

# Numerical Modelling of Gear Tooth Root Fatigue Behaviour

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**ABSTRACT.** *A computational model for determination of service life of gears in regard to bending fatigue in a gear tooth root is presented. The Coffin-Manson relationship is used to determine the number of stress cycles  $N_i$  required for the fatigue crack initiation, where it is assumed that the initial crack is located at the point of the largest stresses in a gear tooth root. The simply Paris equation is then used for the further simulation of the fatigue crack growth, where required material parameters have been determined previously by the appropriate test specimens. The functional relationship between the stress intensity factor and crack length  $K=f(a)$ , which is needed for determination of the required number of loading cycles  $N_p$  for a crack propagation from the initial to the critical length, is obtained numerically in the framework of the Finite Element Method. The total number of stress cycles  $N$  for the final failure to occur is then a sum  $N = N_i + N_p$ .*

## INTRODUCTION

Two kinds of teeth damage can occur on gears under repeated loading due to fatigue; namely the pitting of gear teeth flanks and tooth breakage in the tooth root [1]. In this paper only the tooth breakage is addressed and the developed computational model is used for calculation of tooth bending strength, *i.e.* the service life of gear tooth root.

Several classical standardised procedures (DIN, AGMA, ISO, *etc.*) can be used for the approximate determination of load capacity of gear tooth root. They are commonly based on the comparison of the maximum tooth-root stress with the permissible bending stress [1]. Their determination depends on a number of different coefficients that allow for proper consideration of real working conditions (additional internal and external dynamic forces, contact area of engaging gears, gear's material, surface roughness, *etc.*). The classical procedures are exclusively based on the experimental testing of the reference gears and they consider only the final stage of the fatigue process in the gear tooth root, *i.e.* the occurrence of final failure.

However, the complete process of fatigue failure of mechanical elements may be divided into the following stages [2, 3, 4, 5]: (1) microcrack nucleation; (2) short crack growth; (3) long crack growth; and (4) occurrence of final failure. In engineering applications the first two stages are usually termed as “crack initiation period”, while long crack growth is termed as “crack propagation period”. An exact definition of the

transition from initiation to propagation period is usually not possible. However, the crack initiation period generally account for most of the service life, especially in high-cycle fatigue, see Figure 1. The total number of stress cycles  $N$  can then be determined from the number of stress cycles  $N_i$  required for the fatigue crack initiation and the number of stress cycles  $N_p$  required for a crack to propagate from the initial to the critical crack length, when the final failure can be expected to occur:

$$N = N_i + N_p \quad (1)$$

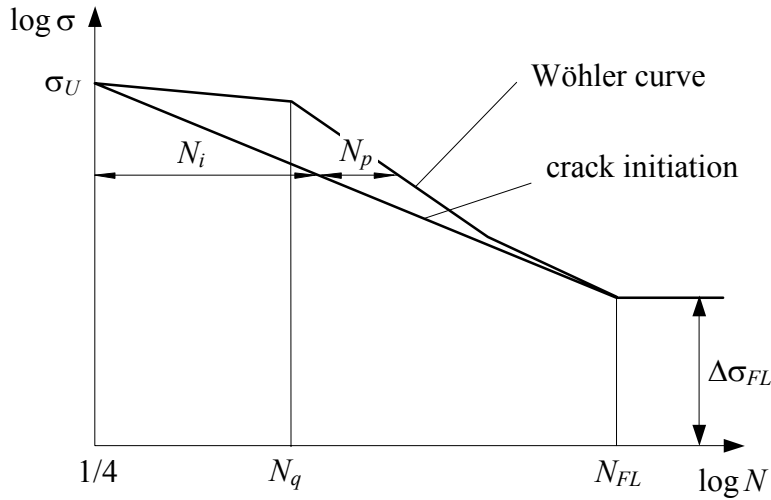


Figure1. Schematic representation of the service life of mechanical elements

## FATIGUE CRACK INITIATION

Presented model for the fatigue crack initiation is based on Coffin-Manson relation between deformations ( $\varepsilon$ ), stresses ( $\sigma$ ) and number of cycles ( $N_i$ ), which can be described as follows [6, 7]:

