# Green's Functions for Hard and Soft Surfaces Derived by Asymptotic Boundary Conditions

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**Abstract - The Green's function for different realizations of soft and hard surfaces are developed by using the asymptotic boundary condition and the spectral domain approach. We consider the following geometries: the ideal PEC/ PMC strip surface, the strip-loaded grounded slab and the corrugated surface. The strips and corrugations are straight in all cases. The asymptotic boundary conditions are valid in the limiting sense, when the period of the strips is approaching zero. The surface waves appear as poles in the developed Green's functions. The fulfillment of both soft and hard boundary conditions is discussed in the near and far field regions. In some cases the appearance of the surface waves prevents the boundary condition in the near field region from being fulfilled.**

## Introduction

Recently, the concept of soft and hard surfaces has been introduced in electromagnetic theory as surfaces along which the power density respectively is zero or has a maximum [1]. It has been shown that such surfaces can be used to control radiation, scattering and propagation characteristics of the waves, and thereby can be used to design better antennas. Two most common realizations are the corrugated surface and the strip-loaded grounded dielectric slab. The latter has the advantages of low cost and light weight in comparison with the corrugated surface.

The rigorous analysis of open soft and hard surfaces has previously only been performed when they are excited by plane waves [2], [3]. This is a sufficiently good model only for analyzing wave propagation in certain waveguides or for open surfaces when the source is far away from the surface. The obtained results are not satisfactory in applications such as hard struts [4], because then the model predicts zero bandwidth of the hard surface for grazing incidence which is not the case in practice. The bandwidth of hard struts may actually be as large as 30% [4]. Furthermore, the plane wave model cannot predict surface waves which may be excited in the slab or along the grating. The first step in making a more general model was the two-dimensional ($2D$) analysis of the open hard surface in [5].

If the source is not a plane wave the rigorous analysis of the artificially soft and hard surfaces is a complicated procedure. For example, the strip-loaded surfaces can be rigorously analyzed by using the periodical property of the structure, thus expanding the fields in Floquet modes. The transverse corrugations are known to be well modeled by the surface impedance approach, see e.g. [6, pp. 440-442], but this does not work for longitudinal corrugations if there are a spectrum of incident waves. The reason is that the surface impedance for the longitudinal case varies strongly with angle of incidence.

One simplification of the problem can be made by using the asymptotic boundary conditions introduced in [7] - [9]. If the width of the strip is narrow and the periodicity of the strips is small compared to the wavelength (which is common in practice), we can treat the structure in an asymptotic way, as if the width and the periodicity of the strips approach zero. This method do not suffer from the complexity of the Floquet mode technique and from the limitations of the surface impedance method. Furthermore, the asymptotic boundary conditions can be easily applied to surfaces of arbitrary shape (not only canonical problems) and to the strips or corrugations which are generally nonperiodical (in general they can be placed randomly provided the distance between them is small enough).

## Green's functions

The procedure of deriving the Green's function in the spectral domain follows the one explained in [10]. For convenience the soft and hard surface is placed in the plane $z = 0$ and the $x'$ and $y'$ coordinate of the source are $x' = y' = 0$. The electric field of the Hertz dipole above the general soft or hard surface has the form

$$\mathbf{G}(x, y, z|z') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\mathbf{G}}(\beta_x, \beta_y, z|z')\, e^{j\beta_x x}\, e^{j\beta_y y}\, d\beta_x\, d\beta_y\ . \quad (1)$$

Here $\tilde{}$ denotes the two-dimensional Fourier transformation with $\beta_x$ and $\beta_y$ as spectral variables.

An ideally soft or hard surface behaves like a perfect electric conductor (PEC) for one polarization and like a perfect magnetic conductor (PMC) for the other polarization [1]. Let us consider the realization consisting of narrow PEC/PMC strips (Fig. 1.a), which is ideally soft and hard when the excitation is a plane wave. The boundary conditions are

$$E_y^{air} = 0 \qquad\qquad H_y^{air} = 0 \tag{2}$$

If the source is located at the interface, the second of these boundary conditions is equal to $H_y^{air} = -J_x^{source}$. For brevity, we give the result only for the case when the source is at the interface (the other expressions are more complex). However, all the main properties of the Green's functions for other positions of the source are found to be the same as when the source is at the interface. The spectral domain Green's functions for the $x$-oriented Hertz dipole are (the observation point is in the air, i.e $z > 0$)

$$\tilde{G}_{xx}(\beta_x, \beta_y, z|0) = -\omega\mu_0 \frac{k_2 e^{-jk_2 z}}{k_0^2 - \beta_y^2} \tag{3}$$

$$\tilde{G}_{yx}(\beta_x, \beta_y, z|0) = 0 \tag{4}$$

$$\tilde{G}_{zx}(\beta_x, \beta_y, z|0) = -\omega\mu_0 \frac{\beta_x e^{-jk_2 z}}{k_0^2 - \beta_y^2} \tag{5}$$

