LOW-NOISE LOW-POWER ALLPOLE ACTIVE-*RC* FILTERS MINIMIZING RESISTOR LEVEL

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ABSTRACT

The design procedure of 2nd- and 3rd-order low-sensitivity low-power allpole active resistance-capacitance (*RC*) filters, using the impedance tapering design method has already been published. Beside sensitivity minimization the impedance tapering was used for output noise minimization and output dynamic range maximization. In this paper it is shown that active resistance-capacitance (*RC*) filters can also be designed for low input thermal noise. The design procedure of filters with minimum input noise is very similar to the design of minimum sensitivity filters in its most important steps. Thus, the judicious selection of component values account for the considerable decrease in sensitivity and in input thermal noise. The minimum noise at the filters input is obtained by minimizing the total resistance of the filter circuit. The noise analysis was performed on the 2nd- and 3rd-order (class 4) Sallen and Key low-pass filter sections using MATLAB.

1. INTRODUCTION

The quality of low signal-level and low-power signal processing, depends, among other factors, on the noise level produced in the circuits. It has already been published in [1] and in [2] (see [2] pp. 337) that the allpole active-*RC* filters of second-order which are designed for minimum sensitivity to component tolerances (see [3]) are also superior in terms of low output thermal noise, when compared with standard designs. In [1] the method of Zurada and Bialko [4] was used to determine output noise spectral density and total RMS output noise of filters. Passive elements and operational amplifiers are represented by substitute noise models. The noise contribution of each device to the output node is calculated using noise transfer functions. The methods used for analysis of the output noise in [1] were used in the analysis of the input noise in this paper, where we reduced the output noise to the input. Considered are Sallen and Key low-pass filters 2nd- and 3rd-order (class 4) as in [5], [6] and [7]. Note that most circuit analyzing programs, such as PSPICE can analyze both the output and the input noise spectral density.

Using the numerical programming tool MATLAB, mathematical calculations of the output noise were performed. An ideal operational amplifier was replaced by a simple positive voltage gain β, combined with voltage and current noise sources.

The transfer function of every noise source to the *output voltage*, is given by

$$T_{i,k}(s)=\frac{V_{OUT}}{I_k};\quad T_{v,l}(s)=\frac{V_{OUT}}{V_l}\,, \tag{1}$$

where V_l is the voltage and I_k is the current noise source of element denoted by *l* and *k*, respectively. For the Sallen and Key 2nd-order LP filter (as presented in Figure 1) the transfer functions in (1) are calculated with help of the symbolic calculation tool MATHEMATICA *Wolfram Research* and are presented in [1]. Once the total output noise is determined, we can refer the noise back to the input to obtain equivalent input noise. This is done by dividing every transfer function from noise source to the output, which is defined by (1) by the filter's transfer function $T(s)=V_{OUT}/V_{IN}=N(s)/D(s)$. For example, we calculate the transfer functions from the noise voltage source to the *input voltage* by

$$T_{v,l,\mathrm{IN}}(s)=\frac{V_{IN}}{V_l}=\frac{V_{OUT}}{V_l}\cdot\frac{V_{IN}}{V_{OUT}}=T_{v,l}(s)\cdot\frac{1}{T(s)}\,. \tag{2}$$

2. DEFINITION OF NOISE FIGURE

Active-*RC* filters consist of resistors, capacitors and operational amplifiers. Thermal noise in resistors is caused by random motion of free charges and is also called Johnson's noise. Noise in real capacitors is also of thermal origin. It is produced within the resistive non-ideal part of a capacitor, and can be neglected. Because thermal noise is stochastic in nature, we describe its influence using the mean-square noise voltage and current within a frequency band Δf as follows:

$$\overline{v_n^2}=4kTR\Delta f\ ;\ \overline{i_n^2}=4kT\frac{1}{R}\Delta f\,, \tag{3}$$

where k=1.38·10^{-23} J/K is Boltzman's constant. Note that thermal noise is proportional to the absolute temperature *T*.

