Optimization of Cargo Transport with a View to Cost Efficient Operation of Container Ship

Zdenka Zenzerović, D.Sc, Svjetlana Bešlić, M.Sc.
Faculty of Maritime Studies, Studentska 2, 51 000 Rijeka
zenzerov@pfri.hr, beslic@pfri.hr

Abstract. *The paper presents a mathematical model addressing the problem of cargo transport by container ship on the selected route, where, certain number of containers of various masses and types are chosen out of total number of containers available in loading port in order to achieve maximum transport profitability, provided maximum payload and transport capacity of container ship. Since the profitability is an index of operation efficiency expressed as a fraction, respective mathematical model can be solved using linear fractional programming. The model was tested on real-life example of feeder ship "Lipa" owned by "Lošinjska plovidba", Rijeka, operating on route Rijeka-Gioia Tauro.*

Keywords. Linear fractional programming, operation efficiency, profitability, transport by container ship

1. Introduction

Operation efficiency in market economy is based on principle of maximum rationality [8, p. 327], i.e., on yielding maximum operating results against a certain amount of investments. In order to achieve maximum results, it is necessary to implement, measure and follow-up realization of economic business principles, and consequently operation efficiency.

In economic literature and practice, the following criteria of efficiency are applied [8, p. 328]: productivity, economic justifiability and profitability, expressing quantitative relations between effect of operating results and quantity of work process elements spent or invested.

In measuring profitability, operating result is the profit or part of the profit, while work process elements are usually considered as average own capital or average funds invested in order to achieve operating results.

Profitability is an index of efficiency [2, p. 685], showing the profit achieved per unit of invested capital, where invested capital can also be time spent in achieving operating results. If operating revenues are greater than operating expenses, a company makes profit, while if operating revenues are smaller than operating expenses, a company suffers loss. In the first case, a company is said to be profit-making, while in the latter case it is unprofitable.

Since above-mentioned indices are expressed as fractions, maximum values of these indices can be determined by using linear fractional programming, provided that constraints are expressed in linear form.

The objective of this paper is to show how to estimate the operation efficiency taking into consideration maximum profitability as criterion function which has to be optimized.

2. Fractional programming in function of operation efficiency optimization

Linear fractional programming is an area of optimization where a linear fractional objective function is optimized subject to linear constraints. There are several methods for solving linear fractional programming problems (Dinkelbach method, Martos method, Charnes-Cooper method, Gilmore-Gomory method and others) [4, p. 31].

Martos method was chosen for this paper, representing a modification of simplex method for linear programs, and using objective function gradient.

One of the ways of defining linear fractional programming problem is shown in the following model [4, p. 32]:

$$Max \frac{C'X + c_0}{D'X + d_0} \tag{1}$$

with constraints:

$$\begin{aligned} AX &\le A_0 \\ X &\ge 0, \end{aligned} \tag{2}$$

where:

$$C=\begin{bmatrix}c_1\\c_2\\\vdots\\c_n\end{bmatrix}, D=\begin{bmatrix}d_1\\d_2\\\vdots\\d_n\end{bmatrix}, X=\begin{bmatrix}x_1\\x_2\\\vdots\\x_n\end{bmatrix}, A_0=\begin{bmatrix}a_1\\a_2\\\vdots\\a_m\end{bmatrix}, \quad (3)$$

while matrix A=[a_{ij}] is of the order (m, n), and c_0 and d_0 are given real scalars.

Considering the practical implementation, the following is assumed [4, p. 32]:

a) Set of possible solutions $S=\{X|AX \le A_0, X \ge 0\}$ is not empty and is limited (in a sense that distance between two arbitrary points from S is less than some positive number given a priori). Also, set S is convex and closed. Consequently, S is a convex polyhedron, it has a finite number of extreme points and each point of S is some convex combination of extreme points.

b) From $D'X + d_0 > 0 \quad \forall X \in S$, it follows that the objective function is continuous and that denominator is positive.

In particular case of some economic problem, set in form of the model as above, condition a) means that none of economic activities can be unlimited, and the second condition b) excludes possibility that program value becomes infinite. Both conditions, therefore, correspond to practical assumptions, i.e., only this case is of practical significance.

Denominator $D'X + d_0$ in (1) can be interpreted as the value of investment for realization of program X. In that case, numerator $C'X + c_0$ represents the economic effect of the investment. The solution of the problem is such a production program X providing, under given conditions, the maximum value of ratio between effect and investment. Therefore, this is the issue of maximizing investment efficiency. For example, if d_j is time spent for transport of cargo unit x_j and c_j is the price for transporting unit of cargo, then the objective function is index of profitability of transport.

