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Embedded Options and Tax Decisions: A Reconsideration of the Traditional vs. Roth IRA Decision

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ABSTRACT

Embedded options arise in many tax-related decisions because of the ability to subsequently alter one's choices in response to changing conditions. This article analyzes one type of embedded option that is especially amenable to being modeled and that is of widespread interest: the decision to contribute to a traditional or Roth IRA, where the embedded option arises from the opportunity to subsequently roll over a traditional IRA to a Roth IRA. The results show that the alternative with which the embedded option is associated (i.e., a traditional IRA contribution) might be incorrectly rejected when the option is ignored.

INTRODUCTION

In many decision contexts, the flexibility to subsequently alter one's choices in response to evolving conditions is valuable and should be taken into account. For example, when making capital budgeting decisions, the ability to subsequently expand or contract a project in response to new information can make the project's value greater than that suggested by a traditional net present value analysis (Trigeorgis 1993). Such embedded options also exist in many tax-related contexts. For example, a U.S. taxpayer deciding whether to repatriate its foreign subsidiary's earnings can, if it chooses not to repatriate them currently, manipulate the timing of their subsequent repatriation (Huddart 1996). Similarly, Sansing (1996) examines the benefits of having the option to postpone an investment on the neutrality of repatriation taxes on a foreign subsidiary's earnings. Constantinides (1984) shows that it is sometimes optimal to sell and repurchase a stock with an accrued long-term capital gain. While such action would accelerate the gain's taxation, it would also give the investor the option to recognize any gain in the subsequent year as a long-term gain by delaying the stock's sale and recognize any loss as a short-term loss by selling it within one year. Finally, a small business owner choosing the C-corporation form generally has the option to later switch to an S-corporation if it becomes advantageous to do so (or vice versa).

Another tax-related decision in which an embedded option exists is one that has received much attention in the financial and popular press because it is one that many individuals face: the choice of contributing to a traditional individual retirement account (IRA) or a Roth IRA. With a traditional IRA, contributions are generally tax-deductible, but distributions are taxable as ordinary income. Contributions to a Roth IRA are not tax-deductible, but distributions are tax-free if certain

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requirements are met. The taxpayer has the flexibility to later roll over a traditional IRA to a Roth IRA, giving the taxpayer the ability to manipulate the timing of the traditional IRA's taxation. When this embedded option is not taken into account, some of the expected benefits of contributing to a traditional IRA are ignored, which may cause one to incorrectly conclude in some circumstances that a Roth IRA is more advantageous than a traditional IRA.

The IRA contribution decision provides an opportunity to examine embedded tax options in a context that, when compared to many other tax contexts such as those discussed above, is relatively easy to understand and quantify. This article extends Seida and Stern's (1998) (hereafter S&S) analysis of the IRA contribution decision by including the embedded rollover option that exists with a traditional IRA.[1] In the model here, future tax rates are assumed to be stochastic rather than known. The benefit of a traditional IRA's embedded rollover option that results from this tax rate variability depends on its stochastic nature. If random variations in the tax rate are transitory and do not affect expectations regarding subsequent tax rates,[2] then this variability creates probable future opportunities to take advantage of a lower tax rate by rolling over a traditional IRA to a Roth IRA. While a particular future tax rate might instead be higher than expected, its transitory nature means that the taxpayer can avoid the higher rate by choosing not to roll over the IRA, with the hope that a lower than expected tax rate will subsequently be realized. At worst, the taxpayer never finds it advantageous to roll over the IRA and pays a high tax rate on its eventual distribution, but the likelihood of this outcome may be small. On the other hand, if random tax rate variations are permanent and change expectations for subsequent tax rates,[3] then the results are quite different. While a particular future tax rate might be lower than expected, the taxpayer might instead encounter a higher future tax rate that will then increase the expectations of all subsequent tax rates. The taxpayer cannot expect to avoid a higher future tax rate by merely choosing not to roll over the IRA and wait for a lower tax rate, so the traditional IRA's embedded option provides no expected benefit.

The next section reviews relevant tax law. S&S's analysis of the annual IRA contribution decision is then summarized. Their analysis is extended to consider the future rollover option embedded in a traditional IRA contribution, with a numerical example to illustrate the model. The effects of relaxing certain model assumptions are then examined, and other factors relevant to the decision are discussed. Finally, conclusions are drawn.

REVIEW OF RELEVANT TAX LAW

A taxpayer may contribute up to $3,000 to a traditional IRA each year. The contribution can generally be deducted in arriving at adjusted gross income (AGI), but this deduction is phased out if the taxpayer (or spouse) is an active participant in an employer-maintained retirement plan and AGI exceeds a statutory amount.[4] A traditional IRA's earnings are not taxed as they accrue; instead, the earnings, plus any deducted contributions, are taxed as ordinary income when distributed.[5]

The taxpayer can instead contribute $3,000 to a Roth IRA, but it cannot be deducted. The opportunity to contribute to a Roth IRA is phased out if AGI exceeds a statutory amount, which

[1] See Anderson and Murphy (1998) and Burman et al. (2001) for analyses similar to that of S&S. Horan et al. (1997) model the decision to make a *nondeductible* traditional IRA contribution or invest outside of an IRA.

[2] For example, business and/or investment income might be unusually high or low. These income fluctuations could affect the tax rate by changing the taxpayer's tax bracket or by moving the taxpayer into or out of the phase-out range for a deduction or credit.

[3] For example, tax rate fluctuations due to legislation might be expected to continue indefinitely.

[4] This phase-out affects only the contribution's deductibility and not the eligibility to contribute. The contribution cannot exceed taxable compensation. The annual contribution limit was $2,000 prior to 2002, and it is scheduled to increase beyond $3,000 after 2004. Additional "catch-up" contributions are allowed for taxpayers age 50 or older. No traditional IRA contribution is allowed for taxpayers age 70½ or older.

[5] If the distribution occurs before the taxpayer attains age 59½, a 10 percent penalty generally is imposed. A taxpayer is required to receive certain minimum distributions from a traditional IRA after attaining age 70½.

is generally much higher than that for a traditional IRA. Like a traditional IRA, a Roth IRA's earnings are not taxed as they accrue, but distributions are tax-free if the taxpayer has attained age 59½ and at least five taxable years have passed since the first contribution to a Roth IRA.[6]

A traditional IRA generally can be rolled over to a Roth IRA. The amount rolled over is taxed as ordinary income, but the 10 percent penalty for distributions before age 59½ is waived.[7] A rollover is not allowed if AGI exceeds $100,000, but AGI for this purpose does not include any gross income that results from the rollover. The amount rolled over does not affect the eligibility to also make a $3,000 annual contribution to a Roth IRA.

