulator, DC motor, LuGre friction model data are given in Tab. 2.

![Fig. 3 Matlab Simulink scheme of two planar robot manipulator controller with LuGre friction model in manipulator joints](image)

Table 2. Technical data of DC motor, LuGre friction model and robot manipulator [1]

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>Armature inductance $L_a$ [H]</td>
<td>$1.4 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>Armature resistance $R_a$ [Ω]</td>
<td>0.6</td>
</tr>
<tr>
<td>Inertia of the motor shaft $J_m$ [kgm²]</td>
<td>$3.66 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Torque constant $K_T$</td>
<td>0.056</td>
</tr>
<tr>
<td>Field constant $K_F$</td>
<td>0.056</td>
</tr>
<tr>
<td>Stiffness of surfaces $\sigma_y$</td>
<td>0.5</td>
</tr>
<tr>
<td>Damping coefficient $\sigma_z$</td>
<td>0.1</td>
</tr>
<tr>
<td>Viscous coefficient $\sigma_z$</td>
<td>0.3</td>
</tr>
<tr>
<td>Coulomb friction level $F_c$ [Nm]</td>
<td>0.285</td>
</tr>
<tr>
<td>Striebeck velocity $v_s$ [rad/s]</td>
<td>0.001</td>
</tr>
<tr>
<td>Stiction level $F_s$ [Nm]</td>
<td>0.335</td>
</tr>
<tr>
<td>Link length of robot manipulator $a_1$, $a_2$ [m]</td>
<td>1</td>
</tr>
</tbody>
</table>

Circle trajectory and velocity of manipulator joints is shown in Fig. 4.

![Fig. 4 Circle trajectory and velocity of first and second joint](image)
Position error link 1, position error link 2, drive torque link 1 and drive torque link 2 are shown in Fig. 5.

![Graphs showing position and drive torques](image)

Fig.5 a) Position error link 1, b) Position error link 2, c) Drive torque link 1, d) Drive torque link 2

**Conclusion**

In this paper the kinematic and dynamic equations of two joint robot manipulator. The friction in robot manipulator joints is modeled based on LuGre friction model which is dynamic friction model. The entire system is modeled using Matlab Simulink software package. The results showed that by adopting the LuGre friction model trajectory of robot manipulator end-effector drastically deviates from ideal trajectory and further investigation should be conducted to verify the results.

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