$$\Delta\varepsilon = \Delta\varepsilon_{el} + \Delta\varepsilon_{pl} = \frac{\sigma'_f}{E} N_i^b + \varepsilon'_f N_i^c \quad (2)$$

where  $\Delta\varepsilon$  is the strain range,  $\Delta\varepsilon_{el}$  and  $\Delta\varepsilon_{pl}$  are the elastic and plastic strain range,  $E$  is the Young's modulus of the material and  $\sigma'_f$ ,  $\varepsilon'_f$ ,  $b$  and  $c$  are the strength coefficient, ductility coefficient, strength exponent and ductility exponent for crack initiation, respectively. The strain range can be obtained numerically (usually by FEM), or by strain gauges measurements in the area of tooth root, where the crack initiation is expected. The material constants  $\sigma'_f$ ,  $\varepsilon'_f$ ,  $b$  and  $c$  are obtained for each material and stress/strain ratio, from strain controlled tests.

In the HCF region commonly applied for gears, where the plastic strain can be neglected, the Coffin-Manson relation reduces only to elastic part and so transforms to an equation of the Basquin type [8, 9]:

$$(\Delta\sigma)^{k_i} \cdot N_i = C_i \quad (3)$$

where  $\Delta\sigma$  is the applied stress range and  $k_i$  and  $C_i$  are the material constants. It is easy to obtain the crack initiation life  $N_i$  using this relation, if we assume that the crack initiation curve passes the same point  $(N_{FL}, \Delta\sigma_{FL})$  as the Wöhler curve, it means at the fatigue limit level the whole fatigue life consists of the crack initiation period:

$$N_i = N_{FL} \cdot \left( \frac{\Delta\sigma_{FL}}{\Delta\sigma} \right)^{k_i} \quad (4)$$

where  $N_{FL}$  is the number of cycles at the knee of the Wöhler curve, see Figure 1. On the basis of the same assumption, the exponent  $k_i$  can be obtained as:

$$k_i = \frac{\log(4N_{FL})}{\log(\sigma_U / \Delta\sigma_{FL})} \quad (5)$$

where  $\sigma_U$  is the ultimate strength, see Figure 1. This relation was found to be in a good correlation with available experimental results [9].

The most important parameter when determining the crack initiation life  $N_i$  according to equation (4) is the fatigue limit  $\Delta\sigma_{FL}$ , which is a typical material parameter and is determined using appropriate test specimen. When determining the fatigue limit for gears, the reference test gears are usually used as the test specimens. According to ISO standard [1], they are spur gears with normal module  $m_n=3$  to 5 mm, tooth width  $B=10$  to 50 mm, surface roughness  $R_z \approx 10 \mu\text{m}$ , etc, which are loaded with repeated pulsating tooth loading. If geometry, surface roughness, gear size and loading conditions of real gears in the praxis deviate from the reference testing, the previously determined fatigue limit  $\Delta\sigma_{FL}$  must be modified through the appropriate correlation factors.

## FATIGUE CRACK PROPAGATION

The application of LEFM to fatigue is based upon the assumption that the fatigue crack growth rate,  $da/dN$ , is a function of the stress intensity range  $\Delta K = K_{\max} - K_{\min}$ , where  $a$  is a crack length and  $N$  is a number of load cycles. In this study the simply Paris equation is used to described of the crack growth rate [10]:

$$\frac{da}{dN} = C[\Delta K(a)]^m \quad (6)$$

where  $C$  and  $m$  are the material parameters. In respect to the crack propagation period  $N_p$  according to Eq.1, and with integration of Eq. 6, one can obtain:

$$\int_0^{N_p} dN = N_p = \frac{1}{C} \cdot \int_{a_0}^{a_c} \frac{da}{[\Delta K(a)]^m} \quad (7)$$