B. Martos was the first who noted that all preconditions were met for solving the model set above, under assumptions a) and b), by using simplex method. According to this method, the starting point is one possible basic solution, and, assuming nondegeneration, one extreme point of set S corresponds to such a solution. From that point, a shift is made to another extreme point where the value of objective function Z is greater, so that one vector is changed in the base. It is the criterion for selecting a new vector in the base that Martos developed and implemented it in the simplex method algorithm, which represents his modification of the simplex method. For details on Martos method, please refer to Martić [4, p. 36, 37].

When solving numerical examples, results of the structural variables are mainly decimal numbers. But, in practice, nature of the problems which are to be optimized demands structural variables to be integer. To find an integer optimal solution "rounding" approach is by no means not recommendable. When decimal numbers in the solution are rounded to integers, it is possible that optimality is lost. Therefore, in these cases, methods for integer fractional programming should be applied, such as branch-and-bound method, or the Gomory "cutting plane" method, adopted to the linear-fractional problem considered, see [1,6,7].

One of the linear fractional programming applications is in defining maximum operating profitability. In that case, profitability function is as follows [4, p. 44]:

$$R=\frac{\sum_{j=1}^{n} c_j x_j - c_0}{\sum_{j=1}^{n} d_j x_j + d_0}, \quad (4)$$

with constraints:

$$\begin{aligned}&\sum_{j=1}^{n} d_j x_j + d_0 \le k\\ &\sum_{j=1}^{n} a_{ij} x_j \le a_{i0} \quad (i=1,2,\ldots,m),\end{aligned} \quad (5)$$

where:

x_j – number of product units of j-th type,
c_j – gross profit per unit of product,
c_0 – fixed costs,
d_j – capital invested per unit of product,
d_0 – fixed capital,
n – number of product types,
k – available funds (capital),
a_{i0} – available capacities, raw materials or some other production constraint factor,
a_{ij} – various technological coefficients.

Following mathematical model (4)-(5), the profitability is defined by ratio between profit yielded and funds invested. The following assumptions are considered: a) some production activities require only proportional costs and funds and b) fixed costs and fixed used funds do not depend on selected production type and size.

3. Modelling of cargo transport by container ship

3.1. Problem description

The example for defining the optimal structure of container transport is illustrated through operation of container ship "Lipa", on the data basis provided by the owner "Lošinjska plovidba".

Namely, in early March 1999, feeder service is established on the line (Rijeka-Ploče-Gioia Tauro-Malta), performed by Croatian owner "Lošinjska plovidba", financially supported from the Budget of the Republic of Croatia [5, p. 238]. The service has been maintained with one ship of approximately 200 TEU capacity (TEU is 20' equivalent unit), scheduled once a week from ports of Rijeka and Ploče to the largest Mediterranean ports of Malta and Gioia Tauro.

Feeder ship "Lipa" of 5615 tons deadweight and transport capacity of 250 TEU, carries four types of containers. Unit mass and revenue per one transported container are shown in Table 1.

Table 1. Unit mass and revenue per one container

Type of container	TEU	Unit mass in tons	Revenue per one container in USD
20' full	1	25.0	150
20' empty	1	2.5	75
40' full	2	30.0	250
40' empty	2	3.5	125

Maximum allowed mass of 20 feet full containers is 30 tons, while for the 40 feet full containers maximum allowed mass is 35 tons. Average masses are considered in solving the problem, 25 and 30 tones for 20 and 40 feet containers, respectively.

The calculation considers one-way voyage on direct line Rijeka-Gioia Tauro, without any other port of call. Voyage takes 1.74 day, for which period respective ship operation costs amount to 16386 USD. Waiting time costs in ports of Rijeka and Gioia Tauro total to 9525 USD, making the aggregate voyage cost of 25911 USD.

Transshipment rate of one container crane is 35 TEU/h, running continuously, on the round-the-clock basis. Two container cranes are used in parallel. Therefore, time for transshipment per one container is 1/420 days.

The task is to determine number of particular container type to be transported that would yield maximum profitability per one voyage considering payload and transport capacity of a container ship.

3.2. Mathematical model formulation

As it can be seen from the problem description, this is the issue of transport service production. Therefore, the profitability of the transport process will be defined as ratio between profit yielded from transport of various types of cargo and funds invested for realization of such transport process.