Let us now define measures by which we can examine the noise performance of the filters under consideration. The most important is the mean square (MS) noise voltage within a specified frequency range $\Delta\omega=\omega_2-\omega_1$, defined as:

$$\left(E_n^2\right)_{ef}=\int_{\omega_1}^{\omega_2}V_n^2(\omega)d\omega\,, \tag{4}$$

where the square of the noise spectral density, derived from all the noise sources and their corresponding transfer functions, is given by:

$$V_n^2(\omega)=\sum_{k=1}^{m}\left|T_{i,k}(j\omega)\right|^2(i_n)_k^2+\sum_{l=1}^{n}\left|T_{v,l}(j\omega)\right|^2(v_n)_l^2\,, \tag{5}$$

where $T_{i,k}(j\omega)$ is the transfer impedance, i.e. the ratio of the output voltage and input current of the k^{th} current noise source $(i_n)_k$, and $T_{v,l}(j\omega)$ is the voltage transfer function, i.e. ratio of output voltage and input voltage of l^{th} voltage source $(v_n)_l$. Those output voltages are then referred to the input node as described above.

For the purpose of noise analysis, appropriate noise models for resistors and operational amplifiers (OAs) must be used. Resistors are represented by the well-known Nyquist current noise model consisting of a noiseless resistor and a noise source as presented in [1] whose value is given by eq. (3). The OA is represented by the noiseless OA combined with voltage and current noise sources [1]. For the TL081/TI (Texas instruments) operational amplifier in the noise analysis we use approximate values: E_n=17nV/$\sqrt{\text{Hz}}$ and I_n=0.01pA/$\sqrt{\text{Hz}}$.

The single equivalent voltage-noise source V_{nin}, as shown in Figure 1, can completely characterize the noise performance of the filter driven by voltage signal source V_g. Observation of the value of equivalent input noise V_{nin} is a powerful tool for analysis of the filter's noise performance, and will therefore be used. It allows a direct comparison between the signal and the noise and can be used to directly compare the performance of various filter circuits regardless of each filter's pass-band gain, input impedance, or transfer function shape. Using an equivalent input voltage-noise (as opposed to the equivalent input current noise) is particularly advantageous for analog active-*RC* filters.

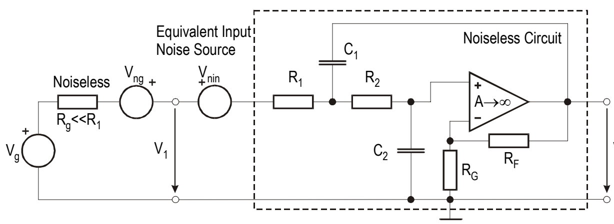

Figure 1. Equivalent input noise source representation on the example of 2nd-order LP Sallen & Key filter section.

The equivalent input noise V_{nin} refers all noise sources to the signal source location and the filter circuit can be regarded as noiseless. Since both the signal and the noise equivalents are then present at that point in the system, the *signal-to-noise* ratio (*S*/*N*) can be easily evaluated. In other words, the single noise source independent of the filter transfer function V_{nin} inserted in series will produce the same total output noise. The *noise factor* is a figure-of-merit for a circuit with respect to noise, defined by:

$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{N_i} \cdot \frac{1}{A_a}, \tag{6}$$

where S_i and N_i are signal and noise powers at input, respectively, while S_o and N_o are the same quantities at output. A_a is system power gain. By definition the *noise figure* is[1]:

$$NF = 10\log F = 10\log\frac{\left(E_{ng}\right)_{ef}^2 + \left(E_{ni}\right)_{ef}^2}{\left(E_{ng}\right)_{ef}^2} = 10\log\left(1 + \frac{\left(E_{ni}\right)_{ef}^2}{\left(E_{ng}\right)_{ef}^2}\right)[\text{dB}], \tag{7}$$

where $\left(E_{ni}\right)_{ef}^2$ represents the power of the noise at input, which can be obtained by integrating $V_{ni}^2(\omega)$ over the frequency range $\Delta\omega$, and $\left(E_{ng}\right)_{ef}^2$ represents the power[2] of the generator noise, which can be obtained by integrating $V_{ng}^2(\omega)$ over the same range $\Delta\omega$. $V_{ni}(\omega)$ is the noise spectral density of the equivalent input noise source, i.e. the noise of the filter circuit reduced to the filter's input, and $V_{ng}(\omega)$ is the noise spectral density of the input resistor R_g of the real voltage signal generator V_g, or the output noise of the previous stage, if the filter is realized in a cascade.

