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EVENT ORIENTED ANALYSIS OF FAIL-SAFE OBJECTS

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Paper
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Summary

The paper tackles the issue of lifetime service uncertainty modeling of redundant engineering objects by event-oriented systems analysis. The objects involved acquire new functional states after configuration changes due to component failures and load redistribution. The article focuses on "fail-safe" object under random loads in order to reveal that the structural redundancy has to account for a number of events as well as for their probability distribution. The redundancy is assessed by employing the conditional entropy of the transition and residual strength. Moreover, it is demonstrated that the entropy of redundant objects is quantifiable and in case of transitive behavior always increases.

Key words: *Engineering, structures, information, probability, redundancy, safety.*

DOGAĐAJIMA USMJERENA ANALIZA «FAIL-SAFE» OBJEKATA

Sažetak

Članak pristupa problemu cjeloživotnog modeliranja neizvjesnosti djelovanja inženjerskih objekata primjenom sustavne analize usmjerene dagađajima. Razmatraju se objekti koji poprimaju nova funkcionalna stanja nakon promjena konfiguracije zbog oštećenja komponenti i preraspodjele opterećenja. Članak se usredotočuje na "fail-safe" objekte izložene slučajnim opterećenjima da bi se pokazalo da strukturna zalihost izdržljivosti mora uzimati u obzir brojne događaje i razdiobe njihovih vjerojatnosti. Zalihost se ocjejnuje primjenom uvjetne entropije prijelaznosti i preostale izdržljivosti. Što više, pokazuje se da se entropija objekata koji raspolažu strukturnom zalihosti izdržljivosti može izračunati, te da je ta entropija u slučaju prijelaznog ponašanja uvijek rastuća.

Ključne riječi: *Inženjerstvo, strukture, informacije, vjerojatnost, zalihost, sigurnost.*

1. INTRODUCTION

Many efforts devoted to defining probabilistic redundancy measures manifest serious difficulties in system modeling of redundant objects. Widely adopted static probabilistic system theories are applicable to time-invariant objects built of discrete components when a failure of one or more components does not change the course of action for the remaining non-failed components [1], [2], [3], [4], [5], [6] and [7]. The probabilistic system theories become much more complicated for time-variant redundant systems where failures can be caused both by complicated redistribution of demands after each component failure and possible changes in the system configuration [8].

System redundancy of engineering objects is often a desired capability comprehended at least as the availability of warning before a total collapse occurs. Redundancy can be classified as local and overall [9]. A majority of the engineering objects intended to be economical and reliable in service act as "fail-safe" systems also described as damage tolerant systems. The remaining members of "fail-safe" systems after some component failures may be able to sustain applied loads, until rescue and reparation take place. Descriptive redundancy characterized by a number of functional levels and by a number of alternative operational events, as well as mere deterministic measures denoted as structural residual resistance factor [10] and redundant factor which relate the ultimate strength of the intact and damaged structures, are insufficient for practical problems. The most widely used probabilistic measure for redundancy is based on the residual or reserve strength viewed as the conditional probability of system survival given if any one failure occurs [11], [12]and [13].

Entropy concept in probability and information theory is known from earlier [14], [15], [16], [17], [18] and [19] and may be used to express the uncertainties of different functional modes of mechanical objects. However, the conditional entropy of important subsystems of events, such as operational and failure modes, provides better insight in system's overall uncertainties merely of the overall system's entropy [20]. For engineering purposes, it may be more appropriate to use relative uncertainty measures [21]. For unobservable or less important events, the usage of the Renyi's entropy [18] for incomplete systems of events is recognized as appropriate [22]. Therefore, it is argued that, when an engineering object operating in an uncertain environment is subjected to event oriented system analysis (EOSA) [22], the system redundancy relates to the conditional entropy of the probability distribution of operational modes [23]. Average uncertainty measures may be useful in some circumstances [24]. The residual strength alone is not sufficient to assess the system redundancy without considering the distribution of the residual strength among the remaining non-failed components. The analysis and design of series structural systems with some events in common, also involving robustness, is accomplished by employing failure modes analysis accompanied by inclusion-exclusion expansion of a union of events [25]. The event-oriented analysis of geometrically over-determinate structures appears simple when no redistribution of loads is considered [25]. A probabilistic model of multi-level systems of events pertinent to hazardous games with transitive events provides an analogy to redundant systems in engineering [26]. The redundancy defined by conditional entropy of failure path is useful in assessment of lifeline systems efficiency in damaged conditions [27]. The article investigates the theoretical and practical benefits of representing a lifetime service of redundant engineering objects, particularly those denoted as "fail-safe" systems, by systems of events. Conclusion offer recognition of the usefulness of event oriented system analysis in technical object lifetime service performance evaluation and in engineering decision in design and maintenance.

2. FAIL-SAFE REDUNDANT SYSTEMS IN ENGINEERING

By operational modes and effects analysis, all N, or at least all observable and important events E_i in a lifetime service of a system, can be determined. The probabilities $p(E_i) = 1,2,...,N$, can be calculated by quantitative methods, where N is the total number of events constituting a system of events $\mathcal{S}$. In addition to the probabilistic analysis, an event oriented system analysis [22], provides a more comprehensive measures based on entropy for redundancy and robustness of anticipated lifetime events [23].

The lifetime functioning of an object in engineering can be represented by events grouped on functional levels, functional states, functional modes and service profiles. The functional status "*s*" according to the common engineering reasoning may have one of the following meanings: *i*-intact, *c*-collapse, *t*-transitive, emerging, *n*-non-transitive, without emergent potentials, *o* -operational, *f*-failure, and combinations.