Material parameters  $C$  and  $m$  can be obtained experimentally, usually by means of a three point bending test as to the standard procedure ASTM E 399-80 [11]. For simple cases the dependence between the stress intensity factor and the crack length  $K = f(a)$  can be determined using the methodology given in [10, 11]. For more complicated geometry and loading cases it is necessary to use alternative methods. In this work the Finite Element Method in the framework of the programme package FRANC2D [12] has been used for simulation of the fatigue crack growth. In this work the determination of the stress intensity factor is based on the displacement correlation method using singular quarter-point elements, Figure 2. The stress intensity factor in mixed mode plane strain condition can then be determined as:

$$K_I = \frac{2G}{(3-4\nu)+1} \cdot \sqrt{\frac{\pi}{2L}} \cdot [4v_d - v_e - 4v_b + v_c] \quad (8)$$

$$K_{II} = \frac{2G}{(3-4\nu)+1} \cdot \sqrt{\frac{\pi}{2L}} \cdot [4u_d - u_e - 4u_b + u_c]$$

where  $G$  is the shear modulus of the material,  $\nu$  is the Poisson ratio,  $L$  is the finite element length on crack face,  $u$  and  $v$  are displacements of the crack tip elements. The combined stress intensity factor is then:

$$K = \sqrt{(K_I^2 + K_{II}^2)} \cdot (1-\nu^2) \quad (9)$$

The computational procedure is based on incremental crack extensions, where the size of the crack increment is prescribed in advance. In order to predict the crack extension angle the maximum tensile stress criterion (MTS) is used. In this criterion it is proposed that crack propagates from the crack tip in a radial direction in the plane perpendicular to the direction of greatest tension (maximum tangential tensile stress). The predicted crack propagation angle can be calculated by, see Figure 5:

$$\theta_0 = 2 \tan^{-1} \left[ \frac{1}{4} \cdot \frac{K_I}{K_{II}} \pm \sqrt{\left( \frac{K_I}{K_{II}} \right)^2 + 8} \right] \quad (10)$$

A new local remeshing around the new crack tip is then required. The procedure is repeated until the stress intensity factor reaches the critical value  $K_c$ , when the complete tooth fracture is expected. Following the above procedure, one can numerically determine the functional relationship  $K=f(a)$ .

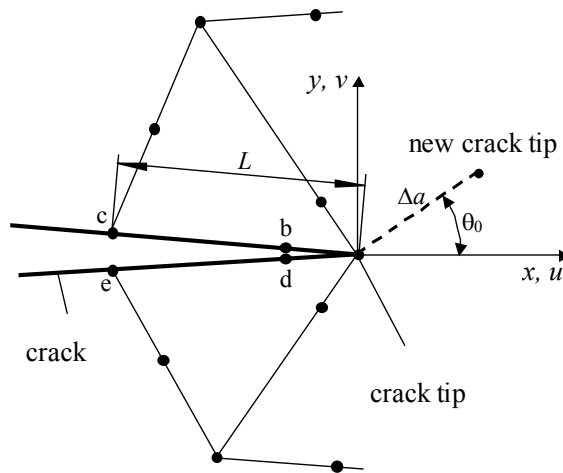


Figure 2. Triangular quarter-point elements around crack tip

## PRACTICAL EXAMPLE

The presented model has been used for the computational determination of the service life of real spur gear with complete data set given in Table 1. The gear is made of high strength alloy steel 42CrMo4 (0.43 %C, 0.22 %Si, 0.59 %Mn, 1.04 %Cr, 0.17 %Mo) with Young's modulus  $E=2.1 \cdot 10^5$  MPa and Poisson's ratio  $\nu=0.3$ . The gear material is thermally treated as follows: flame heated at 810 °C; 2 min, hardened in oil; 3 min and tempered at 180 °C; 2 h.

