A Moment Method Solution for the Input Impedance of a Magnetic Dipole

L. Vegni°, A. Toscano°, and D. Bonefacic+
°Third University of Rome, Department of Electronic Engineering,
Via della Vasca Navale 84, I-00146 Roma, Italy
Fax (+ 39 6) 55 79 078 Phone (+39 6) 55 17 7003 E-mail: vegni@inf.uniroma3.it
+University of Zagreb, Faculty of Electrical Engineering and Computing,
Unska 3, HR-10000 Zagreb, Croatia
Fax (+ 385 1) 61 11 396 Phone (+ 385 1) 61 29 772 E-mail: bona@zea.cc.fer.hr

Abstract - A very efficient yet rigorous method is presented for the analysis of the input impedance of a magnetic dipole. The method is based on the well known Moment Method (MoM) and can be used for the analysis of single elements and arrays as well. The antenna is fed by an electric point-source or it can be excited by an incident wave. We have found a way to evaluate analytically the infinite integrals involved with the moment method. This results in a great reduction of computer time. The results computed by using the presented theory show very good convergence.

I INTRODUCTION

In this article the spatial fields in integrated magnetic dipole antennas are investigated. We have chosen the method of moments in order to perform the analysis. A current loop is chosen to represent the magnetic dipole. The geometry of the structure under examination is shown in Fig.1.

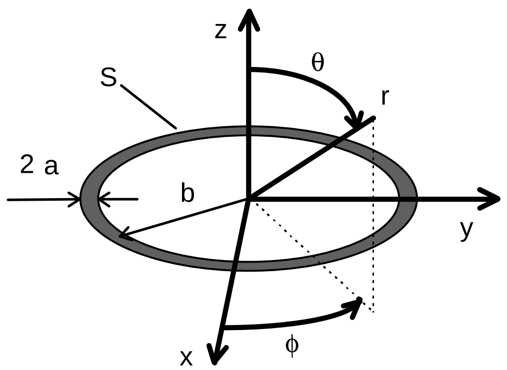

Fig.1. Geometry of the problem.

The loop is located at the z=0 plane. The electric point-source is located at $\phi=\phi_S$ and ε_r is the (complex) permittivity of the substrate material. Applying the method of moments, the integral equation which defines the total electric field outside the loop can be transformed into a set of linear equations by selecting appropriate expansion and test functions for the unknown current on the metallic structure embedded in the substrate [1], [2]. After applying some analytical manipulations we arrive at a matrix equation.

The unknown current on the loop is expanded in entire domain sinusoidal basis functions. We have chosen entire domain basis functions on the loop [3], because only a few of these modes are needed in order to obtain accurate results. If subdomain basis functions were used, the number of basis functions needed would be much larger (>100 typically). Once the mode coefficients are known, the input impedance and the radiation characteristics can be determined.

II THEORETICAL FORMULATION

A great disadvantage of the method of moments is the fact that this method is, from a numerical point of view, very time consuming. The Sommerfeld integral representation of the considered structure is very poorly convergent. The theoretical formulation, that we propose in this contribution, shows how to determine in a closed analytical form the integration needed in the MoM approach. Our approach is appropriate to simplify the MoM type evaluation of the currents induced on arbitrarily shaped magnetic dipole antenna elements and to extract important information to be used in the project, about the influence of the frequency, constitutive parameters, thickness of the slab. Moreover, we remark that, once the modelization of the considered magnetic dipole is obtained, the number of the elements in the array configuration is no longer an obstacle for the analysis.