3. THE PRIMARY ANALYSIS OF FAIL-SAFE SYSTEMS

The only functional state on the first level of "fail-safe" systems is a compound system of at least three important modes, Fig. 1, denoted as the primary system:

$$^{1}\mathcal{S} = \left({}_{1}^{1}\mathcal{S}\right) = \left({}_{1}^{1}\mathcal{S}^{i} + {}_{1}^{1}\mathcal{S}^{t} + {}_{1}^{1}\mathcal{S}^{c}\right).$$

Practically, the majority of "fail-safe" objects provide only one intact event, i.e. ${}^{1}N^{i} = 1$, and therefore, the primary intact mode is represented by a subsystem of one event:

$$^{1}\mathcal{S}^{i} = \begin{pmatrix} {}^{1}E_{1}^{i} \\ p({}^{1}E_{1}^{i}) \end{pmatrix}.$$

The primary transitive functional mode comprises all ${}^{1}N^{t}$ events denoted as transitive, which in the case of the first component failure, yield to new functional states on another functional level, and can be represented by the following subsystem of events:

$$^{1}\mathcal{S}^{t} = \begin{pmatrix} {}^{1}E_{1}^{t} & {}^{1}E_{2}^{t} & \dots & {}^{1}E_{{}^{1}N^{t}}^{t} \\ p({}^{1}E_{1}^{t}) & p({}^{1}E_{2}^{t}) & \dots & p({}^{1}E_{{}^{1}N^{t}}^{t}) \end{pmatrix}.$$

The transitive modes represent the first component failures on the primary level. At the same time, transitive events can emerge new functional states on the next level. However, it must be proven that the new functional states are operational, even in case of redistributed loads. The collapse functional mode comprises all ${}^{1}N^{c}$ primary collapse events:

$$^{1}\mathcal{S}^{c} = \begin{pmatrix} {}^{1}E_{1}^{c} & {}^{1}E_{2}^{c} & \dots & {}^{1}E_{{}^{1}N^{c}}^{c} \\ p({}^{1}E_{1}^{c}) & p({}^{1}E_{2}^{c}) & \dots & p({}^{1}E_{{}^{1}N^{c}}^{c}) \end{pmatrix}.$$

The characteristic of "fail-safe" systems is given as a single functional state of ${}^{1}N = 1 + {}^{1}N^{t} + {}^{1}N^{c}$ events, by an or primary system of events on the first functional level, as:

$$^{1}\mathcal{S} = \begin{pmatrix} {}^{1}E_{1}^{i} & {}^{1}E_{1}^{t} & {}^{1}E_{2}^{t} & \dots & {}^{1}E_{{}^{1}N^{t}}^{t} & {}^{1}E_{1}^{c} & {}^{1}E_{2}^{c} & \dots & {}^{1}E_{{}^{1}N^{c}}^{c} \\ p({}^{1}E_{1}^{i}) & p({}^{1}E_{1}^{t}) & p({}^{1}E_{2}^{t}) & \dots & p({}^{1}E_{{}^{1}N^{t}}^{t}) & p({}^{1}E_{1}^{c}) & p({}^{1}E_{2}^{c}) & \dots & p({}^{1}E_{{}^{1}N^{c}}^{c}) \end{pmatrix}.$$

The system modes are collected in the system of subsystems of events, which are denoted as the primary service profile of intact, transitive and collapse modes, represented as follows:

$$^{1}\mathcal{S}^{itc\,\prime} = \left({}^{1}\mathcal{S}^{i}, {}^{1}\mathcal{S}^{t}, {}^{1}\mathcal{S}^{c}\right) = \begin{pmatrix} {}^{1}\mathcal{S}^{i} & {}^{1}\mathcal{S}^{t} & {}^{1}\mathcal{S}^{c} \\ p({}^{1}\mathcal{S}^{i}) & p({}^{1}\mathcal{S}^{t}) & p({}^{1}\mathcal{S}^{c}) \end{pmatrix}.$$

3.1. THE PRIMARY PROBABILISTIC ANALYSIS OF FAIL-SAFE SYSTEMS

The probability of the primary system to remaining intact amounts to:

$$p({}^{1}\mathcal{S}^{i}) = p({}^{1}E_{1}^{i}) \tag{1}$$

The probability of a transition from the primary level to the possible functional states on the secondary level represents the probability that, in case of the first failure of any one of the redundant components, new operational states at the second functional level emerge. Transformations of system configurations into secondary functional states are feasible only if there are adequate transitive events on the first functional level.

The probability of emergence of new functional states is equal to the probability of the transition from the primary level to the secondary level:

$$p({}^{1}\mathcal{S}^{t}) = \sum_{i=1}^{{}^{1}N^{t}} p({}^{1}E_{i}^{t}) \tag{2}$$

If there were not a redistribution of loads, the probability of transitive mode represents the probability of collapse of the system. The appropriate collapse probability amounts to:

$$p({}^{1}\mathcal{S}^{c}) = \sum_{i=1}^{{}^{1}N^{c}} p({}^{1}E_{i}^{c}) \tag{3}$$

The non-transition functional mode consists of primary intact and collapse events, presented as ${}^{1}\mathcal{S}^{n} = \left({}^{1}\mathcal{S}^{i} + {}^{1}\mathcal{S}^{c}\right)$.

The probability of the non-transition from one level to another represents the probability that the primary system will be either intact or collapsed:

$$p({}^{1}\mathcal{S}^{n}) = p({}^{1}E_{1}^{i}) + \sum_{i=1}^{{}^{1}N^{c}} p({}^{1}E_{i}^{c}) = p({}^{1}\mathcal{S}) - p({}^{1}\mathcal{S}^{t}) \tag{4}$$

The probability of primary observable or significant events, not necessarily equal to unity in case of unobservable events, is defined differently as shown:

$$\begin{aligned} p({}^{1}\mathcal{S}) &= p({}^{1}\mathcal{S}^{i}) + p({}^{1}\mathcal{S}^{t}) + p({}^{1}\mathcal{S}^{c}) = p({}^{1}\mathcal{S}^{i}) + p({}^{1}\mathcal{S}^{f}) = p({}^{1}\mathcal{S}^{o}) + p({}^{1}\mathcal{S}^{c}) = \\ &= p({}^{1}\mathcal{S}^{n}) + p({}^{1}\mathcal{S}^{t}) \end{aligned} \tag{5}$$

The probability conservation equation (5) expresses the fact that not any of the intact, transitive and collapse probabilities can be changed without affecting some of the other complementary probabilities. In all engineering problems, an outstanding importance pertains to the primary intact (1), transitive (2) and collapse (3) probabilities.