Table 1. Basic data of a treated spur gear

module	$m_n = 4.5$ mm
number of teeth	$z = 39$
pressure angle on pitch circle	$\alpha_n = 24^\circ$
coefficient of profile displacement	$x = 0.06$
tooth width	$B = 28$ mm
gear material	42CrMo4
surface roughness	$R_z = 10$ $\mu\text{m}$

### ***Fatigue crack initiation***

The procedure as described in Section 2 has been used to determine the number of stress cycles  $N_i$  required for the fatigue crack initiation. The ultimate tensile strength  $\sigma_u=1100$  MPa, fatigue limit  $\Delta\sigma_{FL}=550$  MPa and number of cycles at the knee of the Wöhler curve  $N_{FL} = 3 \cdot 10^6$  have been taken from [1, 13, 14] for the same material as used in this study. The computational analysis have been done for different values of normal pulsating force  $F$ , which is acting at the outer point of single tooth contact, see Figure 3. As a consequence of  $F$ , the maximum principal stress  $\Delta\sigma$  in a gear tooth root has been determined numerically with the Finite Element Method, where the FE-model shown in Figure 3 has been used.

### **4.2 Fatigue crack propagation**

The FEM-programme package FRANC2D as described in section 3 has been used for the numerical simulation of the fatigue crack growth. The initial crack has been located perpendicularly to the surface at the point of the maximum equivalent stress (calculated after Von Mises) stress on the tensile side of gear tooth.

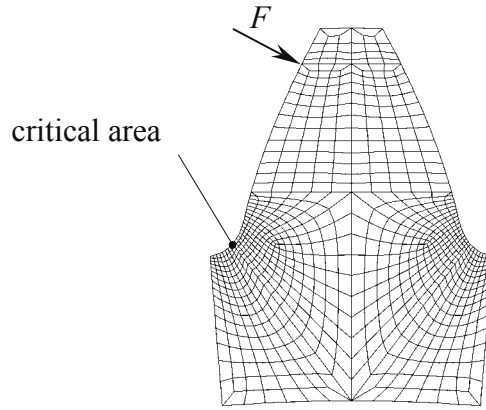


Figure 3. Finite element model

In numerical computations it has been assumed that the initial crack  $a_o$  corresponds to the threshold crack length  $a_{th}$ , below which LEFM is not valid. The threshold crack length may be estimated approximately as [15]

$$a_{th} \approx \frac{1}{\pi} \left( \frac{\Delta K_{th}}{\Delta \sigma_{FL}} \right)^2 \quad (11)$$

determined values of  $K_I$  and  $K_{II}$ . Numerical analysis have shown that the  $K_I$  stress intensity factor is much higher if compared with  $K_{II}$  ( $K_{II}$  was less than 5 % of  $K_I$  for all load cases and crack lengths ). Therefore, the fracture toughness  $K_{Ic}$  can be considered as the critical value of  $K$  and the appropriate crack length can be taken as the critical crack length  $a_c$ . The loading cycles  $N_p$  for the crack propagation to the critical crack

length can then be estimated using equation (7). Figure 4 shows the numerically determined crack propagation path in a gear tooth root.

On the basis of the computational results for crack initiation ( $N_i$ ) and crack propagation ( $N_p$ ) period, the complete service life of gear tooth root can be obtained according to equation (1), see Figure 5. Those computational results for total service life are in a good agreement with the available experimental results, which are taken from [13].