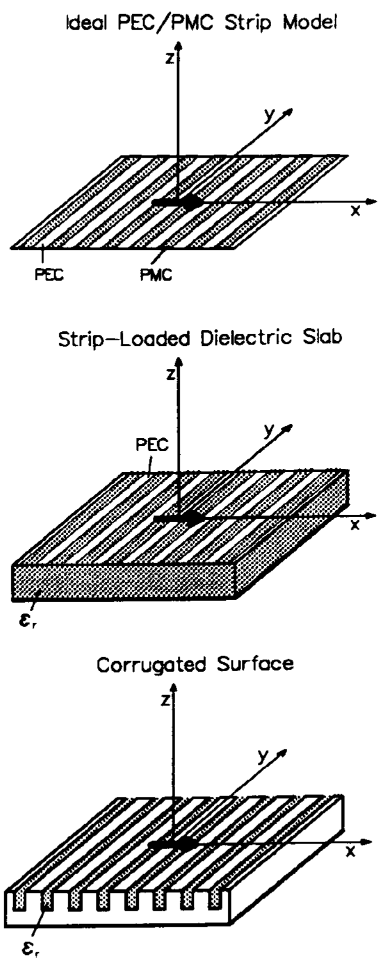


*Figure 1.* Different realizations of surfaces that are soft in the $xz$-plane and hard in the $yz$-plane.

Here $\tilde{}$ denotes the Fourier transformation, $\beta_x$ and $\beta_y$ are the coordinates in the spectral domain, $k_2^2 = k_0^2 - \beta_x^2 - \beta_y^2$, $k_0^2 = \omega^2\mu_0\epsilon_0$ (Im $k_2 \leq 0$).

For the strip-loaded grounded dielectric slab, at the dielectric-air interface, where the strips are located (Fig. 1.b), the asymptotic strip boundary conditions are [8]

$$E_y^{dielectric} = 0 \qquad\qquad E_y^{air} = 0$$

$$E_x^{dielectric} = E_x^{air} \qquad\qquad H_y^{dielectric} = H_y^{air} \tag{6}$$

If the source is located at the interface, the fourth of these boundary conditions is equal to $H_y^{dielectric} - H_y^{air} = J_x^{source}$. Like for the ideally hard and soft surface, we give the result only for the case when the source is at the interface. The spectral domain Green's functions for the $x$-oriented Hertz dipole are (the observation point is in the air)

$$\tilde{G}_{xx}(\beta_x, \beta_y, z|0) = -\omega\mu_0 \frac{k_1 k_2 e^{-jk_2 z}}{k_1(k_0^2 - \beta_y^2) - jk_2(\epsilon_r k_0^2 - \beta_y^2)\cot(dk_1)} \tag{7}$$

$$\tilde{G}_{yx}(\beta_x, \beta_y, z|0) = 0 \tag{8}$$

$$\tilde{G}_{zx}(\beta_x, \beta_y, z|0) = -\omega\mu_0 \frac{\beta_x k_1 e^{-jk_2 z}}{k_1(k_0^2 - \beta_y^2) - jk_2(\epsilon_r k_0^2 - \beta_y^2)\cot(dk_1)} \tag{9}$$

Here $\epsilon_r$ is the relative permittivity of the dielectric slab, $d$ is the thickness of the slab, $k_1^2 = \epsilon_r k_0^2 - \beta_x^2 - \beta_y^2$, (Im $k_1 \leq 0$).

Finally, for the corrugated surface (Fig. 1.c) we assume that the electric field in the corrugations is directed orthogonal to the walls with no variation in that direction (i.e. $\mathbf{E}(x,y,z) = \hat{\mathbf{x}}E_x(y,z)$ for the geometry in Fig. 1.c). In other words, only the fundamental mode of the parallel plate waveguide formed by the walls of the corrugations is excited. The field inside the corrugations, with a discrete variation from corrugation to corrugation, is matched to the outer field which generally varies in the direction perpendicular to the corrugations [7]. In order to obtain enough boundary conditions we have to match both the tangential electric and magnetic fields at the corrugated interface. This can be done by considering a parallel plate waveguide problem with a metal shunt corresponding to the bottom of the corrugations. As a excitation we suppose a plane wave because we derive the Green's functions in spectral domain where the solution is superposition of the plane wave solutions. We use the method described in [7] and [11, pp. 129-130]. For the external field with the $e^{-j\beta_x x}e^{-j\beta_y y}$ variation the vector potential has a form