Consequently, if we consider a multistage filter, where each stage has the corresponding noise factor F_1, F_2, etc., then the overall noise factor F of the cascade is given by the Friis formula [4]

$$F = F_1 + \frac{F_2 - 1}{A_1} + \frac{F_3 - 1}{A_1 A_2} + \ldots, \tag{8}$$

where A_1, A_2, etc., are the maximal power gains of each block. From (8) and A_i>1; (i=1,2,…,N) it follows that to minimize the noise factor F of the whole cascade structure, it is most important to have the first block with minimum noise factor F_1. In what follows we present the design method which will minimize the noise factor of the 2nd- and 3rd-order LP filters.

3. DESIGN OF MINIMUM NOISE FILTERS

The same 2nd- and 3rd-order low-pass filter examples in [3], used for the sensitivity analysis, are used in what follows for the noise analysis. Using the numerical programming tool MATLAB, mathematical calculations of the input noise were performed. The noise factor (*F*) [defined by (6)] is a figure-of-merit for a device or a circuit with respect to noise, but it will not be calculated; instead the curves which represent the input equivalent noise sources' spectral densities will be observed and compared.

In the 2nd- and 3rd-order LP filter examples given in [3], sensitivity performances are dependent only on the values ω_0, r_i, ρ_i; (i=2,…,n) and β. On the other hand, noise is dependent on the resistor values in the circuit and on the operational amplifier noise. With lower resistor values and a "low-noise" amplifier, we obtain lower noise. In what follows, we shall concentrate only on the resistor's noise. To reduce noise of the filter circuit, one can always take as low value resistors as possible, and very high capacitor values. This is always possible for the discrete realization of the filters, where we are not upper-limited with a total capacitance value. But, when we realize filters with integrated circuit techniques, we must calculate resistors and capacitors in a different way, with an important constraint:

$$C_1 + C_2 + \ldots + C_n = C_1\left(1 + \frac{1}{\rho_2} + \ldots + \frac{1}{\rho_n}\right) = C_{\text{TOT}}\,. \tag{9}$$

From ω_0, r_i, ρ_i; (i=2,…,n) and β as given in [3] using constraint (9) we calculate the value of R_1 by:

$$R_1 = \frac{1}{\omega_0 C_1};\text{ where } C_1 = \frac{C_{\text{TOT}}}{1 + 1/\rho_2 + \ldots + 1/\rho_n}\,. \tag{10}$$

3.1 2nd-order LP Filter Example

The 2nd-order LP filter practical examples with f_0=86kHz and q_p=5 as in [3] are repeated here. For the example of total capacitance value C_{TOT}=100pF, we obtain the component values of the filter in Table 1. To concentrate only on the noise contribution of the ladder-network in the filter's positive feedback loop, we choose very small R_G and R_F resistors in the negative feedback loop, which realize the gain β, i.e. they are in [Ω]. The corresponding input noise spectral densities are shown in Figure 2. Observing the input noise spectral density curves in Figure 2, we conclude that the filter with the lowest noise is non-tapered filter no. 1 (r=ρ=1). The second best results are obtained

[1] Note that for calculating noise factor (*F*) and/or noise figure (*NF*), we must include the noise of the input resistor R_g. Because we assumed ideal signal source V_g with R_g=0 the eqs. (6) and (7) are only approximately correct.

[2] More precisely, it is mean square noise voltage, and if used with unity resistive load, it represents the power.

with filters no. 3, which we obtain by tapering only the capacitors, while keeping the resistor values equal (r=1), and no. 4, i.e. the resistively tapered filter with equal capacitors (ρ=1).

Table 1 Component values of 2nd-order LP filters (R_G, R_F in [Ω], other resistors in [kΩ], capacitors in [pF]).