Beginning in 2006, the tax law will allow so-called Roth 401(k) plans. With a traditional 401(k), an employee can elect to contribute up to $15,000 of compensation to the plan, with such contributions being excluded from gross income.[8] The contributions, as well as plan earnings, are taxable upon distribution from the plan, making them similar to traditional IRAs. With the new Roth 401(k), the employee can instead elect to make contributions that are not excluded from gross income, but distributions will be tax-free if the employee has attained age 59½ and at least five years have passed since contributions were first made to the plan.

MODEL OF ANNUAL CONTRIBUTION DECISION

S&S apply the Scholes and Wolfson (1992) investment vehicle formulas to analyze the decision to make an annual contribution (e.g., $3,000) to a traditional or Roth IRA. The model they develop is summarized below, and it is then extended to take into account the rollover option that is embedded in a traditional IRA contribution.

S&S's Analysis

Assume that the taxpayer contributes to an IRA the maximum amount allowed. If the contribution is made to a Roth IRA, the after-tax future value of the contribution is:

$$C(1 + R)^n \tag{1}$$

where:

C = annual contribution limit;
R = annual rate of return on assets inside the IRA; and
n = number of years the IRA will be maintained.

Modeling a deductible traditional IRA contribution is more complicated. Although the tax law imposes the same contribution limit on both types of IRAs, the taxpayer contributes after-tax dollars to a Roth IRA but before-tax dollars to a traditional IRA. The taxpayer can contribute C after-tax to a Roth IRA, but can contribute after-tax to a traditional IRA only C minus the resultant tax savings from deducting C. S&S thus model an investment in a traditional IRA as having two components: (1) the IRA itself and (2) an investment outside the IRA of the contribution's tax savings. The total after-tax future value of these two components is:

$$C(1 + R)^n + C(t_0)(1 + r)^n - C(1 + R)^n(t_n) \tag{2}$$

where C, R, and n are the same as above, and:

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[6] As with a traditional IRA, the contribution is limited to taxable compensation. Unlike a traditional IRA, the taxpayer is not prohibited from contributing to a Roth IRA after attaining age 70½. The minimum distribution rules that apply for traditional IRAs after the taxpayer attains age 70½ do not apply for Roth IRAs.

[7] The amount rolled over from the traditional IRA is not taxed to the extent it represents nondeducted contributions. The 10 percent penalty is waived only for amounts that are rolled over to a Roth IRA. To the extent that taxes and penalties are paid from traditional IRA funds, they are not rolled over and thus can trigger the 10 percent penalty.

[8] Similarly, the tax law will also allow Roth 403(b) plans. The contribution limit for 401(k) and 403(b) plans is gradually increasing from $11,000 in 2002 to $15,000 after 2005, with inflation adjustments beginning in 2007. Additional "catch-up" contributions will be allowed for taxpayers age 50 or older.

t_0 = marginal tax rate in the year of contribution;
t_n = marginal tax rate in the year of the IRA's distribution; and
r = annualized after-tax rate of return on the outside investment.[9]

The first term of Expression (2) is the IRA's pretax value at liquidation; note that it is the same as Expression (1). The other two terms of Expression (2) capture the trade-off inherent with a traditional IRA contribution. There is an immediate tax savings that can be invested elsewhere (the second term of Expression (2)), but it comes with a tax liability when the IRA is liquidated (the third term of Expression (2)). If the second and third terms are a net positive (negative) amount, a traditional (Roth) IRA contribution will be preferable. More formally, setting Expression (1) to be greater than Expression (2) and simplifying, a Roth IRA contribution will result in a larger after-tax future value than an investment in a traditional IRA when:

$$t_0 < t_n \left(\frac{1 + R}{1 + r}\right)^n. \tag{3}$$

The left-hand side of Expression (3) is the tax savings, per dollar of C, of a traditional IRA contribution. Alternatively, it can be interpreted as the amount of tax that must be paid, per dollar of C, if the taxpayer contributes to a Roth IRA, forgoing a deduction. The right-hand side of Expression (3) is the present value of the future tax that will have to be paid, per dollar of C, if the taxpayer contributes to a traditional IRA.[10] In essence, Expression (3) suggests that a Roth IRA contribution should be made if, in present value terms, the tax costs of doing so are less than those of a traditional IRA contribution.

If $R > r$, then Expression (3) will be true if $t_0 \leq t_n$. That is, if the return on the IRA's assets is greater than the after-tax return on non-IRA assets and the tax rate will stay constant or increase, then a Roth IRA contribution results in a greater after-tax accumulation. If the tax rate will decrease, then the particular parameter values will determine whether a Roth or traditional IRA contribution is better.

Extension of Model to Include Embedded Rollover Option

The model above is extended to consider the option to subsequently roll over a traditional IRA to a Roth IRA, using the following notation and assumptions:

- The taxpayer contributes C to an IRA (traditional or Roth), and it is the maximum allowable contribution. A traditional IRA contribution is deductible, and the resulting tax savings are invested outside the IRA.
- R_i is the year i rate of return on the IRA's assets. R_i is unknown *ex ante* and is distributed with mean $R_{\mu i}$ and standard deviation σ_{Ri}. R_i is the same regardless of the type of IRA. For all $i \neq j$, R_i is independent of R_j.
- r_i is the year i annualized after-tax rate of return on non-IRA assets. r_i is unknown *ex ante* and is distributed with mean $r_{\mu i}$ and standard deviation σ_{ri}. For all $i \neq j$, r_i is independent of r_j. No assumption is made regarding the association of R_i and r_i. $R_{\mu i} > r_{\mu i}$.
- t_i is the year i marginal tax rate. The marginal tax rate for the year of contribution, t_0, is known. For all $i > 0$, t_i is unknown *ex ante* and is distributed with mean $t_{\mu i}$ and standard deviation σ_{ti}. For all $i \neq j$, t_i is independent of t_j. In addition, t_i is independent of all returns (both R_i and r_i, for year i and for all other years).[11]

[9] The notation and formulas here differ slightly from S&S's to keep the analysis from becoming too cumbersome. For example, S&S analyze separately a fully taxable, a tax-deferred, and a tax-exempt outside investment. The r term here is meant to represent more generally the after-tax return on the outside investment, and it can represent all three types of outside investments.