The primary system of events can be considered by different partitioning into functional modes. First, let us consider that the primary operational mode includes intact and transitive modes, Fig. 1. The subsystem of all operational events on the primary level is represented as:

$${}^{1}\mathcal{S}^{o} = \left({}^{1}\mathcal{S}^{i} + {}^{1}\mathcal{S}^{t}\right) = \begin{pmatrix} {}^{1}E_{1}^{i} & {}^{1}E_{1}^{t} & {}^{1}E_{2}^{t} & \dots & {}^{1}E_{{}^{1}N^{t}}^{t} \\ p({}^{1}E_{1}^{i}) & p({}^{1}E_{1}^{t}) & p({}^{1}E_{2}^{t}) & \dots & p({}^{1}E_{{}^{1}N^{t}}^{t}) \end{pmatrix}.$$

The reliability with respect to operations on the primary functional level amounts to:

$$p({}^{1}\mathcal{S}^{o}) = p({}^{1}\mathcal{S}^{i}) + p({}^{1}\mathcal{S}^{t}) = p({}^{1}E_{i}^{i}) + \sum_{i=1}^{{}^{1}N^{t}} p({}^{1}E_{i}^{t}) = 1 - p({}^{1}\mathcal{S}^{c}) \tag{6}$$

The primary functional level with respect to primary operational and collapse modes can be represented by the system of events ${}^{1}\mathcal{S} = \left({}^{1}\mathcal{S}^{o} + {}^{1}\mathcal{S}^{c}\right)$ and the appropriate primary functional profile of operational and collapse modes is ${}^{1}\mathcal{S}^{oc\,\prime} = \left({}^{1}\mathcal{S}^{o}, {}^{1}\mathcal{S}^{c}\right)$, Fig. 1.

Let us consider the primary failure mode of transitive and collapse events, Fig. 1:

$$ {}^{1}\mathcal{S}^{f} = \left({}^{1}\mathcal{S}^{t} + {}^{1}\mathcal{S}^{c}\right) = \begin{pmatrix} {}^{1}E_1^t & {}^{1}E_2^t & \ldots & {}^{1}E_{N_t}^t & {}^{1}E_1^c & {}^{1}E_2^c & \ldots & {}^{1}E_{N_t}^c \\ p({}^{1}E_1^t) & p({}^{1}E_2^t) & \ldots & p({}^{1}E_{N_t}^t) & p({}^{1}E_1^c) & p({}^{1}E_2^c) & \ldots & p({}^{1}E_{N_t}^c) \end{pmatrix}. $$

The appropriate primary failure probability accounts for the transitive events as well as for the collapse events and may be comprehended alternatively as the probability of any first failure in the intact object, as follows:

$$ p({}^{1}\mathcal{S}^{f}) = p({}^{1}\mathcal{S}^{t}) + p({}^{1}\mathcal{S}^{c}) = \sum_{i=1}^{{}^{1}N^{t}} {}^{1}E_i^t + \sum_{i=1}^{{}^{1}N^{c}} {}^{1}E_i^c = 1 - p({}^{1}\mathcal{S}^{i}) \tag{7} $$

The primary functional level with respect to the intact and failure mode can be represented by the system ${}^{1}\mathcal{S} = \left({}^{1}\mathcal{S}^{i} + {}^{1}\mathcal{S}^{f}\right)$, and the service profile is ${}^{1}\mathcal{S}^{if}{}' = \left({}^{1}\mathcal{S}^{i}, {}^{1}\mathcal{S}^{f}\right)$.

The first functional level can also be viewed as union of operational and failure modes ${}^{1}\mathcal{S} = \left({}^{1}\mathcal{S}^{o} \bigcup {}^{1}\mathcal{S}^{f}\right)$, with the transitive mode ${}^{1}\mathcal{S}^{t} = \left({}^{1}\mathcal{S}^{o} \bigcap {}^{1}\mathcal{S}^{f}\right)$ in common, Fig. 1.

Finally, the primary functional level can be viewed as a compound of subsystems non-transitive and transitive events ${}^{1}\mathcal{S} = \left({}^{1}\mathcal{S}^{n} + {}^{1}\mathcal{S}^{t}\right)$ with the functional profile ${}^{1}\mathcal{S}^{nt}{}' = \left({}^{1}\mathcal{S}^{n}, {}^{1}\mathcal{S}^{t}\right)$.

3.2. THE PRIMARY UNCERTAINTY ANALYSIS OF FAIL-SAFE SYSTEMS

The total unconditional uncertainty of the primary level is expressed in general by the Renyi/Shannon's entropy [15], [16], [17], [18] and [19], for either complete or incomplete systems as:

$$ H({}^{1}\mathcal{S}) = \frac{1}{p({}^{1}\mathcal{S})}\left[- p({}^{1}E_1^i)\log p({}^{1}E_1^i) - \sum_{i=1}^{{}^{1}N^{t}} p({}^{1}E_i^t)\log p({}^{1}E_i^t) - \sum_{i=1}^{{}^{1}N^{c}} p({}^{1}E_i^c)\log p({}^{1}E_i^c) \right] \tag{8} $$

The unconditional entropy of the primary service profile of intact, transitive and collapse mode is also expressed by the Renyi/Shannon's entropy is calculated as:

$$ H({}^{1}\mathcal{S}^{itc}{}') = \frac{1}{p({}^{1}\mathcal{S})}\left[- p({}^{1}\mathcal{S}^{i})\log p({}^{1}\mathcal{S}^{i}) - p({}^{1}\mathcal{S}^{t})\log p({}^{1}\mathcal{S}^{t}) - p({}^{1}\mathcal{S}^{c})\log p({}^{1}\mathcal{S}^{c}) \right] \tag{9} $$

The conditional uncertainty [18] and [19] of the primary functional level, also denoted as the primary redundancy with respect to the primary intact mode, vanishes due to only one primary intact event of a "fail-safe" system:

$$ H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{i}) = RED({}^{1}\mathcal{S}^{i}) = -\frac{p({}^{1}E_1^i)}{p({}^{1}\mathcal{S}^{i})}\log\frac{p({}^{1}E_1^i)}{p({}^{1}\mathcal{S}^{i})} = 0 \tag{10} $$