If following notation is introduced:

x_j – quantity of j-th type of containers being loaded on board (TEU),

Q_j – quantity of j-th type of containers available for loading on berth (TEU),

f_j – transport price of j-th type of container (in currency units per TEU or per ton),

g_j – mass of j-th type of container (tons),

V – container ship deadweight (tons),

W – ship's transport capacity (TEU),

then problem can be formulated as a linear programming model [7, p. 304-306]:

$$Max \sum_{j=1}^{n} f_j x_j \tag{6}$$

with constraints:

$$\begin{gathered} \sum_{j=1}^{n} g_j x_j \le V \\ \sum_{j=1}^{n} x_j \le W \\ x_j \le Q_j \end{gathered} \tag{7}$$

x_j – nonnegative integer values; $j = 1, \ldots, n.$

If $\sum_{j=1}^{n} Q_j \le W$ and $\sum_{j=1}^{n} g_j Q_j \le V$ the solution is trivial, i.e., all cargo available on berth should be loaded on board.

However, mentioned model (6)-(7) is not real because it does not consider time necessary to load cargo or duration of voyage between the loading port and the port of discharge. In order to formulate a model that would correspond to reality, additional elements are introduced:

a_j – average number of TEUs of j-th type of container that can be loaded on board on daily basis,
C_1 – cost of container ship stay in port (per day),
C_2 – cost of container ship during voyage (per day),
d – distance between ports (in nautical miles),
s – speed of container ship (in knots).

Total duration of voyage and ship's stay in a port during loading can be expressed as [7, p. 305]:

$$T = \frac{d}{s} + \sum_{j=1}^{n} \frac{x_j}{a_j} \quad , \tag{8}$$

while the profit is

$$D = \sum_{j=1}^{n} f_j x_j - C_1 \sum_{j=1}^{n} \frac{x_j}{a_j} - C_2 \frac{d}{s} \quad . \tag{9}$$

Operation efficiency cannot be defined taking into consideration total amount of profit since yielding high profit over a (too) long period may not necessarily be acceptable. Therefore, it is necessary to calculate profit per unit of time, i.e., in this case, D/T. Therefore, profitability of a service line for transport of various container types is expressed by amount of profit per unit of time.

Considering (8) and (9), mathematical model contains fractional function thus becoming a linear fractional programming model:

$$Max \frac{\sum_{j=1}^{n} f_j x_j - C_1 \sum_{j=1}^{n} \frac{x_j}{a_j} - C_2 \frac{d}{s}}{\frac{d}{s} + \sum_{j=1}^{n} \frac{x_j}{a_j}} \tag{10}$$

with constraints:

$$\begin{aligned} &\sum_{j=1}^{n} g_j x_j \le V \\ &\sum_{j=1}^{n} x_j \le W \\ &x_j \le Q_j \end{aligned} \tag{11}$$

x_j – nonnegative integer values; $j = 1, \ldots, n$.

Structure of container ship transport is exclusively made of containers, and possibly transport of RO/RO cargo. Containers loaded on container ships are uniquely standardized (in terms of length, width, height), which enables setting conditions to be met for all cases when formulating a model.

To use such a mathematical model set for solving the problem of transport of containers by sea, it is assumed that maritime market offers enough containers of various types, masses and sizes, i.e., greater than transport capacity of a ship. This assumption is by all means true for feeder ships, having a capacity of up to 1200 TEU, which results from their function of transporting containers from so called hub terminals, where large container ships discharge more than 6000 TEU.

3.3. Analysis of the optimal solution

The mathematical model for problem stated in item 3.1. is as follows:

$$R = \frac{150x_1 + 250x_2 + 75x_3 + 125x_4 - 25911}{1{,}74 + \frac{1}{420}x_1 + \frac{1}{420}x_2 + \frac{1}{420}x_3 + \frac{1}{420}x_4}$$

with constraints:

$$\begin{aligned} &x_1 + 2x_2 + x_3 + 2x_4 \le 250 \\ &25x_1 + 30x_2 + 2.5x_3 + 3.5x_4 \le 5615 \\ &x_1, x_2, x_3, x_4 \ge 0 \text{ and integer.} \end{aligned} \tag{12}$$

Notation x_j, j=1,…,4 refers to container type as per Table 1. The numerator of function R represents the difference between revenue from transport of particular container type and total costs, including voyage costs and cost of ship's stay in a port. The denominator of function R represents duration of voyage and transshipment time in a port for a particular container type, taking into consideration transshipment capacity of container crane, continuous operation and number of cranes simultaneously unloading a ship.

The problem was solved using Martos method, i.e., modified simplex method. According to the optimal program, following results are obtained: 186.5 containers of the first container type and 31.75 containers of the second container type with 4425 USD profit per one day.