[10] The future tax payment per dollar of C is $t_n(1 + R)^n$. If this is discounted to the present for n years at an r annual rate, then the right-hand side of Expression (3) results.

[11] The effects of relaxing these independence assumptions, and some of the other assumptions, are discussed later. R_i, r_i, and t_i could be, but are not required to be, distributed normally.

- The IRA balance (traditional or Roth) will be distributed in year n as a lump sum. The five-year rule for Roth IRAs and the 10 percent penalty for early distributions are ignored.
- The taxpayer will be eligible every year to roll over a traditional IRA to a Roth IRA (i.e., AGI without the rollover will be \$100,000 or less each year). Any tax on the rollover will be paid out of funds that otherwise would be invested outside the IRA.
- The objective is to maximize the expected after-tax future value of the investment.

A decision tree approach is used to analyze the IRA contribution decision. A three-year investment horizon is used for brevity, but the approach can be applied to longer investment horizons. Figure 1 illustrates the structure of the decision tree. A contribution could be made to a Roth IRA, to a traditional IRA that is rolled over at some interim date, or to a traditional IRA that is never rolled over. S&S's analysis considers the first and third outcomes; the analysis here also considers the second possible outcome. The backward induction method is used to solve a decision tree problem (Gordon and Pressman 1978). The latest possible decision is analyzed first, which here is whether to exercise the option to roll over the traditional IRA to a Roth IRA at $i = 2$. The earlier decisions are then analyzed in reverse order, taking into account for each decision the optimal action for all subsequent decisions. Eventually, the initial decision is reached, and its analysis incorporates the future, although contingent, decisions.

Consider first the decision at $i = 2$. If the traditional IRA has not yet been rolled over, then it will have a value of $C(1 + R_1)(1 + R_2)$. If the IRA is rolled over, then the after-tax future value at the end of the investment horizon will be:

$$C(1 + R_1)(1 + R_2)(1 + R_3) + C(t_0)(1 + r_1)(1 + r_2)(1 + r_3)$$
$$- C(1 + R_1)(1 + R_2)(t_2)(1 + r_3). \qquad (4)$$

The first term of Expression (4) is the future value of the Roth IRA, and the second term is the after-tax accumulation from investing the $C(t_0)$ tax savings that resulted from deducting the initial traditional IRA contribution. The third term is the future wealth forgone because a tax was paid at $i = 2$, was not invested for a year, and did not earn an after-tax return of r_3. At $i = 2$, R_3 and r_3 are not yet known, so the expectation of Expression (4) is used instead:

$$C(1 + R_1)(1 + R_2)(1 + R_{\mu 3}) + C(t_0)(1 + r_1)(1 + r_2)(1 + r_{\mu 3})$$
$$- C(1 + R_1)(1 + R_2)(t_2)(1 + r_{\mu 3}). \qquad (5)$$

The only difference between Expressions (4) and (5) is that expectations, rather than actual amounts, are used for R_3 and r_3 in Expression (5).

Rather than rolling over the traditional IRA, the taxpayer instead could choose to keep the funds in it. The after-tax future value of so choosing is:

$$C(1 + R_1)(1 + R_2)(1 + R_3) + C(t_0)(1 + r_1)(1 + r_2)(1 + r_3)$$
$$- C(1 + R_1)(1 + R_2)(1 + R_3)(t_3). \qquad (6)$$

Again, R_3 and r_3 are unknown, and t_3 is also unknown. Taking the expectation of Expression (6), the expected after-tax future value if the traditional IRA is maintained is:

$$C(1 + R_1)(1 + R_2)(1 + R_{\mu 3}) + C(t_0)(1 + r_1)(1 + r_2)(1 + r_{\mu 3})$$
$$- C(1 + R_1)(1 + R_2)(1 + R_{\mu 3})(t_{\mu 3}). \qquad (7)$$

Setting Expression (5) to be greater than Expression (7) and simplifying, the expected after-tax future value of rolling over the traditional IRA at $i = 2$ will be greater than that of not doing so if:

$$t_2 < t_{\mu 3}\left(\frac{1 + R_{\mu 3}}{1 + r_{\mu 3}}\right). \qquad (8)$$

The left-hand side of Expression (8) is the amount of tax, per dollar of $i = 2$ IRA value, that must be paid if it is rolled over. The right-hand side of Expression (8) is denoted as t_2^*, and it can be interpreted as the present value of the expected amount of tax to be paid at $i = 3$, per dollar of $i = 2$ IRA value, if the taxpayer does not roll it over. That is, the taxpayer can pay a tax now on

FIGURE 1
IRA CONTRIBUTION DECISION FOR THREE-YEAR INVESTMENT HORIZON

$i = 0$ $\xrightarrow{\text{Contribute to Roth IRA}}$ $ATFV = C(1 + R_1)(1 + R_2)(1 + R_3)$

$\downarrow$ Contribute to traditional IRA

$i = 1$ $\xrightarrow{\text{Roll over to Roth IRA}}$

$$ATFV = C(1 + R_1)(1 + R_2)(1 + R_3) + C(t_0)(1 + r_1)(1 + r_2)(1 + r_3) - C(1 + R_1)(t_1)(1 + r_2)(1 + r_3)$$

$\downarrow$ Keep funds in traditional IRA

$i = 2$ $\xrightarrow{\text{Roll over to Roth IRA}}$

$$ATFV = C(1 + R_1)(1 + R_2)(1 + R_3) + C(t_0)(1 + r_1)(1 + r_2)(1 + r_3) - C(1 + R_1)(1 + R_2)(t_2)(1 + r_3)$$

$\downarrow$ Keep funds in traditional IRA

$i = 3$ $\xrightarrow{\text{Liquidate traditional IRA}}$

$$\text{ATFV} = C(1 + R_1)(1 + R_2)(1 + R_3) + C(t_0)(1 + r_1)(1 + r_2)(1 + r_3) - C(1 + R_1)(1 + R_2)(1 + R_3)(t_3)$$

C = amount contributed to an IRA (traditional or Roth);
R_i = year i rate of return on IRA assets;
r_i = year i after-tax rate of return on non-IRA assets;
t_i = year i marginal tax rate; and
$ATFV$ = after-tax future value at the end of the three-year period.

the current IRA balance at a known t_2 rate or a tax in one year on an IRA balance that is expected to be $R_{\mu 3}$ percent larger at an expected rate of $t_{\mu 3}$. In essence, the taxpayer will be better off choosing the action with the lower present value of expected tax payments.

