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Standard uncertainty in each measurement result explicit or implicit

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Abstract

Concepts given in the ISO Guide will have the chance to be used widely in science, engineering, industry and commerce only if their realizations will be simple enough. According to the ISO Guide the complete statement of a measurement result should contain information about standard uncertainty of measurement. Because the uncertainty was traditionally expressed as confidence interval, the ISO Guide also allows expression of uncertainty by the so-called expanded uncertainty. The author proposes that expanded uncertainty should not be used because it complicates the estimation of combined uncertainty and can cause misunderstandings. The user of measurement results should multiply the standard uncertainty with the coverage factor which is appropriate for their particular application. The uncertainty of a measurement result should always be expressed by standard deviation only. As the consequence, the estimation of measurement uncertainty will be simpler and the quality of measurement results directly comparable. In everyday routine work measurement result is expressed by measured value (the best approximation of the measurand) without measurement uncertainty. The author proposes the method of rounding off the measurement result on the basis of its total measurement uncertainty. A properly rounded off measurement result contains the information about measurement uncertainty. The user of a measurement result will be able to estimate total measurement uncertainty on the basis of the number of significant digits of measurement result. © 1997 Elsevier Science Ltd.

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1. Introduction

Since no measurement is perfect it is generally accepted that a measurement result is not a single value but a range of values, which is characterized by the best estimate of the specific quantity subject to measurement (measurand), and quantitative statement of its uncertainty (Fig. 1). Such expression allows the user to asses the significance of any difference between measurement results.

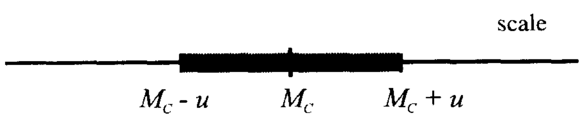

Fig. 1. The result of measurement of quantity M is a range of values expressed by the best estimate of the quantity (M_c) and the uncertainty (u).

Uncertainty is a parameter associated with the result of measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand. Uncertainty should be expressed with one standard deviation (standard uncertainty) [1,2]. The expression of a measurement result without information about uncertainty should be considered incomplete.

2. How to express uncertainty of measurement result

I propose two methods. (i) Uncertainty should be expressed with one standard deviation only, and not with expanded uncertainty. (ii) Expression of measurement results should be adjusted to the application.

3. Why not expanded uncertainty?

Expanded uncertainty defines an interval about the best estimate of measurand that may be expected to encompass a large fraction of possible values that could be attributed to the measurand. This fraction is usually chosen to be approximately 95 or 99%, and is called level of confidence. Expanded uncertainty is the standard deviation multiplied with factor k (coverage factor), which can be 2, 3, 2.58 or something else, because the value of the coverage factor k depends on the number of measurements if the number of measurements is less than 30 (according to Student's distribution or t-distribution; for $n>30$ Student's distribution can be satisfactorily approximated with normal distribution, which is a function of confidence level only and not a function of n). Interval defined by expanded uncertainty was sometimes termed confidence interval. Using the expanded uncertainty one must consider the following complications.

For the results of direct measurements one should state:

- best estimate of measurand;
- expanded uncertainty;
- value of coverage factor k, and
- level of confidence.

For results of indirect measurements the law of propagation of uncertainty generally does not apply to the propagation of confidence intervals [3]. Therefore, one should first calculate the combined standard uncertainty of indirect measurement by means of standard uncertainties of estimates of input quantities, and finally expanded combined uncertainty for selected level of confidence and effective degrees of freedom of combined standard uncertainty. The effective degree of freedom of the estimate of combined standard uncertainty is a measure of its uncertainty, and it is a key factor in determining coverage factor [4]. Effective degrees of freedom of combined uncertainty of indirect measurement result is calculated from the Welch–Satterthwaite formula [2]:

$$\nu = \mathrm{ent}\left[\frac{u_c^4(y)}{\displaystyle\sum_{i=1}^{N} \frac{\left(\dfrac{\partial F}{\partial x_i} u(x_i)\right)^4}{\nu_i}}\right] \tag{1}$$

$u_c(y)$ is combined (standard) uncertainty of the best estimate of measurand ($y=F(x_1,x_2,\dots,x_N)$), $\partial F/\partial x_i$ are sensitivity coefficients, $u(x_i)$ is the (standard) uncertainty associated with the input estimate x_i, and ν_i denotes the degrees of freedom of uncertainty of input estimates x_i. The operator "ent" rounds the real number to the first lower integer [5].

