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TIME OPTIMAL ROBOT CONTROL WITH OBSTACLE AVOIDANCE CONSTRAINTS

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Abstract: *This paper presents a new numerical algorithm for time optimal control of nonlinear multivariable systems with control and state vectors constraints. The algorithm is based on the backpropagation-through-time algorithm (BPTT), which is used as a learning algorithm for recurrent neural networks. The proposed algorithm provides better convergence properties then numerical algorithms based on conversion of optimal control problem into a nonlinear programming one. The algorithm is applied on the problem of time optimal robot control with obstacle avoidance constraints.*
Key words: *time optimal control, nonlinear systems, robot control, gradient algorithm.*

1. INTRODUCTION

There are many cases where the time optimal control has been applied in industry, for example in the control of industrial robots, where increasing of the speed of motion is of primary importance, and minimum-time control is an attractive control strategy for this purpose.

As it is already known from the classical optimal control theory (Bryson & Ho, 1969), the solution of the optimal control problem requires the solution of the first-order stationary conditions (two-point boundary-value problem), which can be solved analytically only for very simple problems. A review of the different approaches to the numerical solution of optimal control problems is given in survey paper (Sargent, 2000). Sargent recognize essentially three approaches to solve numerically optimal control problems: a) numerical solution of the two-point boundary value problem given by the necessary conditions, b) complete discretization of the problem, converting it into a nonlinear programming one, and c) finite parameterization of the control trajectory, again converting the problem into nonlinear programming.

The problem of state vectors constraints considerably complicates the solution of the problem both from the theoretical and numerical aspects. The penalty functions for state vectors constraints can be very complicated and impractical, particularly in robot control (avoidance of obstacles, cooperative robots work, etc).

In this paper a new gradient-based numerical algorithm for time optimal control of nonlinear multivariable systems with control and state vectors constraints is proposed. The basic characteristic of this algorithm is derivation without using the calculus of variations and Lagrange multiplier techniques. This approach provides an obvious geometric interpretation of convergence properties of optimal solution. The approximation of the penalty function gradient means a certain deviation from the exact direction of the overall cost function gradient. However, the approximation of the gradient does not mean the approximation of the optimal solution but only slower convergence toward the optimal solution. This fact provides much easier deal with complicate state vectors constraints what is demonstrated in (Kasac & Novakovic, 2001a) on the example of cooperative work of two robots.

Further, mentioned approach provides numerical solution of a wide class of non-standard optimal control problems like minimum time control where initial and final conditions are parameterized by a coordinate transformation (Kasac & Novakovic, 2001b).

2. PROBLEM FORMULATION

A discrete nonlinear time optimal control problem is considered. The problem is to find control vector $\mathbf{u}(i)$ and sampling interval $t = t_f / N$, where t_f is terminal time, that minimizes the cost function

$$J_0 = \min_{t,\mathbf{u}(i)} t \sum_{i=0}^{N-1} F\left(\mathbf{x}(i), \mathbf{u}(i)\right), \tag{1}$$

subject to the constraints defined by the plant equations

$$\mathbf{x}(i+1) = \mathbf{f}\left(\mathbf{x}(i), \mathbf{u}(i)\right), \tag{2}$$

then the initial and final conditions of the state vector

$$\mathbf{x}(0) = \mathbf{x}_0, \quad \mathbf{x}(N) = \mathbf{x}_f, \tag{3}$$

and subject to the control and state vector inequality and equality constraints

$$\mathbf{g}\left(\mathbf{x}(i), \mathbf{u}(i)\right) \geq 0, \quad \mathbf{h}\left(\mathbf{x}(i), \mathbf{u}(i)\right) = 0, \tag{4}$$

for $i=0, 1,\ldots, N\text{-}1$, where N is number of sampling intervals, $\mathbf{x}(i)$ is n-dimensional state vector, $\mathbf{u}(i)$ is m-dimensional control vector, $\mathbf{g}$ is p-dimensional vector function of inequality constraints, and $\mathbf{h}$ is q-dimensional vector function of equality constraints.

3. NUMERICAL SOLUTION

A standard method for reducing the optimal control problem to a nonlinear programming one is adding the penalty functions for state and control vector constraints and plant equation constraints to the cost function, and optimizing the total cost function according to the control and state vectors. In such a way formulated problem has very slow convergence due to additional equality constraints for plant equation.

A new gradient-based algorithm is proposed in this paper, which avoids inclusion of plant equation constraints into cost function and so provides better convergence properties.

First step in reduction of problem (1)-(4) is elimination of constraints (4) and final boundary condition in (3) by introducing theirs penalty functions. On this way, problem is reduced on extended cost function (1) and only one type of constraints – plant equation (2) with initial condition (3).

The gradient descent algorithm is used for minimizing the cost function

$$\mathbf{u}^{(l+1)}(i) = \mathbf{u}^{(l)}(i) - h^{(l)}(i) \frac{\partial J}{\partial \mathbf{u}^{(l)}(i)}, \tag{5}$$

for $i=0, 1,\ldots, N\text{-}1$, where $h^{(l)}(i)$ is the convergence coefficient, and index l represents the l-th iteration of the gradient algorithm. The main difficulty in exact gradient calculation of cost function (1) is implicit dependence of state vector on control vector in previous time intervals via plant equations (2).