Consider next the decision at $i = 1$, when the traditional IRA's value is $C(1 + R_1)$. If it is rolled over to a Roth IRA, then the expected after-tax future value at $i = 3$ is:

$$C(1 + R_1)(1 + R_{\mu 2})(1 + R_{\mu 3}) + C(t_0)(1 + r_1)(1 + r_{\mu 2})(1 + r_{\mu 3})$$
$$- C(1 + R_1)(t_1)(1 + r_{\mu 2})(1 + r_{\mu 3}) \quad (9)$$

Similar to Expression (5), the first term of Expression (9) is the expected future value of the Roth IRA, the second term is the expected future value of the tax savings from the initial traditional IRA contribution, and the third term is the expected forgone future wealth because a tax was paid on the rollover at $i = 1$.

Alternatively, the taxpayer may choose to maintain the traditional IRA, but the expected after-tax future value is more complicated because it depends on the future action at $i = 2$. There is a P_2 probability that t_2 will be less than t_2^* and the taxpayer will thus choose to roll over the IRA, and there is a $1 - P_2$ chance that it is instead distributed at $i = 3$. Taking into account these probabilistic future outcomes, the expected after-tax future value of maintaining the traditional IRA at $i = 1$ is:

$$C(1 + R_1)(1 + R_{\mu 2})(1 + R_{\mu 3}) + C(t_0)(1 + r_1)(1 + r_{\mu 2})(1 + r_{\mu 3})$$
$$- C(1 + R_1)(1 + R_{\mu 2})E(t_2 \mid t_2 < t_2^*)(1 + r_{\mu 3})(P_2)$$
$$- C(1 + R_1)(1 + R_{\mu 2})(1 + R_{\mu 3})(t_{\mu 3})(1 - P_2) \quad (10)$$

where $E(t_2 \mid t_2 < t_2^*)$ is the expected value of t_2, given that t_2 is less than t_2^*. The first two terms of Expression (10) are nearly identical to the first two terms of Expression (7); they differ only because expectations are used for R_2 and r_2. The third term is the expected tax payment at $i = 2$ if it is optimal to roll over the IRA at that time, multiplied by the after-tax return that the payment was otherwise expected to have earned in year 3 and multiplied by the probability that it will be optimal to roll over the IRA at $i = 2$. The fourth term is the expected tax payment when liquidating the traditional IRA at the end of the investment horizon, multiplied by the probability that it will persist that long.

Setting Expression (9) to be greater than Expression (10) and simplifying, it will be optimal to roll over the traditional IRA at $i = 1$ if:

$$t_1 < \left[E(t_2 \mid t_2 < t_2^*)\left(\frac{1 + R_{\mu 2}}{1 + r_{\mu 2}}\right)(P_2)\right] + \left[(t_{\mu 3})\left(\frac{1 + R_{\mu 2}}{1 + r_{\mu 2}}\right)\left(\frac{1 + R_{\mu 3}}{1 + r_{\mu 3}}\right)(1 - P_2)\right]. \quad (11)$$

Similar to Expression (8), the right-hand side of Expression (11) is denoted as t_1^*. Similar also to Expression (8), Expression (11) compares the taxes paid at $i = 1$ if the traditional IRA is rolled over to the present value of the expected future taxes if it is not rolled over.

Finally, at $i = 0$, the taxpayer must decide whether to contribute to a Roth or traditional IRA. If the former is chosen, then the expected after-tax future value is:

$$C(1 + R_{\mu 1})(1 + R_{\mu 2})(1 + R_{\mu 3}). \quad (12)$$

If the latter is chosen, the expected after-tax future value is:

$$C(1 + R_{\mu 1})(1 + R_{\mu 2})(1 + R_{\mu 3}) + C(t_0)(1 + r_{\mu 1})(1 + r_{\mu 2})(1 + r_{\mu 3})$$
$$- C(1 + R_{\mu 1})E(t_1 \mid t_1 < t_1^*)(1 + r_{\mu 2})(1 + r_{\mu 3})(P_1)$$
$$- C(1 + R_{\mu 1})(1 + R_{\mu 2})E(t_2 \mid t_2 < t_2^*)(1 + r_{\mu 3})(1 - P_1)(P_2)$$
$$- C(1 + R_{\mu 1})(1 + R_{\mu 2})(1 + R_{\mu 3})(t_{\mu 3})(1 - P_1)(1 - P_2) \quad (13)$$

where P_1 is the probability that t_1 will be less than t_1^*, and $E(t_1 \mid t_1 < t_1^*)$ is the expected value of t_1, given that t_1 is less than t_1^*. As with Expressions (7) and (10), the first term of Expression (13) represents the expected before-tax $i = 3$ value of the IRA (whether it is traditional or Roth), and the second term is the expected after-tax future value of the traditional IRA contribution's tax savings. The other three terms represent expectations for the three possible future tax payments resulting from a traditional IRA contribution, each one multiplied by the expected return that is subsequently forgone and by the probability that the payment will be made.

Setting Expression (12) to be greater than Expression (13) and simplifying, the taxpayer is expected to be better off contributing to a Roth IRA rather than to a traditional IRA if:

$$t_0 < \left[E(t_1 \mid t_1 < t_1^*)\left(\frac{1 + R_{\mu 1}}{1 + r_{\mu 1}}\right)(P_1)\right] + \left[E(t_2 \mid t_2 < t_2^*)\left(\frac{1 + R_{\mu 1}}{1 + r_{\mu 1}}\right)\left(\frac{1 + R_{\mu 2}}{1 + r_{\mu 2}}\right)(1 - P_1)(P_2)\right]$$
$$+ \left[(t_{\mu 3})\left(\frac{1 + R_{\mu 1}}{1 + r_{\mu 1}}\right)\left(\frac{1 + R_{\mu 2}}{1 + r_{\mu 2}}\right)\left(\frac{1 + R_{\mu 3}}{1 + r_{\mu 3}}\right)(1 - P_1)(1 - P_2)\right]. \quad (14)$$

The left-hand side of Expression (14) is the amount of tax that must be paid currently, per dollar of C, if the taxpayer contributes to a Roth IRA and thus gets no deduction. The right-hand side is the present value of the expected future tax payments, per dollar of C, if a traditional IRA contribution is made. In short, Expression (14) suggests that a Roth IRA contribution should be made if, in present value terms, the tax costs of doing so are less than the expected future tax costs of contributing to a traditional IRA.

