

# Digital Processing of Modulating Signal for Direct Digital Synthesizer Indirect Analog Phase Modulation

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**Abstract** — Direct digital synthesizers have the capability of the output signal frequency tuning. It is used for DDS output signal linear frequency modulation. FM-DDS compatibility with digital modulating signal sources, linear modulation characteristics and low element aging sensitivity are the major advantages of DDS application for analog FM. DDS direct phase modulation has the same good performances but its implementation is often not possible. Indirect PM generation is dual to the well known indirect analog FM generation. It requires the correction of the modulating signal digital sequence and sampling rate expansion. The correction rule is derived on the single frequency modulating signal example. The implementation on complex modulating signals is discussed. The modulating signal sampling frequency expander characteristics and the corresponding interpolation digital filter design are presented together with the overall system analysis.

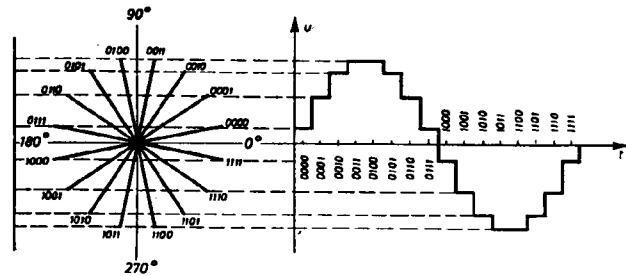


Fig. 1. The 4-bit direct digital synthesis principle

$$f_{out} = F \cdot \frac{f_{clk}}{N} \quad (1)$$

## I INTRODUCTION

Unlike analog LC or crystal oscillators, direct or indirect (PLL) frequency synthesizers all the signal parameters are defined by numbers in direct digital synthesis procedure. Signal synthesis does not begin with an existing signal. Direct digital synthesizer (DDS) takes a small set of parameters (numbers) that describe the desired signal (amplitude, frequency and phase for sine waveform) and generates a number sequence which represents the signal samples. This number sequence usually undergoes a digital-to-analog conversion to produce a final analog signal, [1].

## II DIRECT DIGITAL SYNTHESIZER MODULATION CAPABILITIES

Direct digital synthesizer arrangement allows digital tuning of the synthesizer output sine waveform frequency by applying the so called frequency number  $F$  or the *tuning word* to the tuning terminal. Frequency can be tuned in steps the size of which depend on the clock frequency  $f_{clk}$  and the size of the sine waveform look-up table  $N$  which can also be expressed in the corresponding number of bits in the phase accumulator. Minimal frequency step equals  $f_{clk}/N$  and it is often called the DDS *quantization frequency*. Fig. 1 illustrates the basic DDS principle for the 4-bit system ( $N=16$ ). Output frequency is integer multiple of  $f_{clk}/N$  and it is selected by the frequency number  $F$ :

Factor  $F$  sets the phase increment in reading the sine waveform look-up table. DDS can be directly frequency modulated by adding the so called modulation number  $M(j)$  to the carrier frequency number  $F_c$ . The resulting frequency modulation of the output signal may have digital as well as analog character. This digital method of analog FM generation uses all the advantages of digital circuitry resulting in improved carrier frequency stability, modulation linearity and low elements aging sensitivity. That is why it attracted scientists' attention. Several reports on the applications to the broadcasting FM exciter design were published, [5, 6]. Digital method of analog FM generation compatibility with digital audio sources, CDs for example, is a very strong motive for further research.

Analog mobile FM-systems use analog phase modulation on the transmitter side in order to achieve 6 dB/octave preemphasis characteristics in the output FM-signal. The attempt to modulate the phase of the DDS output signal faces several problems. Analog PM can be simply generated by keeping the frequency register content unchanged ( $F_c$ ) and by adding the phase modulation number  $M(j)$  to the phase register content in each clock interval. However, the input port of the phase register is not accessible in many single-chip direct digital synthesizers. There is an indirect method of analog PM generation by using a FM-modulator. It is dual to the well known indirect FM generation from the analog modulation theory. The respective digital signal processing procedure, that has to be applied to the digital form of the analog modulating signal, is developed in the following sections. For the simplicity reasons the analysis is carried out for the sinusoidal form of the modulating signal  $u_m$ .