No.	Filter	r	ρ	R_1	C_1	R_{TOT}	C_{TOT}	β	R_G	R_F
1.	Non Tapered	1	1	37	50	74	100	2.8	1	1.8
2.	Impedance Tapered	4	4	23.1	80	116	100	2.05	1	1.05
3.	Part. Tapered (r=1)	1	4	46.3	80	92.5	100	5.6	1	0.4
4.	Part. Tapered (ρ=1)	4	1	18.5	50	92.5	100	1.4	1	4.6
5.	r=1 and min. GSP	1	5.53	51.4	84.7	103	100	1.28	1	0.28
6.	R-Taper, min. GSP	4	13.5	36.5	93.1	183	100	1.26	1	0.26

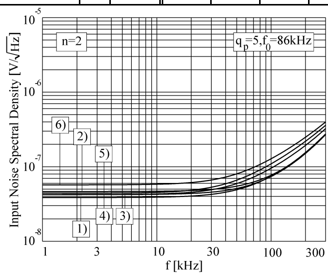

Figure 2. Input noise of 2nd- LP filter circuits in Table 1.

The worst results are obtained with ideally impedance tapered filter no. 2 and filters no. 5, 6, i.e. partially tapered filters with min. GSP [8]. Those results can be concluded from the R_{TOT} column in Table 1. It is obvious that best noise performances is given by filters with minimum total resistance R_{TOT}. *In summary, for the general second-order allpole low-pass filter, minimum noise is obtained with non-tapered circuits, and total resistance minimization.*

3.2 3rd-order LP Filter Example

Using the filter examples in [3] having specifications in Table 2

Table 2 Specifications and coefficients for 3rd-LP filter.

No.	Approx.	Spec. [dB; kHz]	$a_0 \cdot 10^{18}$	$a_1 \cdot 10^{12}$	$a_2 \cdot 10^6$	q_p
1.	Butterworth	0.5/25; 80/300	0.364	1.02	1.49	1.0
2.	Chebyshev	0.04/25; 80/300	0.33	0.91	1.2	1.236
3.	Chebyshev	0.5/38; 75/300	0.0749	0.341	0.59	1.706

and total capacitance value C_{TOT}=100pF, we obtain the component values of the filters in Table 3, and Table 4. The corresponding input noise characteristics are shown in Figure 3. Observing those diagrams we conclude that the best noise performance is given by circuits with equal resistors. This is logical, since "R-equal" circuits have minimum total resistance R_{TOT}, as can be seen from Table 3.

Table 3 Component values of capacitively- and resistively-tapered 3rd-order LP filters (resistors in [kΩ], capacitors in [pF], note: R_G, R_F in [Ω]).

No.	Spec.	Filter	$\omega_0 10^5$	R_1	R_{TOT}	C_1	C_{TOT}	β
1.	Chebyshev no. 3, Table 2	C-equal	2.51	119	314	33.3	100	2.0
2.		C-taper: 5	2.179	56.9	745	80.6	100	2.0
3.		R-equal	1.817	87.4	262	63	100	4.0
4.		R-taper: 5	2.468	43.8	1359	92.4	100	4.0

Table 4 Dependence of R_i, C_i (i=1,2,3) and β on selection of ω_0, for capacitively-tapered 3rd-order LP filter.

No.	Spec; Filter	$\omega_0 10^5$	R_1	R_2	R_3	R_{TOT}	C_1	C_{TOT}	β
1.	Chebyshev no. 3, Table 2 C-taper: 3	2.25	64.2	93.4	181	339	69.2	100	1.69
2.		2.5	57.8	124	151	333	69.2	100	1.48
3.		2.6	55.6	145	135	335	69.2	100	1.41
4.		2.75	52.3	200	103	356	69.2	100	1.31
5.		2.9	49.8	425	51.3	526	69.2	100	1.27

Note that C_2=23.1 and C_3=7.69 in Table 4. Observing the curves in Figure 3(a) and (b) we conclude that both resistive and capacitive tapering deteriorate the noise performance transformed to the input. The proper choice of the design frequency ω_0, while capacitive-tapering, improves noise characteristics. Observing Figure 3(c) the filter with the lowest input-noise is filter no. 3, which we obtain by tapering the capacitors, while keeping the R_2 and R_3 resistor values approximately equal ($R_2 \approx R_3$). *Note that according to [3] this coincides with the minimum sensitivity circuit.*