Recall that Expressions (2) and (13) are the after-tax future values of a traditional IRA contribution without and with, respectively, the embedded rollover option. A comparison of these expressions is complicated by the fact that Expression (2) assumes that t_n is known *ex ante* and that R and r are also known *ex ante* and do not change from year to year. If it is modified to relax these assumptions and to reflect a three-year investment horizon, then the result is:

$$C(1 + R_{\mu 1})(1 + R_{\mu 2})(1 + R_{\mu 3}) + C(t_0)(1 + r_{\mu 1})(1 + r_{\mu 2})(1 + r_{\mu 3})$$
$$- C(1 + R_{\mu 1})(1 + R_{\mu 2})(1 + R_{\mu 3})(t_{\mu 3}). \quad (15)$$

Expression (13) is greater than Expression (15). See the Appendix for proof. That is, when the embedded rollover option is taken into account, the expected after-tax future value of a traditional IRA contribution is higher, reflecting the additional flexibility that the taxpayer has. Stated differently, this means that the benefits of a traditional IRA contribution are understated when the rollover option is ignored, and it could cause one to incorrectly conclude in some circumstances that a Roth IRA contribution is preferable. One such circumstance might occur when tax rates are expected to be constant or to increase. If the rollover option is ignored, then a Roth IRA will appear to be better (Expression (3) will be true). In contrast, Expression (14) could be true or false because there is a tension between two factors whose net effect is indeterminate. Disregarding any variability in the future tax rate for a moment, the future tax liability associated with a traditional IRA contribution is expected to grow more quickly than the invested tax savings. However, future tax rate variability creates the embedded rollover option and increases the probability of a low tax rate applying to the traditional IRA's distribution.[12]

NUMERICAL EXAMPLE

The example below illustrates the model and the effect that a traditional IRA's embedded rollover option can have on the annual contribution decision. The following parameters values are used:

- The maximum IRA contribution (C) is \$3,000.
- For all i, the return on IRA assets (R_i) is distributed normally with mean 0.12 ($R_{\mu i}$) and standard deviation 0.108 (σ_{Ri}), and the after-tax return on non-IRA assets (r_i) is distributed normally with mean 0.10 ($r_{\mu i}$) and standard deviation 0.09 (σ_{ri}).[13]
- The year 0 marginal tax rate (t_0) is 0.25. For all subsequent years, the marginal tax rate (t_i) is distributed normally with mean 0.25 ($t_{\mu i}$) and standard deviation 0.02 (σ_{ti}).

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[12] Note that $E(t_i \mid t_i < t_i^*)$ is less than $t_{\mu i}$ since the former excludes the upper part of t_i's distribution. The extent to which $E(t_i \mid t_i < t_i^*)$ is less than $t_{\mu i}$ depends on σ_{ti}.

[13] Since Expressions (12) and (13) do not contain σ_{Ri} and σ_{ri}, these standard deviations are not relevant to the computation of the expected after-tax future values, but they do affect the standard deviation of the possible actual after-tax future values. This third standard deviation is irrelevant to a taxpayer whose assumed objective is to maximize the expected after-tax future value, but the effect of allowing risk-aversion is discussed later.

Applying these parameter values to Expression (8) to determine t_2^*, it will be optimal to roll over a still-existing traditional IRA at $i = 2$ if t_2 is less than 25.45 percent.[14] That is, per dollar of $i = 2$ IRA value, the taxpayer can pay a tax of t_2 currently or an expected tax of 0.28 (0.25 × 1.12) in one year, with the present value of the expected future tax being 0.2545 (0.28 ÷ 1.10). Given the distribution of t_2, the probability that it will be less than 0.2545 is 0.5899, and the expected value of t_2, given that it is less than 0.2545, is 23.68 percent.[15] These values are used in Expression (11) to determine t_1^*, the year 1 marginal tax rate below which it is optimal to roll over the IRA, and it equals 24.85 percent.[16] The expected value of t_1, given that it is less than t_1^*, is 23.31 percent, and there is a 47.06 percent chance that t_1 will be less than 0.2485. Finally, Expression (14) indicates that it is optimal for this taxpayer to contribute to a Roth IRA if t_0 is less than 24.56 percent.[17] Since t_0 is 25 percent, it is optimal for this taxpayer to contribute to a traditional IRA. In contrast, a Roth IRA appears to be better if the rollover option is ignored since t_0 is less than 26.39 percent.[18]

The expected after-tax future value of this traditional IRA contribution is $4,232,[19] while that of a Roth IRA contribution is only $4,215.[20] Without the rollover option, the expected after-tax future value of a traditional IRA contribution is $4,159.[21] The embedded rollover option that comes with the traditional IRA contribution thus increases the expected after-tax future value by $73 ($4,232 − $4,159), which, here, more than offsets the Roth IRA's advantage of being able to receive larger after-tax contributions. There is a possibility that the taxpayer contributing to a traditional IRA will face a high t_3 and have to pay it because t_1 and t_2 were also high, but the $4,232 amount takes into account this possibility. There is a 65 (57) percent chance that a traditional IRA will provide a larger actual after-tax future value than a Roth IRA in this case if R_i and r_i are perfectly positively correlated (uncorrelated).[22]

Similar to a stock option, the embedded option has a smaller (larger) effect as the standard deviation of t_i decreases (increases). In this example, if σ_{ti} were less than 0.0128, then the expected after-tax future value of a traditional IRA contribution would be less than $4,215, which is the expected after-tax future value of a Roth IRA contribution. Furthermore, the rollover option becomes more valuable as the investment horizon increases, making it similar to a stock option in another way. For the short, three-year horizon used here, the option's $73 effect is relatively small, increasing the expected after-tax future value by only 1.8 percent ($73 ÷ $4,159), but the effect

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[14] 0.25 × (1.12 ÷ 1.10).