Reporting the result of an indirect measurement with expanded uncertainty may be the same as for results of direct measurements, but it is useful to additionally report effective degrees of freedom [2].

Results of all international comparisons and other work performed under the auspices of the Comite International des Poids et Mesures (CIPM) are given with standard uncertainty ($k=1$) [2]. Measurement results of fundamental constants are also given with standard uncertainty ($k=1$). Results of some calibration laboratories (Western European Calibration Cooperation (WECC) [6], USA National Institute of Standards and Technology (NIST) [4]) are given with expanded uncertainty ($k=2$). Often, no coverage factor is stated (for example, for its calibrators and standards, the manufacturer "Fluke" uses confidence intervals with a 99% level of confidence and $k=2.58$, yet this is not stated in the specifications, but in the literature [7]).

One can conclude that expanded uncertainty:

- does not allow direct comparisons of quality of measurement results;
- can be misleading (if not properly interpreted);
- is complicated for calculation, and
- is complicated for reporting.

It is better and more simple to express the uncertainty with one standard deviation ($k=1$). The user, subsequently, can multiply the standard uncertainty with the coverage factor that is appropriate for the particular conclusion or decision.

4. Three levels of expressing measurement results

When reporting the result of a measurement according to the ISO Guide one should:

- describe clearly the methods used to calculate the measurement result and its uncertainty;
- list all uncertainty components and document fully how they were evaluated;
- present the data analysis in such a way that each of its important steps can be followed and the calculation of the reported result can be repeated, and
- give all corrections and constants used in the analysis and their sources.

Such lengthy reporting of a measurement result is justified in science and metrology, but it is impractical in everyday routine work. Reporting should be more simple and adapted to application needs. To this purpose I propose the following three levels for expressing measurement results.

High level (for example in scientific reports, metrology reports, etc.): the measurement result should contain all the relevant data that enable application, verification and reproduction of a measurement result and its uncertainty [2].

Medium level (for example in expert reports, reports of industrial laboratories, etc.): the measurement result should contain the best estimate of the measurand (M_c), the standard uncertainty (u), measurement unit $[M]$ and the effective degrees of freedom (*d.f.*) if larger than zero:

$$M=\{M_c\pm u\}[M] \quad (d.f.) \tag{2}$$

Low level (for example in everyday routine work): the measurement result is expressed by the best estimate of measurand only, but with such number of digits that the uncertainty of rounding off is equal or less than one fourth of the total measurement uncertainty.

Medium level would usually be used to express measurement results that have been obtained by standardized measurement methods, where the repeatability and reproducibility are known. The advantage of expressing the measurement results on M(edium) level is its simplicity in comparison with H(igh) level, and at the same time it is still informative enough to enable decisions on specified risk level.

On L(ow) level the results are expressed only with rounded off number and measurement unit:

$$M=\{M_c\}[M] \tag{3}$$

4.1. *How to round off a measurement result properly*

If a measurement result consists of too many digits it is not easily surveyed and leaves a wrong impression of high accuracy. If a result is rounded off to too small a number of digits, it will lose part of the information about the measurand.

The problem lies in that there are clear mathematical rules for rounding off a number if the place value is chosen [8], but no method exists for the determination of the place value on which a measurement result should be rounded off.