### III DDS INDIRECT PHASE MODULATION

In each clock interval The PM-signal instantaneous phase equals:

$$\Phi(j) = 2\pi f_c T_0 \cdot j + \Delta\Phi \cdot u_m(j), \quad (2)$$

where  $T_0 = 1/f_{clk}$  is clock interval,  $f_c$  is carrier frequency and  $\Delta\Phi$  the phase deviation. In direct PM generation the phase register stores the digital equivalent of the instantaneous phase that has the form:

$$\Phi[j] = F_c \cdot j + \delta_\Phi \cdot u_m[j]. \quad (3)$$

Integers  $F_c$  and  $\delta_\Phi$  are normalized carrier frequency and phase deviation (carrier frequency number and phase deviation number) respectively:

$$F_c = \frac{N}{f_{clk}} \cdot f_c, \quad (4)$$

$$\delta_\Phi = \frac{\Delta\Phi}{2\pi} \cdot N. \quad (5)$$

Equation (3) describes the direct method of PM-generation. Modulation number, that has to be added to the phase register content is then  $M[j] = \delta_\Phi \cdot u_m[j]$ . The instantaneous frequency of the PM-signal is derived using (2):

$$f(j) = \frac{\Phi(j) - \Phi(j-1)}{1}. \quad (6)$$

If the modulating signal is a single tone of the frequency  $f_m$ , then it can be expressed as:

$$u_m(j) = \sin\left(\frac{2\pi F_m}{N_m} \cdot j\right), \quad (7)$$

where:

$$F_m = N_m \cdot \frac{f_m}{f_{clk}}, \quad (8)$$

is the modulating frequency number. Symbol  $N_m$  represents the number of the modulating signal samples per period. If  $u_m$  is generated by another DDS then  $N_m$  equals the corresponding DDS look-up table size. The PM-signal instantaneous frequency is derived by putting (7) into (2) and the result into (6):

$$f(j) = \frac{2\pi F_c}{N} + 2\Delta\Phi \sin\left(\frac{\pi F_m}{N_m}\right) \times \cos\left[\frac{2\pi F_m}{N_m}\left(j - \frac{1}{2}\right)\right]. \quad (9)$$

This equation shows that if DDS is to be used for the PM-signal generation then the frequency number, that is stored in the frequency register, should be updated after each clock interval to have the following form:

$$F[j] = F_c + 2 \text{SIN}\left\{\frac{F_m}{2}\right\} \delta_\Phi \text{COS}\left\{F_m \cdot \left(j - \frac{1}{2}\right)\right\}. \quad (10)$$

It shows that digital numbers corresponding to the sinusoidal modulating signal samples levels have to be corrected with the digital factor  $2 \text{SIN}\{F_m/2\}$ . Numerical value of this factor can be taken from the modulating signal sine-wave look-up table at the location the address of which is the number corresponding to  $F_m/2$ . The operator  $\text{SIN}\{x\}$  in (10) represents the look-up table content at the location the address of which is  $x$ .

If  $F_m$  is sufficiently below the Nyquist rate ( $F_m < N_m/2$ ) the approximation:

$$\text{SIN}\left\{\frac{F_m}{2}\right\} \approx \frac{F_m}{2}, \quad (11)$$

can be introduced in (10) resulting in:

$$F[j] = F_c + F_m \cdot \delta_\Phi \text{COS}\left\{F_m \cdot \left(j - \frac{1}{2}\right)\right\}. \quad (12)$$

If  $F_m < N_m/4$  the approximation error introduced in the second part of (12) is less than 10%. One half sample delay required by the last factor in (10) and (12) can be produced by doubling the size of the respective look-up table and by introducing a time delay of one sample.

Equation (12) has the counterpart in the analog modulation theory. It shows that the elements of the modulating signal digital sequence  $u_m[j]$  have to be multiplied by the modulating frequency number  $F_m$ , and the resulting product has to be added to the carrier frequency number  $F_c$ . The resultant digital sequence  $F[j]$  corresponds to the PM-signal instantaneous frequency. It should then be fed to the DDS tuning terminal.

### IV DESIGN CONSIDERATIONS

For complex modulating signals a digital filter is needed to perform the modulating sequence correction according to (12).

It might be expected that the modulating signal section operates at lower clock rate. However, both clocks must be synchronous. Integer clock rates ratio  $M$  is preferred. The modulating signal sampling rate has to be expanded by the factor  $M$ . This is done by keeping the modulating sequence elements constant during  $M$  clock intervals. The corresponding digital interpolation filter impulse response (fig. 3) and the system function are then:

$$h(k) = \begin{cases} 1, & \text{for } 0 \leq k \leq M-1, \\ 0, & \text{elsewhere,} \end{cases} \quad (13)$$