In what follows, we shall try to optimize "C-equal" and "R-equal" circuits to minimize the noise performances, by an appropriate choice of design frequency ω_0. Using the Chebyshev third-order transfer function satisfying the filter specifications as in the previous example, for 5 different values of ω_0, we obtain 5 different third-order circuits with equal resistors, with the design values listed in Table 5. Corresponding curves for total resistance R_{TOT}, vs. design frequency ω_0, are shown in Figure 4(b) for three values of total capacitance C_{TOT}, i.e. C_{TOT}=100, 200 and 300pF. (In our examples we calculate only with C_{TOT}=100pF). In Figure 4(a) are the noise characteristics of the "C-equal" case.

Observing Figure 4(a), i.e. for the "equal-C" case, we note that there is a minimum of noise for filter with approximately R_{TOT}=277kΩ. Furthermore, in Figure 4(b) for the "equal-R" case, the minimum of noise is found in circuit no. 2 with R_{TOT}=249kΩ. The optimal "equal-R" case has slightly better noise performances than the optimal "equal-C" case. This is because it has a lower total resistance value.

We can conclude that the noise of the circuit is directly proportional to the total resistance of the circuit R_{TOT}.

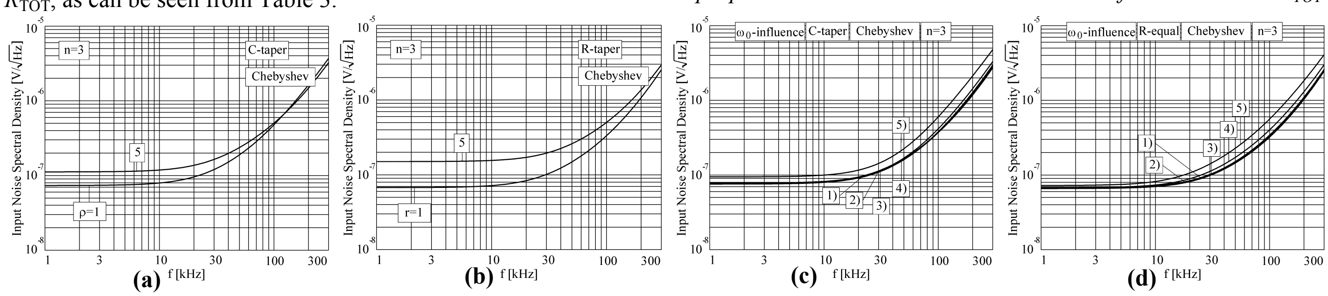

Figure 3. Input noise spectral density of 3rd-order low-pass filters, with component values given in **(a)** Table 3 filters no. 1,2. **(b)** Table 3 filters no. 3,4. **(c)** Table 4, and **(d)** Table 5.

Table 5 Component values of equal-resistor 3rd-order low-pass filter. Dependence on selection of ω_0.

No.	Spec; Filter	$\omega_0 10^5$	R_1	R_{TOT}	C_1	C_2	C_3	C_{TOT}	β
1.	Chebyshev no. 3, Table 2 *R*-equal	1.8	88.0	264	63.1	12.9	23.9	100	4.21
2.		2.0	82.9	249	60.3	22.0	17.6	100	2.32
3.		2.1	83.4	250	57.1	28.9	13.9	100	1.77
4.		2.2	87.8	263	51.8	38.3	9.95	100	1.39
5.		2.3	101	303	43.0	51.2	5.87	100	1.15

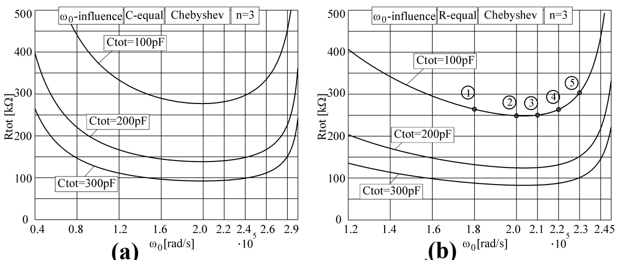

Figure 4. Total resistance R_{TOT} of 3rd-order low-pass filters, versus design frequency ω_0. For **(a)** "equal-*C*" case. **(b)** For "equal-*R*" case and components in Table 5.