The conditional entropy of the primary functional level [18] and [19], also denoted as the primary redundancy with respect to the transitive mode:

$$ H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{t}) = RED({}^{1}\mathcal{S}^{t}) = -\sum_{i=1}^{{}^{1}N^{t}} \frac{p({}^{1}E_i^t)}{p({}^{1}\mathcal{S}^{t})}\log\frac{p({}^{1}E_i^t)}{p({}^{1}\mathcal{S}^{t})} \tag{11} $$

The conditional entropy of the primary functional level, also denoted as the primary robustness with respect to the collapse mode, is calculated as shown:

$$ H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{c}) = ROB({}^{1}\mathcal{S}^{c}) = -\sum_{i=1}^{{}^{1}N^{c}} \frac{p({}^{1}E_i^c)}{p({}^{1}\mathcal{S}^{c})}\log\frac{p({}^{1}E_i^c)}{p({}^{1}\mathcal{S}^{c})} \tag{12} $$

The unconditional entropy (8) overestimates the uncertainty because it does not account for the uncertainty of the service profile (9).

The general primary uncertainty of redundant objects is more precisely expressed by the conditional entropy of the first functional level with respect to the primary service profile of intact, transitive and collapse modes. This conditional entropy is calculated as a weighted sum of the primary conditional entropy of collapse and transitive modes with weights equal to the appropriate probability of transition and collapse probability:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{itc}{}') = p({}^{1}\mathcal{S}^{t})RED({}^{1}\mathcal{S}^{t}) + p({}^{1}\mathcal{S}^{c})ROB({}^{1}\mathcal{S}^{c}) = p({}^{1}\mathcal{S})[H({}^{1}\mathcal{S}) - H({}^{1}\mathcal{S}^{itc}{}')] \quad (13)$$

The conditional entropy (13) reflects the diminution of overall system unconditional uncertainty (8) for the uncertainty of the service profile (9).

The terms in the sequel are simplified without loss of generality, by assuming complete systems of events, implying that all the events are observable. The system completeness is expressed by the term $p({}^{1}\mathcal{S}) = 1$.

4. SECONDARY ANALYSIS OF FAIL-SAFE SYSTEMS

Secondary functional states of "fail-safe" objects after load redistribution, comprise one intact mode and several collapse modes, Fig. 1, and can be regarded as series systems:

$${}_{j}^{2}\mathcal{S} = \left({}_{j}^{2}\mathcal{S}^{i} + {}_{j}^{2}\mathcal{S}^{c}\right) = \begin{pmatrix} {}_{j}^{2}E_{1}^{i} & {}_{j}^{2}E_{1}^{c} & {}_{j}^{2}E_{2}^{c} & \cdots & {}_{j}^{2}E_{{}_{j}^{2}N^{c}}^{c} \\ p({}_{j}^{2}E_{1}^{i}) & p({}_{j}^{2}E_{1}^{c}) & p({}_{j}^{2}E_{2}^{c}) & \cdots & p({}_{j}^{2}E_{{}_{j}^{2}N^{c}}^{c}) \end{pmatrix}, \; j = 1,2,\ldots,{}^{1}N^{t}.$$

4.1. THE SECONDARY PROBABILISTIC ANALYSIS OF FAIL-SAFE SYSTEMS

Reliabilities and probabilities of the collapse of secondary functional states account for redistribution of loads in a modified system configuration and are calculated as follows:

$$p({}_{j}^{2}\mathcal{S}^{i}) = p({}_{j}^{2}E_{1}^{i}) \quad (14)$$

$$p({}_{j}^{2}\mathcal{S}^{c}) = \sum_{i=1}^{{}_{j}^{2}N^{c}} p({}_{j}^{2}E_{i}^{c}) \quad (15)$$

Secondary functional states are complete systems of events since $p({}_{j}^{2}\mathcal{S}) = p({}_{j}^{2}\mathcal{S}^{i}) + p({}_{j}^{2}\mathcal{S}^{c}) = 1$.

The emergence of functional states is symbolically presented by subsystems of compound secondary and primary transitive events, as follows:

$${}_{j}^{2}\mathcal{S} \cap {}^{1}E_{j}^{t} = \begin{pmatrix} {}_{j}^{2}E_{1}^{i} \cap {}^{1}E_{j}^{t} & {}_{j}^{2}E_{1}^{c} \cap {}^{1}E_{j}^{t} & \cdots & {}_{j}^{2}E_{{}_{j}^{2}N^{c}}^{c} \cap {}^{1}E_{j}^{t} \\ p({}_{j}^{2}E_{1}^{i}/{}^{1}E_{j}^{t})\,p({}^{1}E_{j}^{t}) & p({}_{j}^{2}E_{1}^{c}/{}^{1}E_{j}^{t})\,p({}^{1}E_{j}^{t}) & \cdots & p({}_{j}^{2}E_{{}_{j}^{2}N^{c}}^{c}/{}^{1}E_{j}^{t})\,p({}^{1}E_{j}^{t}) \end{pmatrix}.$$

Since the emergence in no way affects the probabilities of transitive events, this independence can in general be presented as $p({}_{j}^{2}E_{l}^{s}/{}^{1}E_{j}^{t}) = p({}_{j}^{2}E_{l}^{s})$. For new functional modes $p({}_{j}^{2}\mathcal{S}/{}^{1}E_{j}^{t}) = p({}_{j}^{2}\mathcal{S}) = 1$. The compound probabilities of emergence of functional states in case of transition and load redistribution, equals the probabilities of transition:

$$p({}_{j}^{2}\mathcal{S} \cap {}^{1}E_{j}^{t}) = p({}_{j}^{2}\mathcal{S}/{}^{1}E_{j}^{t})p({}^{1}E_{j}^{t}) = p({}_{j}^{2}E_{1}^{i})p({}^{1}E_{j}^{t}) + \sum_{i=1}^{{}_{j}^{2}N^{c}} p({}_{j}^{2}E_{i}^{c})p({}^{1}E_{j}^{t}) = p({}^{1}E_{j}^{t}) \quad (16)$$

The secondary functional level of a "fail-safe" object has ${}^{2}N = 1 + {}^{1}N^{t} + \sum_{j=1}^{{}^{1}N^{t}} {}_{j}^{2}N^{c} + {}^{1}N^{c}$ events, composed of primary non-transitive modes and of new second level modes can be jointly presented as ${}^{2}\mathcal{S} = \left({}^{1}\mathcal{S}^{i} + {}_{1}^{2}\mathcal{S} \cap {}^{1}E_{1}^{t} + {}_{2}^{2}\mathcal{S} \cap {}^{1}E_{2}^{t} + \cdots + {}_{{}^{1}N^{t}}^{2}\mathcal{S} \cap {}^{1}E_{{}^{1}N^{t}}^{t} + {}^{1}\mathcal{S}^{c}\right)$.