$$F_z(y,z) = \left(F^+ e^{-j\sqrt{\epsilon_r k_0^2 - \beta_y^2}\,z} + F^- e^{j\sqrt{\epsilon_r k_0^2 - \beta_y^2}\,z}\right) e^{-j\beta_y y}, \tag{10}$$

where $\epsilon_r$ is the relative permittivity of the dielectric material, $k_0^2 = \omega^2\epsilon\mu_0$ and $d$ is the depth of corrugations (Fig. 1.c). After calculating the ratio of the constants

$F^+$ and $F^-$ we get the boundary conditions at the interface

$$E_y^{air} = 0$$

$$E_x^{dielectric} = E_x^{air} \qquad H_y^{dielectric} = H_y^{air}$$

$$\frac{E_x^{dielectric}}{H_y^{dielectric}} = -j\omega\mu_0 \frac{1}{\sqrt{\epsilon_r k_0^2 - \beta_y^2}} \tan(d\sqrt{\epsilon_r k_0^2 - \beta_y^2}) \ . \tag{11}$$

As before we obtain (the $x$-oriented Hertz dipole is located at the interface, the observation point is in the air)

$$\tilde{G}_{xx}(\beta_x, \beta_y, z|0)$$
$$= -\omega\mu_0 \frac{k_2 e^{-jk_2 z}}{(k_0^2 - \beta_y^2) - jk_2(\sqrt{\epsilon_r k_0^2 - \beta_y^2})\cot(d\sqrt{\epsilon_r k_0^2 - \beta_y^2})} \tag{12}$$

$$\tilde{G}_{yx}(\beta_x, \beta_y, z|0) = 0 \tag{13}$$

$$\tilde{G}_{zx}(\beta_x, \beta_y, z|0)$$
$$= -\omega\mu_0 \frac{\beta_x e^{-jk_2 z}}{(k_0^2 - \beta_y^2) - jk_2(\sqrt{\epsilon_r k_0^2 - \beta_y^2})\cot(d\sqrt{\epsilon_r k_0^2 - \beta_y^2})} . \tag{14}$$

## E- and H-plane fields

The near and far fields are plotted in Fig. 2 for different realizations of hard and soft surfaces ($\theta$ and $\phi$ are respectively the polar and azimuth angles in the spherical coordinate system). The free-space wavelength is defined by $\lambda_0^{HARD} = 4d\sqrt{\epsilon_r - 1}$, and at this frequency the hard boundary condition is fulfilled in the $2D$ case, i.e. when the excitation is an infinitely long constant filament current [5]. In the present $3D$ case, the hard boundary condition is fulfilled expectedly for all structures in the far field in $H$ plane, while the near field pattern predicts surface waves.

The presence of the surface waves is important for the fulfillment of the hard boundary condition. This can be seen in Fig 3, where the $E_x$ component of the near field is shown for the corrugated surface excited by a constant filament current which has a length of $2\lambda_0^{HARD}$ along the $x$ axis. The different curves show the field evaluated at different heights $z$ above corrugations. We see that the hard boundary condition is fulfilled in the near field region at those corrugations which are located under the source. Similar results are obtained in the case of the ideal PEC/PMC strip surface. However, for the strip-loaded dielectric slab there are two types of surface waves: a strip grating wave propagating along the strips and an ordinary surface wave [12]. The latter is needed in order to realize the hard boundary condition. The former has a propagation constant which is different from the one in free-space.

The results of the ideal PEC/PMC strip surface are not changing with frequency. The soft boundary condition is fulfilled in $E$ plane in both the near and far field regions (Fig 2). For the other structures the radiation patterns are similar to the ideal case although at the frequency defined by $\lambda_0^{HARD}$ the soft boundary condition is not fulfilled. For the corrugated surface the soft boundary condition occurs when $\lambda_0^{SOFT} = d/4\sqrt{\epsilon_r}$, and changes slowly with frequency. In the case of the strip-loaded dielectric slab we see that the field peaks up close to the boundary in the $E$ plane but looks otherwise similar to the ideal case. The peak corresponds to a surface wave which has a propagation constant different from in free-space. This surface wave is needed in order to create the soft form of the field pattern, but it also destroys the soft boundary condition exactly at the interface. The surface wave can be removed by using lossy dielectric. In practical applications with finite lengths of the surface, the effect of the surface waves can be removed by preventing it from radiating from the end of the slab [13].