$$H(z) = \sum_{j=0}^{M-1} z^{-j} = \frac{1 - z^{-M}}{1 - z^{-1}}. \quad (14)$$

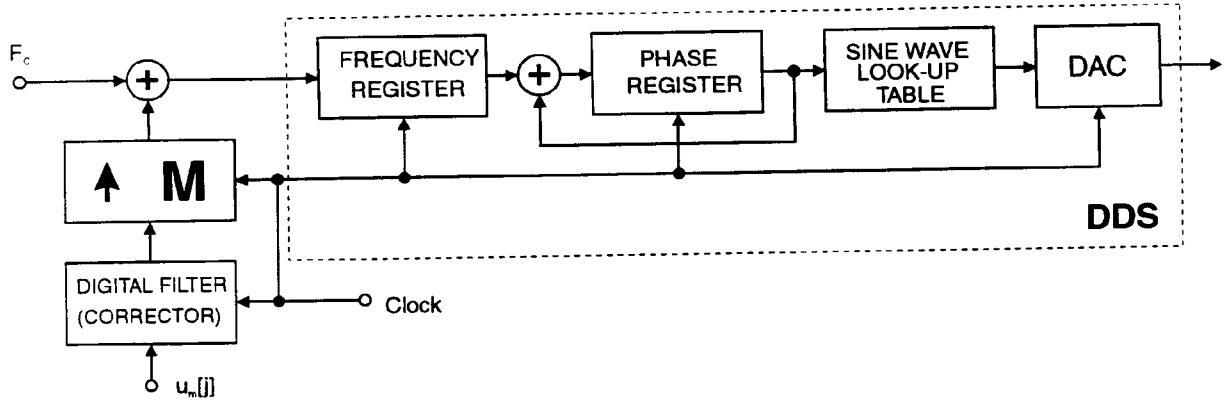


Fig. 2. The block diagram of the phase modulated DDS

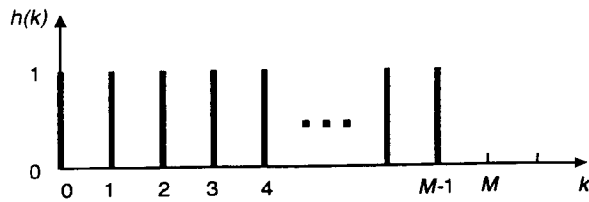


Fig. 3. Impulse response of the interpolating filter

The filter frequency response is:

$$\begin{aligned}
 H(f) &= \frac{1 - e^{-j2\pi M \frac{f}{f_{clk}}}}{1 - e^{-j2\pi \frac{f}{f_{clk}}}} = \\
 &= \frac{\sin \pi M \frac{f}{f_{clk}}}{\sin \pi \frac{f}{f_{clk}}} \cdot e^{-j\pi M \frac{f}{f_{clk}}}
 \end{aligned} \quad (15)$$

The zeros of (15) are at:

$$\begin{aligned}
 f &= k \cdot \frac{f_{clk}}{M} = k \cdot f_{clkM}, \\
 &\text{for } k = 1, 2, \dots,
 \end{aligned} \quad (16)$$

where  $f_{clkM}$  denotes the modulating section clock rate ( $f_{clkM} = f_{clk}/M$ ). If the modulating sequence  $u_m[j]$  corresponds to a sine waveform of frequency less than  $f_{clkM}/2$  the interpolated signal spectrum will be the input sequence spectrum replicated  $M$  times between 0 and  $f_{clk}$  and shaped by the filter response (15). However if the modulation index is kept small relative to the ratio  $M = f_{clk}/f_{clkM}$ , then the interpolating harmonics will be located close to the zeros of the interpolation filter response.

Fig. 2 shows the system block diagram. It has been assumed that the clock rate divider is part of the digital correction filter and the reduced clock rate is distributed to all modulation section units. The clock  $clk$  can also be slaved by the clock  $clkM$ .

## V CONCLUSION

Direct digital synthesis is inherently a continuous-phase technique. Its output waveform calculation always proceeds from the present point, whether or not any parameter changes. Therefore, DDS completely eliminates switching transients, overshoot and undershoot. DDS output signal phase continuity even by abrupt frequency changes makes it highly interesting for applications in frequency-hopping systems. Continuous or discrete phase shifts can be created for analog PM or discrete PSK. Simultaneous amplitude modulation permits very complex data communication constellations. Phase and frequency deviations are independent of carrier frequency and can be held constant over the entire synthesizer tuning range.

The output signal spur levels associated with digital-to-analog amplitude truncation and nonlinearities are, in general, much larger than any phase quantization errors of the DDS device. The largest spurs are due to harmonics and aliases. DAC with unsatisfactory dynamic characteristics is followed by a sample-and-hold (S/H) circuit. In this way the DAC dynamic characteristics is replaced by those of the S/H circuit which is generally better.

Each harmonic, including the fundamental, has an associated alias product. Due to the hold function of the DAC, their amplitudes will be reduced by the  $[\sin(\pi f/f_{clk})]/[\pi f/f_{clk}]$  response. For signal frequencies below  $1/4$  the clock rate all spurious signals are typically 70 dB below the desired signal when 12-bit DACs are used [4].

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