[15] The 0.2545 value corresponds to a z-score of 0.2273 ((0.2545 − 0.25) ÷ 0.02, which differs from 0.2273 because of rounding). The value of the standard normal's cumulative distribution function for 0.2273 is 0.5899. The 0.2368 amount was determined by simulation.

[16] $[(0.2368)(1.12 \div 1.10)(0.5899)] + [(0.25)(1.12 \div 1.10)^2(0.4101)]$. That is, if the IRA is not rolled over, a tax will be paid at either $i = 2$ or $i = 3$. If it is rolled over at $i = 2$, then the expected tax, per dollar of $i = 1$ IRA value, is 0.2652 (0.2368 × 1.12), with a present value of 0.2411 (0.2652 ÷ 1.10). If the traditional IRA is maintained until $i = 3$, then the expected tax, per dollar of $i = 1$ IRA value, is 0.3136 (0.25×1.12^2), which has a present value of 0.2592 ($0.3136 \div 1.10^2$). There is thus a 58.99 (41.01) percent chance that a tax with a present value of 0.2411 (0.2592) will be paid, making the present value of the expected future tax if the IRA is not rolled over equal to 0.2485 ((0.2411 × 0.5899) + (0.2592 × 0.4101)).

[17] $(0.2331)(1.12 \div 1.10)(0.4706) + (0.2368)(1.12 \div 1.10)^2(0.5294)(0.5899) + (0.25)(1.12 \div 1.10)^3(0.5294)(0.4101)$. The three components of this calculation are the present values of the expected future tax payments at $i = 1$, 2, and 3, with each one being multiplied by the probability that a tax will be paid at that time.

[18] See Expression (3): $0.25 \times (1.12 \div 1.10)^3$.

[19] See Expression (13): $3000(1.12)^3 + 3000(0.25)(1.10)^3 - 3000(1.12)(0.2331)(1.10)^2(0.4706) - 3000(1.12)^2(0.2368)(1.10)(0.5294)(0.5899) - 3000(1.12)^3(0.25)(0.5294)(0.4101)$.

[20] See Expression (12): $3000(1.12)^3$.

[21] See Expression (15): $3000(1.12)^3 + 3000(0.25)(1.10)^3 - 3000(1.12)^3(0.25)$.

[22] In contrast, if the embedded rollover option were ignored, the Roth IRA would then have a 72 (58) percent chance of providing a larger after-tax return if R_i and r_i are perfectly positively correlated (uncorrelated).

can become more substantial for longer investment horizons. For example, if the horizon were lengthened to 20 years, then the embedded rollover option increases the traditional IRA's expected after-tax future value by 8.9 percent.[23]

ALTERNATIVE ASSUMPTIONS FOR MODEL AND OTHER FACTORS

The results above were obtained on the basis of various assumptions, and these assumptions may not always reasonably portray reality. This section discusses how the analysis could be modified to relax some of these assumptions and the effect that it has on the numerical example. In addition, it discusses other factors that may be relevant to the IRA contribution decision that were not taken into account above.

t_i Follows a Random Walk

The year i marginal tax rate was assumed to be independent of all other years' tax rates. Assume instead that tax rates follow a random walk: the year i expected tax rate equals the actual tax rate for the preceding year. Substituting t_2 for $t_{\mu 3}$ in Expression (8), it simplifies to $R_{\mu 3} > r_{\mu 3}$, which is assumed to be true. It will always be optimal at $i = 2$ to roll over a traditional IRA, so P_2 equals 1 and $E(t_2 \mid t_2 < t_2^*)$ equals $t_{\mu 2}$. Similarly, since $t_{\mu 2}$ equals t_1 at $i = 1$, Expression (11) simplifies to $R_{\mu 2} > r_{\mu 2}$, which is true and indicates that the traditional IRA should always be rolled over at $i = 1$. Finally, at the time of the initial contribution, $t_{\mu 1}$ equals t_0, and Expression (14) simplifies to $R_{\mu 1} > r_{\mu 1}$, which is true and indicates that a Roth IRA contribution is always optimal.

If t_i follows a random walk, then the expectation for all future tax rates is the current tax rate. When this expectation occurs in the main analysis, the optimal IRA contribution form is indeterminate. The difference between that analysis and this one is whether deviations of the actual tax rate from the expected tax rate are transitory or permanent. In the main analysis, they were transitory; any difference between the actual and expected tax rates was not expected to affect subsequent tax rates. If a positive deviation occurs, it can be avoided by doing nothing (except for the last year of the investment horizon). If a negative deviation occurs, then the taxpayer can take advantage of the temporarily lower tax rate and roll over the IRA. In contrast, with the random walk assumption a random fluctuation in the tax rate is expected to persist because it affects the expectations for all subsequent tax rates. While the taxpayer might be fortunate and face a lower future tax rate, a higher tax rate may instead be realized, with this higher rate then being expected to persist.

t_i is Correlated with Investment Returns

Another assumption was that t_i is independent of the investment returns R_i and r_i. With a progressive tax rate structure, this assumption may not be reasonable because a larger cumulative return prior to distributing a traditional IRA might make a higher tax rate more likely. If this assumption is relaxed to allow t_i to depend partially on the investment returns for years 1 through i, then the analysis would differ somewhat. More specifically, when deciding the optimal action for year i, the expectation for the tax rate of a subsequent year j must be made jointly with the returns on the IRA assets for years $i + 1$ through j. For example, in Expression (14), the expectation of t_3 should be made jointly with R_1, R_2, and R_3. If t_i is positively correlated with investment returns, then relaxing this independence assumption would decrease the expected after-tax future

[23] The expected after-tax future values of a traditional IRA contribution with and without the rollover option are $29,118 and $26,750, respectively. The expected after-tax future value of a Roth IRA contribution is $28,939. Although the value of the option increases from $73 to $2,368 ($29,118 − $26,750) when the investment horizon increases from 3 to 20 years, the investment's annualized return decreases from 12.15 percent to 12.03 percent. This occurs because the option value grows more quickly, relative to the rest of the investment, during the first three years than it does during the next 17 years.

value of a traditional IRA contribution and not affect that of a Roth IRA contribution, although the former would not necessarily be less than the latter.[24]