As it can be seen, decimal results were obtained and at the same time structural variables are number of containers which has to be integer. In this case the branch-and-bound method is used for getting integer results, due to practical reasons. For the authors, very helpful was branch-and-bound method incorporated into the program package WinGULF 3.1, see [1], for linear and linear-fractional problems.

With this program, the result is as follows: x_1=186 containers, x_2=32 containers, and the optimal objective value R=4413.66 USD.

Such a combination of cargo offers full exploitation of transport capacity and with 5 tons of unused payload of container ship.

Maximum profitability of service line Rijeka-Gioia Tauro for transport of obtained combination of two types of full containers, expressed as profit per unit of time, i.e., profit per one day, amounts to 4413.66 USD.

Introducing feeder service was an exceptionally important decision for the Republic of Croatia because, at the time, the foreland of Croatian ports was running a risk of disappearing, it was nevertheless saved. Also, transport policy decision-makers became aware of actual position of Croatian container ports within world wide system of containerization and in respect to economic needs of Croatia and hinterland.

However, organization of the feeder service requires considerable funds and therefore it is not realistic to expect owners to achieve large profits with this quantity and permanency of container transport. Considering the fact that this single one ship achieved over 50% of container traffic in Port of Rijeka in 1999, the role of feeder service in operation of Port of Rijeka is unquestionable.

Since the function of a feeder ship is to deliver all containers from home port to all other ports in respective area, i.e., load all available cargo regardless of profit yielded, actual situation is generally worst than obtained optimal program. The optimal program shows the potential maximum profitability if container types were selected at will. Therefore, state subvention granted to such a service, which is case in the Republic of Croatia, is quite justified. Besides, such a financing maintains links between Croatian container ports and Mediterranean transshipment port centers. Permanent presence of the ships at terminals enables port service users to plan their operations better and establish permanent connections with container terminal.

Considering the above-mentioned features of feeder service, the model presented in this paper could be base to the shipowner "Lošinjska plovidba" at defining amount of subvention, which is annually granted from the Budget of the Republic of Croatia. Namely, the difference between the maximum profitability obtained in the model and effectuated or expected amount of profitability from container transport by ship is so-called "lost profit", that should go to shipowner.

Therefore, obtained optimal solution represents the quantitative basis for analysis of results achieved by a container ship in the previous period and the estimate of expected results in the future.

4. Conclusion

In real life, it is often necessary to calculate the optimal productivity, economic justifiability and profitability as indices of operation efficiency.

Profitability is an especially important index of operation efficiency, representing the ratio between profit yielded and resources invested or consumed, i. e., time spent in realization of the business result.

If these indices are chosen as criteria, then the respective objective function in mathematical model is the fractional function and the problem is solved using linear fractional programming.

In this paper the mathematical model for determining the optimal structure of container ship when the optimality criterion is maximum profitability is presented.

The model was tested on real-life example of feeder ship "Lipa" owned by "Lošinjska plovidba", Rijeka, operating on route Rijeka-Gioia Tauro.

Presented model can be used in operative planning when making respective business decisions referring to the structure of cargo and by other transport means, aiming to maximum profitability.

5. Acknowledgements

The authors would like to acknowledge Erik Bajalinov for his support related to program package WinGULF 3.1, developed by himself especially for linear and linear-fractional problems.

6. References

[1] Bajalinov E. http://www.math.klte.hu/~bajalinov

[2] Dragičević A. Ekonomski leksikon. Zagreb: Informator; 1991.

[3] Kos S, Zenzerović Z. Modelling the Transport Process in Marine Container Technology. Promet 2003; 15(1): 13-17

[4] Martić Lj. Nelinearno programiranje. Zagreb: Informator; 1973.

[5] Mrnjavac E, urednik. Optimizacija sustava hrvatskih kontejnerskih luka. Rijeka: Visoka pomorska škola u Rijeci i Fakultet za turistički i hotelski menadžment, Opatija; 2001.

[6] Petrić J. Nelinearno programiranje. Beograd: IŠRO "Privredno finansijski vodič"; 1979.

[7] Petrić J, Zlobec S. Nelinearno programiranje. Beograd: Naučna knjiga; 1983.

[8] Ravlić P, urednik. Ekonomika poduzeća. Zagreb: Ekonomski fakultet Zagreb; 1993.

[9] Zenzerović Z, Bešlić S. Contribution to the Optimisation of the Cargo Transportation Problem. Promet 2003; 15(2): 65-72