The tangential component of the total spatial electric field must vanish on the metallic surface S of the magnetic dipole (Fig.1.):

$$\mathbf{n}\times\mathbf{E}^{s} = -\mathbf{n}\times\mathbf{E}^{i} \qquad \text{on} \quad S \quad , \qquad (1)$$

where $\mathbf{E}^{i}$ and $\mathbf{E}^{s}$ are the incident and scattered field and $\mathbf{n}$ is the unity normal vector on S. To analyze the current loop we suppose that the conductor of which the loop is formed is thin, i.e. that the radius of the conductor a is much smaller than the loop radius b (Fig.1.). Subsequently, the current on the loop is supposed to be confined on the conductor axis, and the boundary condition is applied only to the axial component of the electric field on S. Now (1), in a cylindrical coordinate system, can be rewritten as:

$$E_{\phi}^{s} = -E_{\phi}^{i} \qquad . \qquad (2)$$

Equation (2) can be formally rewritten in an operator form:

$$L[I] = E_{\phi}^{i} \qquad , \qquad (3)$$

which can then be solved by using the moment method procedure. For the geometry in Fig. 1. it can be shown that the integro-differential operator L is:

$$L[I]=\int_0^{2\pi}\left\{\omega\mu b\ \cos(\phi-\phi')\,I(\phi')-\frac{1}{j\omega\varepsilon b}\cdot\right.$$
$$\left.\cdot\left[\frac{\partial}{\partial\phi'}I(\phi')\right]\frac{\partial}{\partial\phi}\right\}\frac{e^{-jkR}}{4\pi R}\,d\phi' \quad , \tag{4}$$

where:

$$R=\sqrt{2b^2[1-\cos(\phi-\phi')]+a^2}\ =$$
$$=\ b\sqrt{4\ \sin\left(\frac{\phi-\phi'}{2}\right)+\left(\frac{a}{b}\right)^2} \quad , \tag{5}$$

and k is the wave number in the dielectric medium in which the magnetic dipole is placed.

As basis functions we choose functions of the form:

$$f_n=e^{jn\phi} \quad . \tag{6}$$

Now, the electric current on the loop can be expanded in Fourier series in terms of the basis functions given in (6):

$$I(\phi)=\sum_n I_n\,f_n\ =\ \sum_n I_n\ e^{jn\phi} \quad . \tag{7}$$

I_n are the unknown coefficients to be determined. By substituting (7) into (3), because of the linearity of the operator L, we have:

$$L\left[\sum_n I_n\ f_n\right]=\sum_n I_n\,L[f_n]\ =E^i_\phi \quad . \tag{8}$$

For the weighting functions we choose:

$$w_m=f^*_m=e^{-jm\phi} \quad . \tag{9}$$

By applying the inner product with the weighting functions on (8) we obtain:

$$\sum_n I_n\ \langle w_m,L[f_n]\rangle=\langle w_m,E^i_\phi\rangle \quad . \tag{10}$$

Introducing the notation:

$$Z_{mn}=\langle w_m,L[f_n]\rangle=\int_0^{2\pi}e^{-jm\phi}\ L[e^{jn\phi'}]\,b\,d\phi \tag{11}$$

and

$$V_m=\langle w_m,E^i_\phi\rangle=\int_0^{2\pi}e^{-jm\phi}\ E^i_\phi\ b\ d\phi \quad , \tag{12}$$

(10) can be written in a matrix form:

$$[Z_{mn}]\cdot[I_n]=[V_m] \quad , \tag{13}$$

which is then solved for the unknown coefficients I_n. To evaluate (11) it is necessary to determine how L transforms the basis functions (6). It can be shown that:

$$L\{\ e^{jn\phi'}\}=$$
$$=\frac{j}{2b}\left[\frac{\omega\,\mu\,b}{2}\left(K_{n-1}+K_{n+1}\right)-\frac{n^2}{\omega\varepsilon\,b}\ K_n\right]e^{jn\phi} \quad . \tag{14}$$

In (14), K_n are the coefficients of the Fourier series given by:

$$K_n\ (=K_{-n})=\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{b}{R}e^{-jkR}\right)e^{-jn\phi}\ b\ d\phi \quad , \tag{15}$$

where R is given by (5).