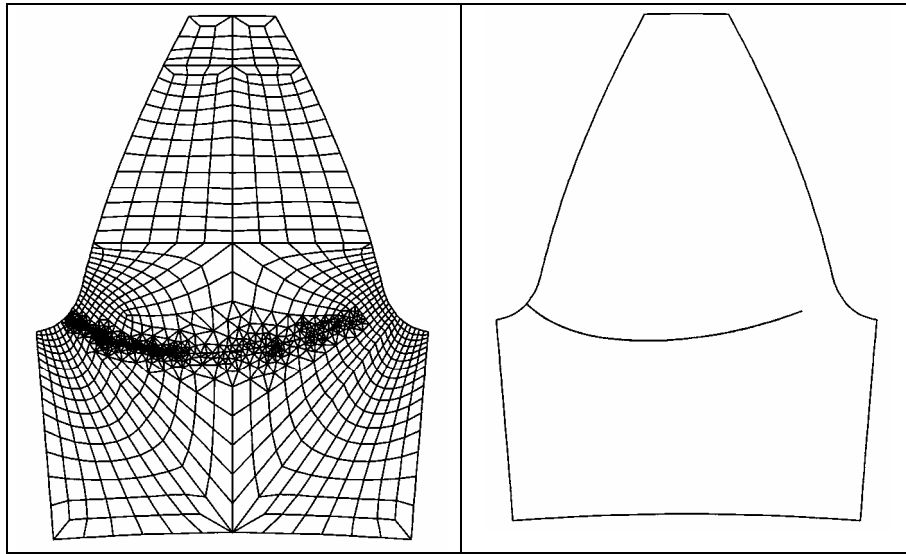


Figure 4: Crack propagation path in a gear tooth root

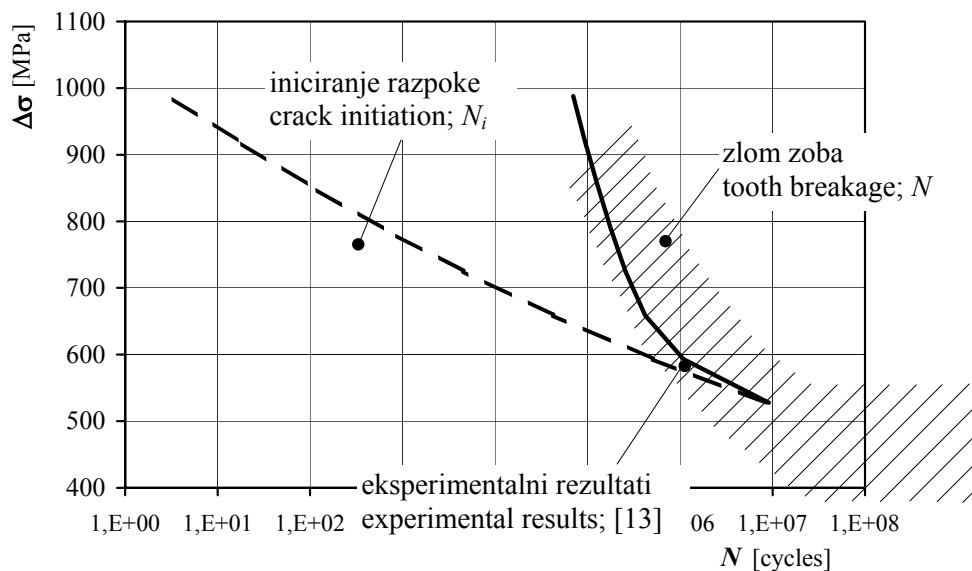


Figure 5. The computed service life of treated gear

## CONCLUSIONS

The paper presents a computational model for determination of service life of gears in regard to bending fatigue in a gear tooth root. The fatigue process leading to tooth breakage in a tooth root is divided into crack initiation ( $N_i$ ) and crack propagation ( $N_p$ ) period, which enables the determination of total service life as  $N = N_i + N_p$ . The simple Basquin equation is used to determine the number of stress cycles  $N_i$ . In the model it is assumed that the crack is initiated at the point of the maximum principal stress in a gear tooth root, which is calculated numerically using FEM. The displacement correlation method is then used for the numerical determination of the functional relationship between the stress intensity factor and crack length  $K=f(a)$ , which is necessary for consequent analysis of fatigue crack growth, i.e. determination of stress cycles  $N_p$ .

The model is used to determine the complete service life of spur gear made from high strength alloy steel 42CrMo4. The final results of the computational analysis are shown in Figure 7, where two curves are presented: the crack initiation curve and the curve of tooth breakage, which at the same time represents the total service life. The results show that at low stress levels near fatigue limit almost all service life is spent in crack initiation. It is very important cognition by determination the service life of real gear drives in the praxis, because majority of them really operate with loading conditions close to the fatigue limit.

The computational results for total service life are in a good agreement with the available experimental results. However, the model can be further improved with additional theoretical and numerical research, although additional experimental results will be required to provide the required material parameters.

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