The secondary service profile accounting for primary non-transitive modes and emerged modes comprises $2+{}^1N^t$ subsystems ${}^2\mathcal{S}^{itc\prime} = \left({}^1\mathcal{S}^i, {}_1^2\mathcal{S}\cap{}^1E_1^t, {}_2^2\mathcal{S}\cap{}^1E_2^t, \cdots, {}_{{}^1N^t}^{2}\mathcal{S}\cap{}^1E_{{}^1N^t}^t, {}^1\mathcal{S}^c\right)$.

The secondary level is a complete system since $p({}^2\mathcal{S})=1$.

The emergence of the secondary level does not affect the probabilities of transition and therefore is $p({}^2\mathcal{S}/{}_1^1\mathcal{S}^t)=p({}^2\mathcal{S})=1$.

The compound probability of emergence and transition equals that of the transition:

$$p({}^2\mathcal{S}\cap{}_1^1\mathcal{S}^t)=p({}^2\mathcal{S}/{}^1\mathcal{S}^t)\,p({}^1\mathcal{S}^t)=p({}^1\mathcal{S}^t) \tag{17}$$

The appropriate compound probabilities of secondary intact and collapse modes are:

$$p({}_j^2\mathcal{S}^i\cap{}^1E_j^t)=p({}^1E_j^t)\,p({}_j^2E_1^i)=p({}^1E_j^t)\,p({}_j^2\mathcal{S}^i) \tag{18}$$

$$p({}_j^2\mathcal{S}^c\cap{}^1E_j^t)=p({}^1E_j^t)\cdot\sum_{i=1}^{{}_j^2N^c}p({}_j^2E_i^c)=p({}^1E_j^t)\,p({}_j^2\mathcal{S}^c) \tag{19}$$

However, from the engineering point of view it is more important to consider separately the probabilities and uncertainties of all emerged secondary intact modes denoted as

${}^2\mathcal{S}^i=\left({}_1^2\mathcal{S}^i\cap{}^1E_1^t+\cdots+{}_j^2\mathcal{S}^i\cap{}^1E_j^t+\cdots+{}_{{}^1N^t}^{2}\mathcal{S}^i\cap{}^1E_{{}^1N^t}^t\right)=\left({}_1^2E_1^i+\cdots+{}_j^2E_1^i+\cdots+{}_{{}^1N^t}^{2}E_1^i\right)$

and of collapse modes denoted as

${}^2\mathcal{S}^c=\left({}_1^2\mathcal{S}^c\cap{}^1E_1^t+\cdots+{}_j^2\mathcal{S}^c\cap{}^1E_j^t+\cdots+{}_{{}^1N^t}^{2}\mathcal{S}^c\cap{}^1E_{{}^1N^t}^t\right)$.

In addition, the collapse profile may be presented as follows:

${}^2\mathcal{S}^{c\prime}=\left({}_1^2\mathcal{S}^c\cap{}^1E_1^t,\cdots,{}_j^2\mathcal{S}^c\cap{}^1E_j^t,\cdots,{}_{{}^1N^t}^{2}\mathcal{S}^c\cap{}^1E_{{}^1N^t}^t\right)$.

The compound probabilities of only the secondary intact and collapse modes with the transitive mode, denoted as secondary reliability and failure probability, respectively, are:

$$p({}^2\mathcal{S}^i)=\sum_{j=1}^{{}^1N^t}p({}^1E_j^t)\,p({}_j^2\mathcal{S}^i) \tag{20}$$

$$p({}^2\mathcal{S}^c)=\sum_{i=1}^{{}^1N^t}p({}^1E_j^t)\,p({}_j^2\mathcal{S}^c) \tag{21}$$

The secondary level is alternatively partitioned into primary and secondary intact and collapse modes, as ${}^2\mathcal{S}=\left({}^1\mathcal{S}^i+{}^2\mathcal{S}^i+{}^2\mathcal{S}^c+{}^1\mathcal{S}^c\right)$.

The alternative secondary service profile of primary non-transitive events, as well as of all emerged intact and collapse modes, is presented as ${}^2\mathcal{S}^{ic\prime}=\left({}^1\mathcal{S}^i,{}^2\mathcal{S}^i,{}^2\mathcal{S}^c,{}^1\mathcal{S}^c\right)$.

The secondary service profile accounting for the collapse profile can be represented as ${}^2\mathcal{S}^{ic\prime\prime}=\left({}^1\mathcal{S}^i,{}^2\mathcal{S}^i,{}_1^2\mathcal{S}^c\cap{}^1E_1^t,\cdots,{}_j^2\mathcal{S}^c\cap{}^1E_j^t,\cdots,{}_{{}^1N^t}^{2}\mathcal{S}^c\cap{}^1E_{{}^1N^t}^t,{}^1\mathcal{S}^c\right)$. The secondary level can also be viewed as the compound of two modes ${}^2\mathcal{S}=\left({}^2\mathcal{S}^o+{}^2\mathcal{S}^f\right)$, where the operational mode is ${}^2\mathcal{S}^o=\left({}^1\mathcal{S}^i+{}^2\mathcal{S}^i\right)$ and the failure mode is ${}^2\mathcal{S}^f=\left({}^2\mathcal{S}^c+{}^1\mathcal{S}^c\right)$.

The secondary service profile of operational and failure modes is presented as ${}^2\mathcal{S}^{of\prime}=\left({}^2\mathcal{S}^o+{}^2\mathcal{S}^f\right)$.