Because of the rotational symmetry of the ring and the bi-orthogonality of f_n and w_m, the matrix $[Z_{mn}]$ is diagonal with elements given by:

$$Z_{mn}\ =0 \qquad ; \qquad m\neq n$$

and

$$Z_{nn}=j\pi\eta b\left[\frac{1}{2}K_{n+1}+\frac{1}{2}K_{n-1}-\left(\frac{n}{kb}\right)^2\ K_n\right] \ . \tag{16}$$

So $[Z_{mn}]$ can be easily inverted to find the unknown current coefficient vector $[I_n]$ from (13). Once the current coefficients are determined, the current distribution on the loop can be found form (7). So, for an Dirac delta source with amplitude V_s placed at ϕ_s=0°, the current distribution is:

$$I(\phi)=\sum_n\frac{V_n}{Z_{nn}}\ e^{jn\phi}\ =\ V_s\sum_{n=-\infty}^{\infty}\frac{1}{Z_{nn}}\ e^{jn\phi}\ =$$
$$=V_s\left(\frac{1}{Z_{00}}\ +2\sum_{n=1}^{\infty}\frac{\cos\,(n\,\phi)}{Z_{nn}}\right) \quad . \tag{17}$$

Also the input admittance of the current loop - magnetic dipole can be calculated. The admittance at $\phi=\phi_s$=0° is:

$$Y_{in}=\frac{I(0)}{V_s}=\sum_{n=-\infty}^{\infty}\frac{1}{Z_{nn}}=\frac{1}{Z_{00}}+2\sum_{n=1}^{\infty}\frac{1}{Z_{nn}} \quad . \tag{18}$$

To calculate the coefficients K_n given by (15) a following recursion relation can be used:

$$K_{n+1}=K_n+\Delta K_n \quad , \tag{19}$$

where ΔK_n is given by:

$$\Delta K_n=\Omega_{2n+1}(2\,k\,b)+j\ J_{2n+1}(2\,k\,b) \quad . \tag{20}$$

Here J_n is the Bessel function of the first kind of order n, and Ω_n is the Lommel-Weber function defined as:

$$\Omega_n(x) = \frac{1}{\pi}\int_0^{\pi} \sin(x\,\sin\theta - n\theta)\,d\theta \quad . \qquad (21)$$

To start the recursion given by (19) the coefficient K_0 can be calculated as:

$$K_0 = \frac{1}{\pi}\ln\left(\frac{8b}{a}\right) - \frac{1}{2}\int_0^{2kb}\Omega_0(x)\,dx - \frac{j}{2}\int_0^{2kb}J_0(x)\,dx \quad . \qquad (22)$$

When a magnetic dipole is used as a receiving antenna, it is interesting to study the current distribution on the loop induced by a plane wave incident on the dipole. The elements of the voltage excitation vector are again given by (12). For a plane wave with a unitary amplitude at the origin (Fig.1.), with electric field vector oriented only in the direction transverse to z axis, and incident from the direction given by (ϕ_i ,θ_i) the excitation vector elements are:

$$V_n = \pi\, b\, j^{n+1} e^{-jn\phi_i}\left[J_{n+1}(k\,b\sin\theta_i) - J_{n-1}(k\,b\,\sin\theta_i)\right] \quad . \qquad (23)$$

In the case that the magnetic field vector is oriented only in the direction transverse to z axis, the excitation vector elements are:

$$V_n = \pi\, b\, j^{n+1} e^{-jn\phi_i}\cos\theta_i \;\cdot\; \left[J_{n+1}(k\,b\sin\theta_i) + J_{n-1}(k\,b\,\sin\theta_i)\right] \quad . \qquad (24)$$

III NUMERICAL RESULTS

The properties of a magnetic dipole have been calculated using the theory described in section II. Also the efficiency and convergence of this approach have been tested.

To calculate an exact input admittance, according to (18), an infinite sum should be made. In practical applications this sum should be truncated at a certain finite number of terms n. The criterion for determining n at which the sum will be truncated is the required accuracy of the computed result.

The input admittance was calculated for different n. The difference in percent between successive results for increasing n at three normalized frequencies are shown in Figs.2 and 3. It can be seen that the convergence is very good and that for $n \geq 9$ an error of less than 2% can be expected. Nevertheless, it should be said that by increasing the ratio a/b (the wire becomes thicker), the convergence is slower and in some cases the imaginary part of the input admittance shows oscillatory behavior. This can be ascribed to the fact that the theory was developed starting from the thin wire approximation.