A simple method for determining the place in the numerical value (L) at which the measurement result should be rounded is given by Eq. (4):

$$L=\mathrm{ent}\left[\lg\left(\frac{\{u_{\mathrm{at}}\}}{1.2}\right)\right] \tag{4}$$

$\{u_{\mathrm{at}}\}$ is the numerical value of absolute total uncertainty of the measurement result. The operator "ent" rounds the real number to the first lower integer [5]. The digit on the place value (10^L) obtained using Eq. (4) should be the last retained digit of the measurement result. This digit should be rounded in accordance with the known mathematical rules. Therefore, the absolute total uncertainty of a measurement result determines the place value on which the result should be rounded off (see Appendix).

If the numerical value of a measurement result is rounded off in the proposed manner, one can estimate the total uncertainty of the measurement result from the number of significant digits (see Appendix).

4.2. *Example*

Transformer no-load losses of 12650 W are measured. The estimated total uncertainty of measurement is 1.7%. How should we express the result of measurement both on L level and M level?

The absolute overall uncertainty of the measurement is 215 W, and consequently, according to Eq. (4), $L=2$. The measurement result expressed on L level is 12.6 kW, and on M level is (12.57 ± 0.22) kW.

5. Conclusion

Concepts given in the ISO Guide [2] will have the chance to be used widely in science, engineering, industry and commerce only if their realizations are simple enough. Proposed methods of using standard uncertainty only and expressing measurement results on Medium or Low level satisfy this condition.

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Appendix

According to the ISO Guide [2], measurement uncertainty should be rounded off to two significant digits. This may cause maximal error of rounded off uncertainty of up to -4.8% (10.5 is rounded off to 10). The next largest symmetrical errors are $+4.5\%$ and -4.5% for numbers 10.500...1 and 11.499... which are rounded off to 11. For all numbers between 10.500...1 and 11.499... errors are uniformly distributed between $+4.5\%$ and -4.5%, and one can estimate maximal uncertainty owing to rounding off to two significant digits as 2.6%, or approximately 3.0%.

The condition can be set that the component of absolute uncertainty of measurement result, owing to rounding off the measurement result (u_{ar}), shall not increase the total uncertainty of measurement result (u_{at}) by more than 3.0%. The uncertainty component related to the rounding off must then be smaller than 1/4 of total uncertainty, because:

$$u_{at}^{*}=\sqrt{u_{at}^{2}+u_{ar}^{2}}=u_{at}\sqrt{1+\left(\frac{1}{4}\right)^{2}}=1.03\times u_{at} \tag{A1}$$

where u_{at}^{*} is absolute total uncertainty of rounded off measurement result. Accordingly, a condition to be satisfied is:

$$\{u_{ar}\}\leq\frac{\{u_{at}\}}{4} \tag{A2}$$

where $\{u_{ar}\}$ is the numerical value of absolute uncertainty owing to rounding off measurement result, and $\{u_{at}\}$ is the numerical value of absolute total uncertainty of the measurement result.

Generally, a decimal number is represented by a set of digits whose position represents their weight in orders of magnitude of powers of ten:

$$D=\sum_{-m}^{n-1} d_i\times 10^{i} \tag{A3}$$

Since any fraction of the last remaining digit of a rounded off number is equally probable in the range from -0.5 to $+0.5$ (uniform or rectangular distribution), the numerical value of absolute uncertainty owing to rounding is

$$\{u_{ar}\}=\frac{0.5}{\sqrt{3}}\times 10^{L}\approx 0.3\times 10^{L} \tag{A4}$$

where 10^{L} is the place value of the last remained digit. Condition Eq. (A2) combined with Eq. (A4) gives

$$0.3 \times 10^L \leq \frac{u_{at}}{4} \tag{A5}$$

and finally Eq. (4) follows, which is a simple method for determination of place (L) in numerical values of measurement results at which measurement results with uncertainty $\{u_{at}\}$ should be rounded.

The estimate of absolute total uncertainty of measurement result expressed on Low level is a range of values

$$1.2 \times 10^L \leq \{u_{at}\} \leq 12 \times 10^L \tag{A6}$$

and the best estimate of absolute total uncertainty of measurement result expressed on Low level is

$$\{u_{at}\} = 10^L \times \sqrt{1.2 \times 12} = 3.8 \times 10^L \tag{A7}$$