Five-Year Rule

The analysis so far has ignored the requirement that at least five taxable years have passed since the taxpayer first contributed to a Roth IRA or rolled over funds into it in order for a distribution from it to be tax-free. By ignoring this rule, the analysis ignores the fact that, for an n-year investment horizon, a rollover from a traditional to a Roth IRA will not subsequently earn tax-free returns if the roll over occurs in years $n - 4$ through $n - 1$. The analysis can be modified by setting P_i (the probability that a rollover in year i is optimal because $t_i < t_i^*$) equal to zero for these years. This effectively begins the backward induction method at the decision node for year $n - 5$. This change reduces the expected after-tax future value of a traditional IRA contribution since it reduces the number of opportunities to take advantage of a low tax rate and roll it over. However, this reduction is small in many circumstances because the probability of rolling over the IRA before year $n - 4$ is high. For example, if the numerical example above is extended to a ten-year investment horizon and continues to ignore the five-year rule, then the expected after-tax future values are \$9,385 for a traditional IRA and \$9,318 for a Roth IRA, and there is an 88 percent probability that a rollover will occur in years 1 through 5. If the rollover opportunities for years 6 through 9 are eliminated, then the expected after-tax future value of a traditional IRA contribution decreases by \$12 to \$9,373, while the Roth IRA's is unchanged.

\$100,000 AGI Rule

It was assumed that AGI without any gross income from the rollover is less than or equal to \$100,000 each year so that the taxpayer is always eligible to roll over a traditional IRA. The analysis can be modified to relax this assumption by including additional parameters that capture the probability each year that AGI does not exceed \$100,000. Applying this to the numerical example, assume that there is a one-third probability each year that AGI will exceed \$100,000 and that this probability is unrelated to the other parameters.[25] At $i = 2$, the taxpayer is facing the last opportunity to roll over the IRA, so the probability that year 3 AGI will exceed \$100,000 is irrelevant and t_2^* is still 0.2545 (and the expected value of t_2, given that t_2 is less than t_2^*, is still 0.2368). The probability that t_2 will be less than t_2^* is still 0.5899, but the probability that t_2 will be less than t_2^* *and* AGI will be \$100,000 or less is 0.3933 (0.5899 × ⅔). This means that t_1^* will be 0.2521 (rather than 0.2485).[26] The determination of the level of t_0 below which it is optimal to contribute to a Roth IRA can similarly be modified, and the resulting expected after-tax future value is now \$4,213 (it was \$4,232 before this assumption was relaxed, and the expected after-tax future value of a Roth IRA contribution is unchanged at \$4,215). Unsurprisingly, relaxing this assumption decreases the expected after-tax future value of a traditional IRA contribution since it reduces the opportunities to later roll it over. Taken to the extreme, if the probability that AGI will exceed \$100,000 is 100 percent every year, the analysis becomes similar to S&S's. That is, if there

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[24] Note that any meaningful correlation between t_i and investment returns should be taken into account even if the embedded rollover option were completely ignored. A related issue is the desirability of rolling over only part of the traditional IRA. By assuming in the original analysis that t_i is independent of investment returns, such a partial rollover is suboptimal (except when $t_i = t_i^*$): either a rollover of the entire IRA or none of it is optimal. However, if t_i increases as the amount rolled over increases, then it would be beneficial to roll over the IRA to the extent that it does not increase t_i beyond t_i^*, with the remaining balance being retained in the traditional IRA and being available for rollover in subsequent years. The partial rollover would have to occur by the end of year i, though, and the taxpayer may not have complete information about t_i until April 15 of the next year (or later).

[25] This latter assumption is made to avoid the complexities of having to specify a functional form of the relationship between this probability and the other parameters.

[26] See Expression (11): $(0.2368)(1.12 \div 1.10)(0.3933) + (0.25)(1.12 \div 1.10)^2(0.6067)$.

is no possibility that the taxpayer will be able to roll over the IRA, there is no embedded rollover option.

Risk Aversion

It was assumed that the taxpayer's objective was to maximize the expected after-tax future value of the investment, which implicitly assumes that the taxpayer is risk-neutral. Many taxpayers are actually risk-averse and may be willing to accept a lower expected after-tax future value if the standard deviation of the possible after-tax future values is sufficiently lower. Incorporating risk aversion into the model would require specifying the trade-off that the taxpayer is willing to make between the mean and standard deviation of the possible after-tax future values. The effect of doing so will not necessarily make a Roth IRA contribution more attractive relative to a traditional IRA contribution or vice versa. For example, in the numerical example above, a traditional IRA contribution provided a higher expected after-tax future value than a Roth IRA contribution ($4,232 vs. $4,215). In addition, the standard deviation of the former is less than that of the latter ($698 vs. $707).[27] The traditional IRA thus provides in this example a larger expected return with less risk, even after taking into account future tax rate uncertainty.[28] However, a larger standard deviation for a traditional IRA could occur instead. In the numerical example, if σ_{Ri} were less than 0.06 (rather than equaling 0.108) and R_i and r_i were perfectly positively correlated, then the expected after-tax future values would be unchanged, but the Roth IRA's standard deviation would be less than the traditional IRA's standard deviation.

Minimum Distribution Rules

The model above assumes that the IRA balance would be distributed in year n, but the taxpayer's year n consumption needs are uncertain at the time of the IRA contribution decision. With a traditional IRA, the taxpayer is required to receive certain minimum distributions after attaining age 70½, but these rules do not apply for Roth IRAs. There is thus an embedded option associated with a Roth IRA: the option to retain in the IRA and grow tax-free funds that would have to be distributed if they were in a traditional IRA because of the minimum distribution rules. While this option is obviously relevant for a Roth IRA contribution, it is also relevant for a traditional IRA contribution because the IRA may someday be rolled over to a Roth IRA (either prior to or in year n).

Estate-Planning Considerations

Roth IRAs can have estate-planning advantages over traditional IRAs. For example, by "prepaying" the tax by forgoing a deduction, a Roth IRA can reduce the amount of tax on the taxpayer's estate (Lesser 1998). In addition, the lack of minimum distribution rules for Roth IRAs and the

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[27] These standard deviations were determined by simulation under the assumption that R_i and r_i are perfectly positively correlated. If R_i and r_i are instead assumed to be uncorrelated, then the standard deviation for a traditional IRA decreases to $630 and is unchanged for a Roth IRA.