4.2. THE TRADITIONAL PROBABILISTIC REDUNDANCY DEFINITION

The probability of the primary residual strength equals the probability of the transitive mode and can be expressed on two ways due to the ambiguity of the transitive modes, as:

$$p({}^1\mathcal{S}^t) = p({}^1\mathcal{S}^f) - p({}^1\mathcal{S}^c) = p({}^1\mathcal{S}^o) - p({}^1\mathcal{S}^i) \tag{22}$$

The growth of the probability of residual strength $p({}^1\mathcal{S}^t)$ (22) can be realized according to the probability conservation equation (5), only by diminution of the probability of the intact mode as an unwanted option, and of the collapse mode as a desired option. These two conflicting consequences of the increase of residual strength require engineering judgement about the notion of redundancy index with respect to the importance of intact and collapse modes. The traditional probabilistic redundancy index is defined as the system's primary residual strength conditioned by any first component failure [11]. Such an index can be calculated in terms of the probabilities of primary level modes (2, 3, and 7), as:

$$R_I = p({}^1\mathcal{S}^t / {}^1\mathcal{S}^f) = \frac{p({}^1\mathcal{S}^t)}{p({}^1\mathcal{S}^f)} = 1 - R_R = \frac{R_F}{1 + R_F} \tag{23}$$

The complement R_R of the probabilistic redundancy index in (23) is the system's primary collapse, conditioned by any first component failure [12]. The probabilistic redundancy factor R_F can also be defined [13], as the system's primary residual strength, conditioned by the collapse of the system.

4.3. THE MINIMAL SAFETY REQUIREMENTS ON "FAIL-SAFE" SYSTEMS

A "fail-safe" system service cannot be affirmed without checking the secondary operational modes after component failures. Secondary reliability limits are based on common engineering reasoning about acceptable rates of safety of each alternative operational mode required for a "fail-safe" system in a damaged condition with redistributed loads, as:

$$p({}^2_j\mathcal{S}^i) = p({}^2_jE^i_1) \geq p_{acc}({}^2_j\mathcal{S}^i), \; j = 1,2,..., {}^1N^t \tag{24}$$

It may be more appropriate to express the minimal safety requirements (24) for all operational modes in terms of acceptable safety indices as $p_{acc}({}^2_j\mathcal{S}^i) = \Phi(-{}^2_j\beta_{acc})$.

4.4. THE UNCERTAINTY OF EMERGENCE

The unconditional entropy of independent secondary functional states express the emerged system's overall uncertainty, regardless of the transitivity of redundant objects, as:

$$H({}_j^2\mathcal{S}) = -p({}^2_jE^i_1)\log p({}^2_jE^i_1) - \sum_{i=1}^{{}^2_jN^c} p({}^2_jE^c_i)\log p({}^2_jE^c_i), \; j = 1,2,..., {}^1N^t \tag{25}$$

The conditional entropy of the secondary level with respect to the emerged states equals the unconditional entropy of secondary functional states (25), as:

$$H\left[{}^2\mathcal{S} \;/({}^2_jS \cap {}^1E^t_j)\right] = H({}_j^2\mathcal{S}), \quad j = 1,2,..., {}^1N^t \tag{26}$$

The unconditional entropy of the secondary level is calculated by definition as:

$$H({}^2\mathcal{S}) = -\sum_{all\; E\in {}^2\mathcal{S}} p(E)\log p(E) = H({}^1\mathcal{S}) + H({}^2\mathcal{S}/{}^1\mathcal{S}) \tag{27}$$

The conditional entropy of the secondary level with respect to the primary level expresses the uncertainty due to the emergence of new functional states and equals the difference between the secondary and the primary unconditional entropy:

$$H({}^2\mathcal{S}/{}^1\mathcal{S}) = \sum_{j=1}^{N_t} p({}_j^2\mathcal{S} \cap {}^1E^{ot}_j) H\left[{}^2\mathcal{S} /({}_j^2\mathcal{S} \cap {}^1E^t_j)\right] = \sum_{j=1}^{N_t} p({}^1E^t_j) H({}_j^2\mathcal{S}) = H({}^2\mathcal{S}) - H({}^1\mathcal{S}) \tag{28}$$

The unconditional entropy of the secondary service profile is calculated as follows:

$$H({}^2\mathcal{S}^{itc}{}') = -p({}^1\mathcal{S}^{i})\log p({}^1\mathcal{S}^{i}) - \sum_{i=1}^{{}^1N^t} p({}^1E_i^t)\log p({}^1E_i^t) - p({}^1\mathcal{S}^{c})\log p({}^1\mathcal{S}^{c}) = \tag{29}$$
$$= H({}^1\mathcal{S}^{itc}{}') + p({}^1\mathcal{S}^{t})RED({}^1\mathcal{S}^{t})$$

The relations (27, 29) prove that whenever the system functional level raises the unconditional system entropy and the entropy of the service profile of redundant object increases. The conditional entropy of the secondary functional level with respect to the service profile of transitive events is:

$$H({}^2\mathcal{S}/{}^2\mathcal{S}^{itc}{}') = p({}^1\mathcal{S}^{c})ROB({}^1\mathcal{S}^{c}) + \sum_{j=1}^{{}^1N^t} p({}^1E_j^t)H({}_j^2\mathcal{S}) = \tag{30}$$
$$= H({}^1\mathcal{S}/{}^1\mathcal{S}^{itc}{}') + \sum_{j=1}^{{}^1N^t} p({}^1E_j^t)H({}_j^2\mathcal{S}) - p({}^1\mathcal{S}^{t})RED({}^1\mathcal{S}^{t}) = H({}^2\mathcal{S}) - H({}^2\mathcal{S}^{itc}{}')$$

The conditional entropy (30) underestimates the overall system uncertainty since it does not account for secondary intact and collapse modes.