The calculated real and imaginary part of the input impedance in function of the b/λ ratio is shown in Figs. 4 and 5. Here b is the loop radius and λ is the wavelength.
In this case it was chosen n=9, what is a compromise between result accuracy and time required for computation.

The real and imaginary part of the calculated current distribution on the loop for the case of Dirac delta excitation at $\phi=\phi_s=0°$ are shown in Figs. 6 and 7 respectively. The calculation was performed for four resonant cases, i.e. kb=0.5 ; 1; 1.5 and 2.

The current distribution induced by a plane wave excitation is shown in Figs. 8 and 9.

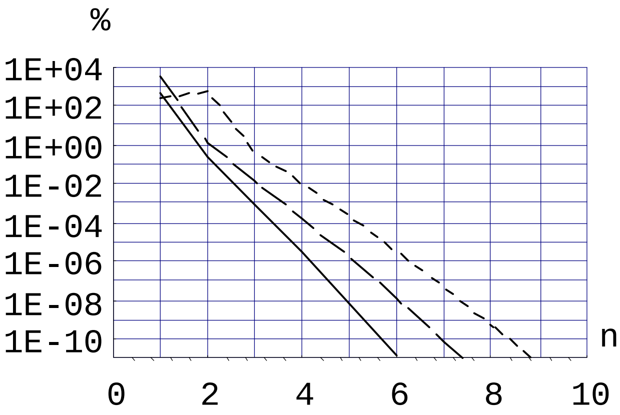

Fig.2. Difference in percent between the successive values of the real part of the input admittance calculated for increasing number n of terms in the series (18) for the cases b/λ=0.1 (solid), 0.2 (dashed), 0.3 (dotted), a/b=0.01 .

Fig.3. Difference in percent between the successive values of the imaginary part of the input admittance calculated for increasing number n of terms in the series (18) for the cases b/λ=0.1 (solid), 0.2 (dashed), 0.3 (dotted), a/b=0.01 .

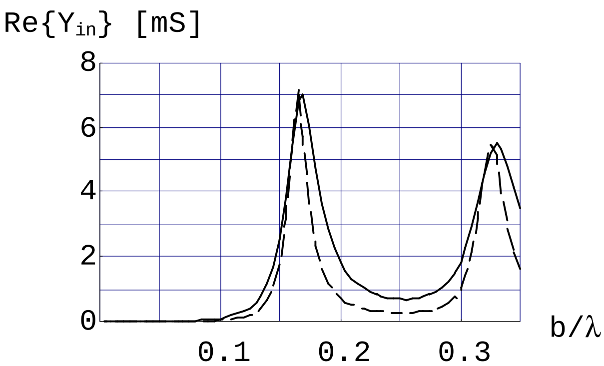

Fig. 4. Real part of the input admittance as a function of b/λ, n=9, a/b=0.01 (solid), a/b=0.001 (dashed) .

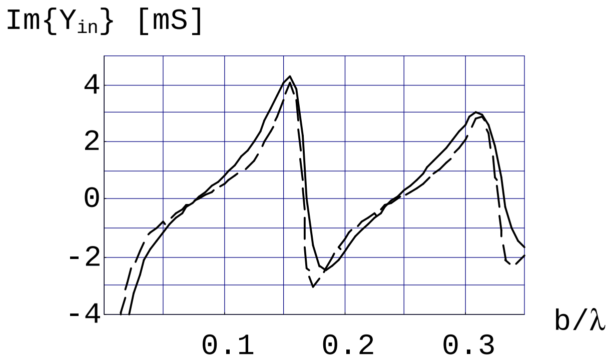

Fig. 5. Imaginary part of the input admittance as a function of b/λ, n=9, a/b=0.01 (solid), a/b=0.001 (dashed).