If we choose higher value of total capacitance C_{TOT}, for example C_{TOT}=200 or 300pF, we obtain a lower minimum of total resistance R_{TOT}, which improves noise performance.

Furthermore, it can be shown that comparing optimized "equal-*C*" case with "*C*-tapered" case, we note that, although appropriate choice of ω_0 minimizes the noise in both cases, impedance tapering worsens the input noise performances, thus the "equal-*C*" case is better. The same could be concluded if we compared "equal-*R*" and "*R*-tapered" cases. It is fortunate that, in optimizing the noise performances by appropriate choice of ω_0, we minimize the sensitivity to the component tolerances, as well. The sensitivity can be minimized further by applying impedance tapering, but it will worsen the noise characteristics. *The choice therefore results in a compromise, which filter best suits our needs, i.e. "C-taper" for minimum sensitivity or "R-equal" for minimum noise and still very low sensitivity.*

3.3 Dependence of noise on pole Q-factor

For three approximations of 3rd-order LP filters shown in Table 2, (note three pole-Q factor values) we calculate filter examples with "*R*-equal" and choose design frequency ω_0 for minimum total resistance R_{TOT} and minimum noise. The input noise spectral densities and R_{TOT} vs. ω_0 curves are shown in Figure 5.

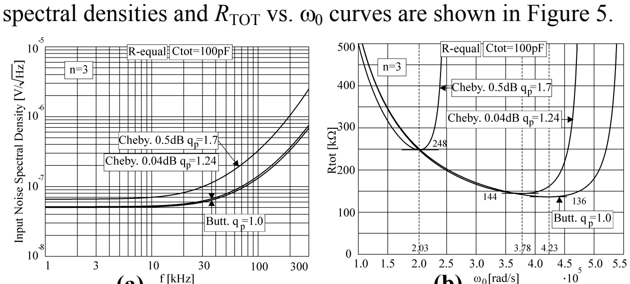

Figure 5. Equal-R 3rd-order LP filter optimized for min. noise (min. R_{TOT}), with different filter approximations. **(a)** Input noise. **(b)** Dependence of R_{TOT} on selection of ω_0 for different filter approximations.

Observing the curves in Figure 5, we note that, for higher values of pole Q-factors, q_p, we obtain higher noise contributions of the resistors in the filter's *RC*-ladder network, and, as would therefore be expected, a higher value of total resistance R_{TOT}. Thus, the higher the pole Q is, and the filter order *n*, the higher the noise of the filter circuit. In [3] it was also shown that, the higher the pole Q and the filter order *n* are, the higher the sensitivity of the filter circuit is.

In summary, for the third-order low-pass filters, minimum noise of the circuit is obtained with an equal-resistor circuit and appropriate choice of design frequency ω_0, which will minimize the total resistance R_{TOT}, for given total capacitance C_{TOT}. This rule can be extended to higher-order filters, as well.

4. CONCLUSIONS

Unlike sensitivity, which only depends on the ratios of component values, thermal noise performance was shown to depend on the values of filter components themselves, particularly the resistors in the filter's ladder network. Realization of the filters, using integrated circuits, caused a constraint that the total capacitance on the chip is limited by an upper value C_{TOT}, thus the design procedure can influence the filter's noise performance. It was shown that the most important design parameter is the design frequency $\omega_0=(R_1C_1)^{-1}$. Optimization of the filter's noise performance concentrates on a search for the optimum value of ω_0, for which we obtain the minimum total resistance in the circuit R_{TOT}. It was shown that, by application of impedance tapering, we worsen the noise performance, (while improving the sensitivity performance) thus the best choice is an equal-*R* circuit, with optimum ω_0 for min. R_{TOT}. Furthermore, the noise generated by the circuit is proportional to the filter's order *n* and to the poles Q-factors. Thus, beside the proposed design technique we should obey some other rules, such as: build filters with *i*) pole *Q*-factors as low as possible, and *ii*) filter order *n* as low as possible, to reduce the input thermal noise power of the circuit. Consequently, the extension to high- and band-pass filters would follow the same principles, concerning the minimization of total resistance R_{TOT}.

5. REFERENCES

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