4.5. THE UNCERTAINTIES OF SYSTEM INTACT AND COLLAPSE MODES

The uncertainties of the individual secondary functional states $j = 1,2,...,N^t$, of a "fail-safe" object relative to intact modes, expressed by secondary conditional entropy, vanish:

$$H({}_j^2\mathcal{S}/{}_j^2\mathcal{S}^{i}) = RED({}_j^2\mathcal{S}^{i}) = -\frac{p({}_j^2E_1^i)}{p({}_j^2E_1^i)}\log\frac{p({}_j^2E_1^i)}{p({}_j^2E_1^i)} = 0 \tag{31}$$

The uncertainty of the individual, independent secondary functional states, relative to collapse modes, denoted as secondary mode robustness, are expressed by conditional entropy:

$$H({}_j^2\mathcal{S}/{}_j^2\mathcal{S}^{c}) = H({}^2\mathcal{S}/({}_j^2\mathcal{S}^{c}\cap{}^1E_j^t)) = ROB({}_j^2\mathcal{S}^{c}) = -\sum_{i=1}^{{}_j^2N^c}\frac{p({}_j^2E_i^c)}{p({}_j^2\mathcal{S}^{c})}\log\frac{p({}_j^2E_i^c)}{p({}_j^2\mathcal{S}^{c})} \tag{32}$$

The conditional entropy of the second level with respect to secondary intact state, also denoted as the secondary state redundancy, accounts for all the secondary intact events:

$$H({}^2\mathcal{S}/{}^2\mathcal{S}^{i}) = RED({}^2\mathcal{S}^{i}) = -\sum_{j=1}^{{}^1N^t}\frac{p({}^1E_j^t)p({}_j^2E_1^i)}{p({}^2\mathcal{S}^{i})}\log\frac{p({}^1E_j^t)p({}_j^2E_1^i)}{p({}^2\mathcal{S}^{i})} \tag{33}$$

The conditional entropy of the second level with respect to collapse state, denoted as the secondary collapse state robustness, accounts for all the secondary collapse events:

$$H({}^2\mathcal{S}/{}^2\mathcal{S}^{c}) = ROB({}^2\mathcal{S}^{c}) = -\sum_{j=1}^{{}^1N^t}\sum_{i=1}^{{}_j^2N^c}\frac{p({}^1E_j^t)p({}_j^2E_i^c)}{p({}^2\mathcal{S}^{c})}\log\frac{p({}^1E_j^t)p({}_j^2E_i^c)}{p({}^2\mathcal{S}^{c})} \tag{34}$$

The uncertainty of the collapse profile is expressed as follows:

$$H({}^2\mathcal{S}^{c}/{}^2\mathcal{S}^{ic}{}') = ROB({}^2\mathcal{S}^{c}{}') = -\sum_{j=1}^{{}^1N^t}\frac{p({}^1E_j^t)p({}_j^2\mathcal{S}^{c})}{p({}^2\mathcal{S}^{c})}\log\frac{p({}^1E_j^t)p({}_j^2\mathcal{S}^{c})}{p({}^2\mathcal{S}^{C})} \tag{35}$$

The expression (34) can be rewritten in terms of partial results (32) and (35) as follows:

$$ROB({}^2\mathcal{S}^{c}) = \frac{1}{p({}^2\mathcal{S}^{c})}\left[p({}^2\mathcal{S}^{c})ROB({}^2\mathcal{S}^{c}{}') + \sum_{j=1}^{{}^1N^t} p({}^1E_j^t p({}_j^2\mathcal{S}^{c})ROB({}_j^2\mathcal{S}^{c})\right] \tag{36}$$

The unconditional entropy of the secondary service profile of primary non-transitive and emerged intact and collapse modes is calculated as follows:

$$H({}^{2}\mathcal{S}^{ic}{'}) = -p({}^{1}\mathcal{S}^{i})\log p({}^{1}\mathcal{S}^{i}) - p({}^{2}\mathcal{S}^{i})\log p({}^{2}\mathcal{S}^{i}) - p({}^{2}\mathcal{S}^{c})\log p({}^{2}\mathcal{S}^{c}) - p({}^{1}\mathcal{S}^{c})\log p({}^{1}\mathcal{S}^{c}) \quad (37)$$

The conditional entropy of the secondary collapse mode with respect to collapse profile relates the state and mode robustness, as shown:

$$H({}^{2}\mathcal{S}^{c}/{}^{2}\mathcal{S}^{c}{'}) = \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t})\, p({}_{j}^{2}\mathcal{S}^{c})\, ROB({}_{j}^{2}\mathcal{S}^{c}) = p({}^{2}\mathcal{S}^{c})\left[ROB({}^{2}\mathcal{S}^{c}) - ROB({}^{2}\mathcal{S}^{c}{'})\right] \quad (38)$$

The unconditional entropy of the secondary service profile of primary non-transitive and emerged intact and collapse modes accounting for all secondary collapse events is:

$$\begin{aligned} H({}^{2}\mathcal{S}^{ic}{''}) = & -p({}^{1}\mathcal{S}^{i})\log p({}^{1}\mathcal{S}^{i}) - p({}^{2}\mathcal{S}^{i})\log p({}^{2}\mathcal{S}^{i}) + p({}^{2}\mathcal{S}^{c})ROBp({}^{2}\mathcal{S}^{c}{'}) - \\ & - p({}^{2}\mathcal{S}^{c})\log p({}^{2}\mathcal{S}^{c}) - p({}^{1}\mathcal{S}^{c})\log p({}^{1}\mathcal{S}^{c}) \end{aligned} \quad (39)$$

The conditional entropy of the secondary level with respect to the service profile of all primary and secondary intact and collapse modes, relates the primary and secondary redundancy and robustness as follows:

$$H({}^{2}\mathcal{S}/{}^{2}\mathcal{S}^{ic}{'}) = p({}^{1}\mathcal{S}^{c})ROB({}^{1}\mathcal{S}^{c}) + p({}^{2}\mathcal{S}^{i})RED({}^{2}\mathcal{S}^{i}) + p({}^{2}\mathcal{S}^{c})ROB({}^{2}\mathcal{S}^{c}) = H({}^{2}\mathcal{S}) - H({}^{2}\mathcal{S}^{ic}{'}) \quad (40)$$

The conditional entropy of the secondary level with respect to the service profile of all primary and secondary intact and collapse modes for all secondary collapse events is calculated as follows:

$$H({}^{2}\mathcal{S}/{}^{2}\mathcal{S}^{ic}{''}) = p({}^{1}\mathcal{S}^{c})ROB({}^{1}\mathcal{S}^{c}) + p({}^{2}\mathcal{S}^{i})RED({}^{2}\mathcal{S}^{i}) + \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t})p({}_{j}^{2}\mathcal{S}^{c})ROB({}_{j}^{2}\mathcal{S}^{c}) = H({}^{2}\mathcal{S}) - H({}^{2}\mathcal{S}^{ic}{''}) \quad (41)$$