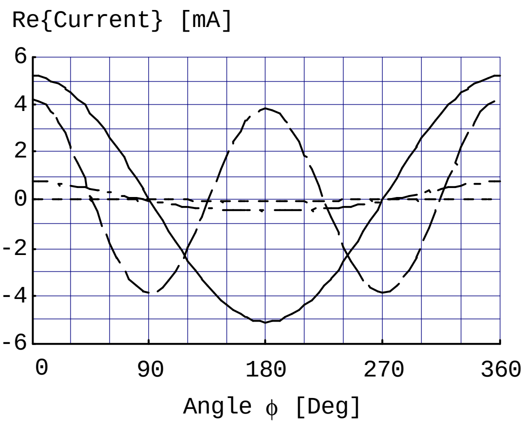

Fig.6. Real part of the current excited on the loop by a Dirac delta source (V_s= 1 V) at $\phi=\phi_s=0°$ for the cases kb=0.5 (dotted), 1.0 (solid), 1.5 (dot-dashed) and 2 (dashed); a/b=0.01, n=9

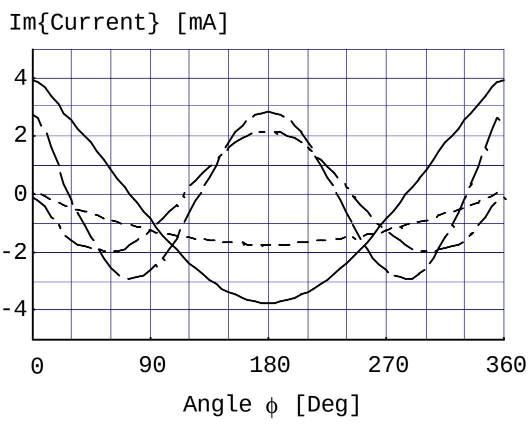

Fig.7. Imaginary part of the current excited on the loop by a Dirac delta source (V_s= 1 V) at $\phi=\phi_s=0°$ for the cases kb=0.5 (dotted), 1.0 (solid), 1.5 (dot-dashed) and 2 (dashed); a/b=0.01, n=9

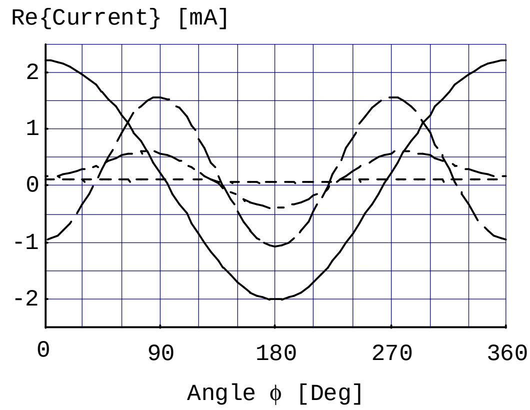

Fig.8. Real part of the current induced on the loop by an incident wave ($\phi_i=0°$; $\theta_i=45°$) with the electric field transverse to z axis for the cases kb=0.5 (dotted), 1.0 (solid), 1.5 (dot-dashed) and 2 (dashed); a/b=0.01, n=9 .

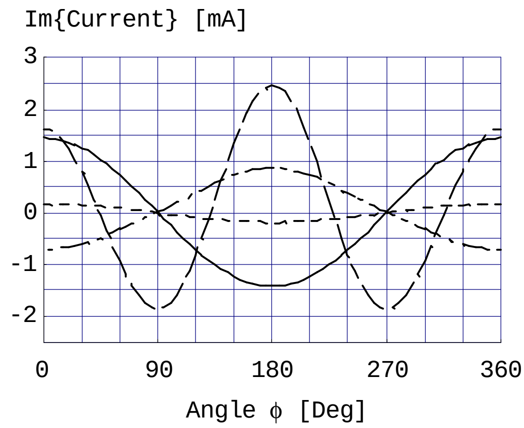

Fig.9. Imaginary part of the current induced on the loop by an incident wave ($\phi_i=0°$; $\theta_i=45°$) with the electric field transverse to z axis for the cases kb=0.5 (dotted), 1.0 (solid), 1.5 (dot-dashed) and 2 (dashed); a/b=0.01, n=9 .

IV CONCLUSIONS

An efficient and accurate method for calculating the properties of a current loop - magnetic dipole has been presented. The calculation is based on the moment method procedure, yet a way to calculate the associate integrals in a closed form has been used. This allowed a very efficient and accurate calculation with very good convergence. So only a few members of the resulting series have to be added to achieve accurate results.

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