A solution of this problem lies in the chain rule for ordered derivatives (which follows from (2)),

$$\frac{\partial x_r(i)}{\partial u_k(j)} = \sum_{l=1}^{n} \frac{\partial f_r(i-1)}{\partial x_l(i-1)} \frac{\partial x_l(i-1)}{\partial u_k(j)}, \tag{6}$$

($r=1, 2, \ldots, n$, $k=1, 2, \ldots, m$, $j=0, 1, \ldots, N-1$, $i=j+2, \ldots, N-1$), resulting in a backward in time iterative algorithm, similar like backpropagation through time (BPTT) algorithm (Werbos, 1990), which is mostly used as a learning algorithm for recurrent neural networks.

The above-mentioned algorithm has been derived for the fixed terminal time t_f. A heuristic approach is applied for time optimal control, which uses the properties of penalty functions,

$$t^{(l+1)} = t^{(l)} - \Delta t H^{-}\left(J_C(t^{(l)}) - J_0\right), \qquad (7)$$

where J_C is sum of penalty functions, J_0 is minimal value of J_C and a measure of accuracy of the solution, Dt is a decreasing constant factor of $t^{(l)}$, and H^- is Heaviside step function. This structure of the algorithm guarantees numerical stability and convergence toward $t_{\min}$, because it does not change the value of $t^{(l)}$, until the value of function J_C falls below the given, sufficiently low value of J_0.

4. TIME OPTIMAL CONTROL OF 2-DOF ROBOT WITH CONSTRAINTS

The mentioned algorithm is applied to minimum-time control of a robot with two degrees of freedom (translation and rotation in horizontal plane). The problem is the transformation of the initial robot state into the final state for minimum time, with the control constraints, and including the condition of avoiding the obstacle in the form of a circle.

Minimum time, t_{min}=1.71 s, and optimal control and state vectors (Fig. 1 and Fig. 2) are obtained using the mentioned algorithm.

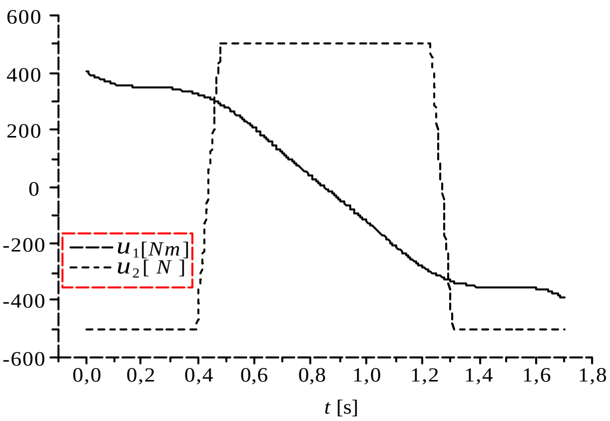

Fig. 1. Time dependence of control variables.

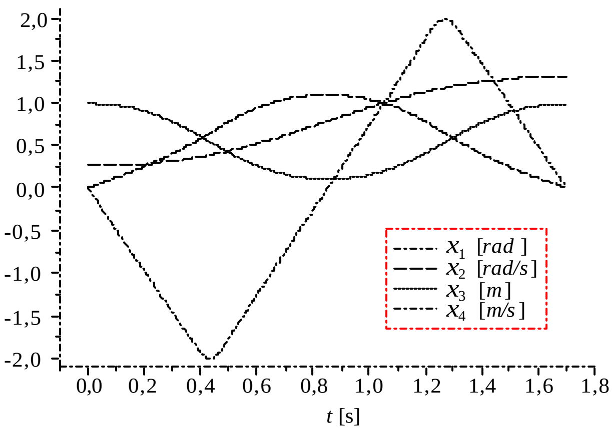

Fig. 2. Time dependence of state variables.

The trajectory of the robot hand in the plane x-y is shown in Fig. 3. The obstacle is a circle with the radius R=0.21 m including the given minimum distance DR=0.01 m. It can be seen that the optimal trajectory (trajectory 1) touches the circle with the radius R, i.e. avoids the obstacle by reaching minimum distance in one point. The figure also shows the trajectory for the same minimum time in the case when conditions for obstacle avoidance do not exist (trajectory 2).

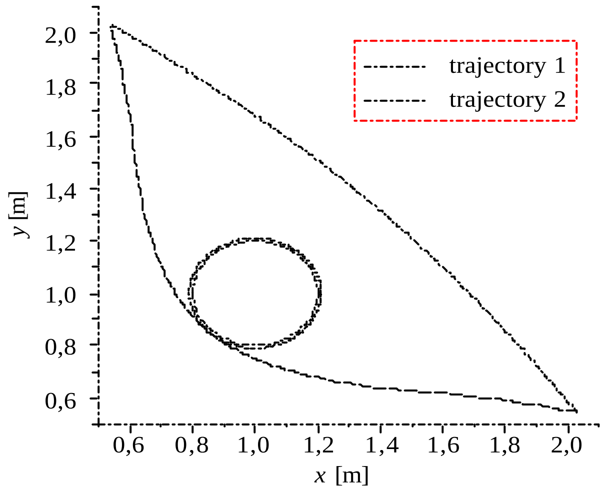

Fig. 3. The trajectory and obstacle in x-y coordinate plane.

5. CONCLUSION

The paper presents a new gradient-based approach to solution of the time optimal control problem, which is especially suitable for treating complicate state vector constraints. This is demonstrated on the problem of the time optimal robot control with state vector constraints in the form of obstacle avoidance, where only geometrical determination of the cross-section of trajectory with obstacle is needful. The future research will be focused on the problem of worst-case analysis for the problem of robot parameter uncertainty.

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