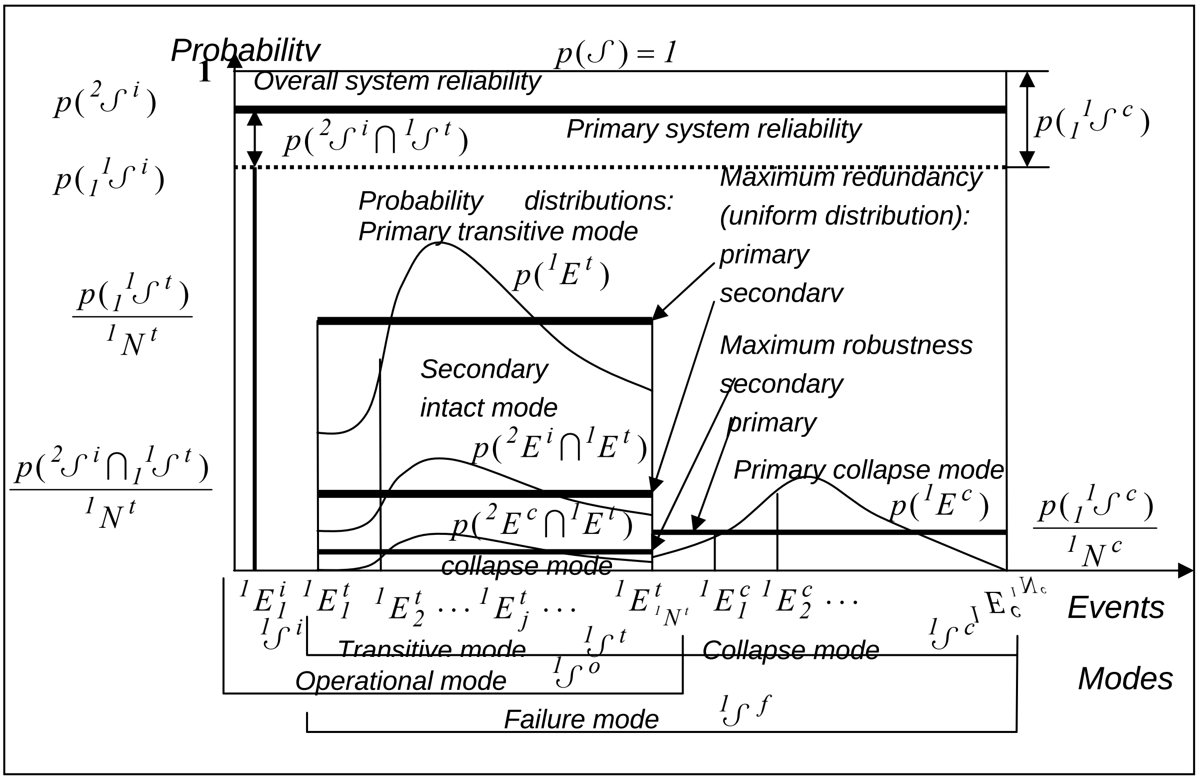

Fig. 1 Single picture example

Slika 1. Primjer smještaja jedne slike

5. CONCLUSION

This paper indicates that the event-oriented analysis of redundant objects exposed to successive component failures, which change the system configuration and provoke a redistribution of demands and capabilities, is a complicated but feasible task. However, neither the system configuration nor the system residual strength can affirm that an object performs its service as a “fail-safe” system, without checking the reliabilities of secondary operational modes after component failures. Moreover, a sound analysis of redundant systems is not possible by taking into account only the residual strength on the primary level and overlooking the probability distribution of secondary operational modes.

The event oriented system analysis identifies transitive and emerging events, as well as intact, operational, failure and collapse modes. In addition, EOSA provides probabilities of successive operational levels and functional states, regardless of the ordering and succession of events. The redundancy measures expressed by the conditional entropy of transitive modes account for the redistribution of loads, as well as for numbers of events and the distribution of their probabilities, independent of the system reliability and residual strength. The traditional probabilistic redundancy index based on the system’s primary residual strength accounts neither for the probability distribution nor for the load redistribution in secondary functional states. However, the residual strength in case of load redistribution has to account for the secondary operational modes.

The article demonstrates that the entropy of redundant objects, pertinent to transition into another functional level, always increases, and moreover, the increase of the entropy due to system redundancy is quantifiable. On the other hand, the implementation of additional knowledge about the system service profile in EOSA reduces uncertainties. High redundancy expressed by entropy indicates a uniform distribution of probabilities, also a more economical allocation of system capabilities with respect to the system performance.

EOSA is applicable either globally to the whole object, that is to all observable and relevant events, or locally, to the subset of events pertinent to components and groups of components with a common purpose, as well as to logical subsystems of events with distinguished meanings or relevance.

Engineers are motivated to use their knowledge and experience to maximize their chances against nature. The designers can intervene on the physical properties of planned objects and artificially change the probabilities, as well as the uncertainties of transitive, operational and failure modes, in order to design system behaviour to everyone's satisfaction.

The event-oriented analysis is performed entirely in the event space. The relation between the event space and the physical world of an engineering object can be defined empirically by statistical methods or theoretically by employing random variable models. Random variable models can only come close to complex non-linear probabilistic problems. The engineering conviction in appropriate relations between probabilistic models and real systems relies on the accuracy of statistical information about model parameters, as well as on the belief that mathematical models suit the practical situation. However, often there is no opportunity to verify the probabilistic approach in practice, since most of the complex systems usually exist in a single or, at the most, only in several specimens.

The event-oriented analysis of large systems may require significant engineering exertion in the identification of transitive, operational and failure modes. The difficulties arising from problem size can hopefully be managed by problem partitioning pertinent to the event oriented system analysis. This paper concludes that the event oriented system analysis, where applicable, provides much more useful information for engineering decisions about complex redundant systems than a mere reliability analysis.

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