## Conclusion

We have derived the Green's functions for different realizations of hard and soft surfaces. The structures are considered in an approximate way by using the asymptotic boundary conditions which turn out to be a good approximation if the periodicity of the structure is small compared to the wavelength. By using the asymptotic boundary condition approach we are able to predict surface waves, which properties can be determined by considering the poles of the spectral Green's function. This approach is in particular advantageous in comparison with the plane wave model because the latter cannot describe the properties of surface waves. There are three different types of surface waves: a strip grating wave propagating along the strips of the strip-loaded dielectric slab, an ordinary dielectric slab surface waves and surface waves occurring due to presence of the corrugated surface and the ideal PEC/PMC surface. Two of them are desired in order to obtain the hard boundary condition. However, the strip grating wave of the strip-loaded dielectric slab prevents the hard boundary condition in the near field region from being fulfilled, and thus is undesired.

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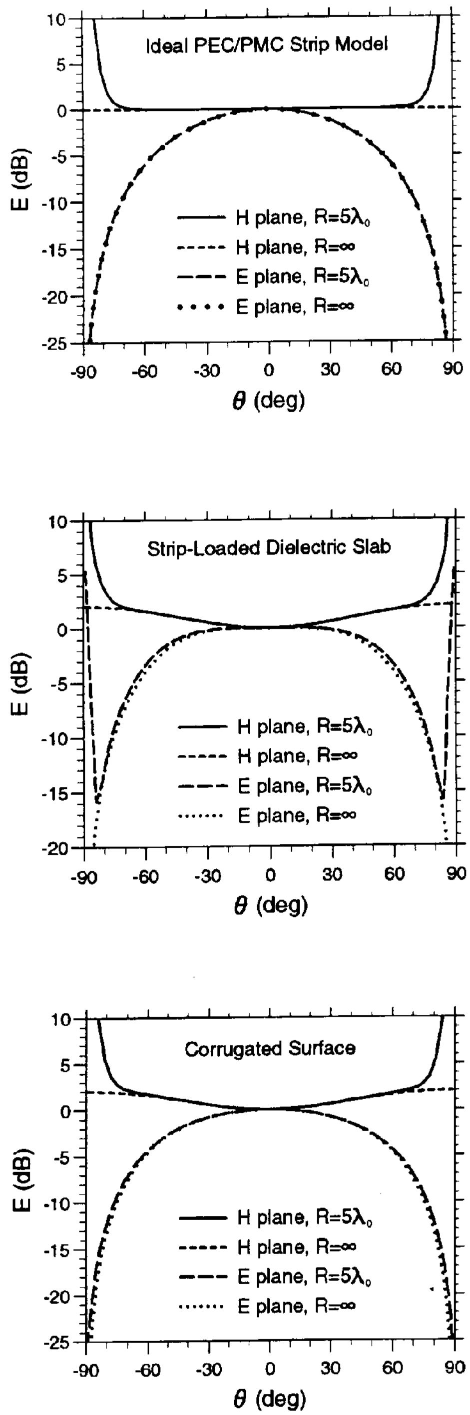


*Figure 2.* Comparison of the near and far electric field in $H$ and $E$ plane of the geometries in Figure 1. $E_\phi$ component in $H$ plane and $E_\theta$ component in $E$ plane are shown at a radial distance of $R = 5\lambda_0^{HARD}$ (i.e. near field) and in the far field. For both the strip-loaded slab and the corrugated surface the results are at the frequency defined by $\lambda_0^{HARD} = 4d\sqrt{\epsilon_r - 1}$, $\epsilon_r = 2.52$.


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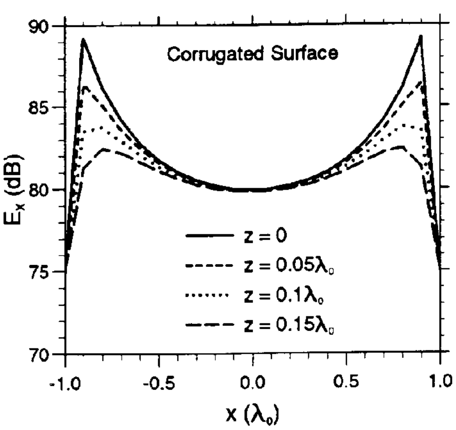


*Figure 3.* The $E_x$ component of the near electric field in the plane defined by $y = 5\lambda_0^{HARD}$. The corrugated surface is excited by a constant filament current which has a length of $2\lambda_0^{HARD}$ along the $x$ axis. The results are at the frequency defined by $\lambda_0^{HARD} = 4d\sqrt{\epsilon_r - 1}$, $\epsilon_r = 2.52$.