A lower standard deviation for a traditional IRA than for a Roth IRA might seem counterintuitive because the former contains more random variables than the latter. It is true that the first term of Expression (13) has the same variance as all of Expression (12), and the other four terms add more variance. However, the variance of Expression (13) is also affected by two times any covariances between its terms, and there is a substantial negative covariance between the first term and the aggregate of the third, fourth, and fifth terms (i.e., a better return on the IRA's assets is associated with a larger tax payment when it is distributed). Likewise, there is also a negative covariance between the second term and the aggregate of the third, fourth, and fifth terms.

[28] One of the assumptions on which this is based is that the marginal tax rate for year i is independent of those for all other years. One source of future tax rate uncertainty is legislative changes, which could change expectations for subsequent tax rates. The effect of this type of future tax rate uncertainty is discussed above (i.e., t_i follows a random walk).

ability to contribute to them after attaining age 70½ can result in larger estates passing to heirs (S&S).

Nontax Costs

The traditional IRA's embedded rollover option not only confers expected tax benefits, but it also has nontax costs associated with it. One such cost is the opportunity cost of the taxpayer's (or tax adviser's) time that is needed to monitor the tax circumstances from year to year and determine whether a rollover is optimal in any particular year. In addition, a rollover may entail transaction costs to complete and will create additional complexity on the taxpayer's tax return. In situations such as the numerical example, where the traditional IRA's expected after-tax future value is only slightly higher than the Roth IRA's, the nontax costs may make the Roth IRA a better choice for the taxpayer.

CONCLUSION

There are many tax choices that have embedded options because they bring the flexibility to make subsequent decisions. Ignoring such an option might cause one to incorrectly reject the decision alternative with which the option is associated. One decision context that is especially amenable to being modeled is examined in the research here: the decision to contribute to a traditional or Roth IRA, where the embedded option arises from the opportunity to subsequently roll over a traditional IRA to a Roth IRA. This decision is also one that is of widespread interest to individuals, and this interest is likely to continue, and perhaps increase, as a result of recent tax legislation that increased IRA contribution limits and allow so-called Roth 401(k) plans beginning in 2006. In other tax decision contexts, the embedded options may be more difficult to quantify, but they nonetheless may still exist. If taxpayers value this flexibility and act accordingly, then tax researchers studying behavior in these contexts may be able to strengthen their empirical tests by taking these embedded options into account. Tax educators might find the IRA contribution decision to be a conducive setting in which to introduce their students to the notion of embedded tax options and the value of subsequent flexibility in many tax-planning decisions.[29]

APPENDIX
PROOF THAT EXPRESSION (13) IS GREATER THAN EXPRESSION (15)

For notational simplicity, define Φ_i (φ_i) to equal $1 + R_{\mu i}(1 + r_{\mu i})$. Expressions (13) and (15), respectively, are thus:

$$[C\Phi_1\Phi_2\Phi_3] + [Ct_0\varphi_1\varphi_2\varphi_3] - [C\Phi_1 E(t_1 \mid t_1 < t_1^*)\varphi_2\varphi_3 P_1]$$
$$- [C\Phi_1\Phi_2 E(t_2 \mid t_2 < t_2^*)\varphi_3(1 - P_1)P_2] - [C\Phi_1\Phi_2\Phi_3 t_{\mu 3}(1 - P_1)(1 - P_2)] \quad \text{(A.1)}$$

and:

$$[C\Phi_1\Phi_2\Phi_3] + [Ct_0\varphi_1\varphi_2\varphi_3] - [C\Phi_1\Phi_2\Phi_3 t_{\mu 3}] \quad \text{(A.2)}$$

Subtracting Expression (A.2) from Expression (A.1), the difference can be expressed as:

$$[t_{\mu 3}\Phi_2\Phi_3 - E(t_1 \mid t_1 < t_1^*)\varphi_2\varphi_3]C\Phi_1 P_1$$
$$+ [t_{\mu 3}\Phi_3 - E(t_2 \mid t_2 < t_2^*)\varphi_3]C\Phi_1\Phi_2(1 - P_1)P_2 \quad \text{(A.3)}$$

Recall from Expression (11) that t_1^* equals:

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[29] However, in a classroom setting, there is likely a great benefit to first analyzing the IRA contribution decision without the embedded rollover option (i.e., S&S's analysis) because of the complexity that the option adds. The instructor could then introduce the notion of the rollover option, examining it at an intuitive level or in a simplified mathematical form. The instructor could also augment the discussion by considering other tax contexts in which embedded tax options exist.

$$\left[E(t_2 \mid t_2 < t_2^*)\left(\frac{\Phi_2}{\varphi_2}\right)P_2\right] + \left[t_{\mu 3}\left(\frac{\Phi_2}{\varphi_2}\right)\left(\frac{\Phi_3}{\varphi_3}\right)(1 - P_2)\right] \quad (A.4)$$

Therefore, $t_{\mu 3}\Phi_2\Phi_3$ equals:

$$[t_1^*\varphi_2\varphi_3] - [E(t_2 \mid t_2 < t_2^*)\Phi_2\varphi_3P_2] + [t_{\mu 3}\Phi_2\Phi_3P_2] \quad (A.5)$$

Substituting Expression (A.5) into Expression (A.3) for $t_{\mu 3}\Phi_2\Phi_3$ and simplifying results in:

$$[t_1^* - E(t_1 \mid t_1 < t_1^*)]C\Phi_1\varphi_2\varphi_3P_1 + [t_{\mu 3}\Phi_3 - E(t_2 \mid t_2 < t_2^*)\varphi_3]C\Phi_1\Phi_2P_2 \quad (A.6)$$

Recall from Expression (8) that t_2^* equals:

$$t_{\mu 3}\left(\frac{\Phi_3}{\varphi_3}\right) \quad (A.7)$$

Therefore, $t_{\mu 3}\Phi_3$ equals $t_2^*\varphi_3$. Substituting $t_2^*\varphi_3$ into Expression (A.6) for $t_{\mu 3}\Phi_3$ and simplifying results in:

$$[t_1^* - E(t_1 \mid t_1 < t_1^*)]C\Phi_1\varphi_2\varphi_3P_1 + [\, t_2^* - E(t_2 \mid t_2 < t_2^*)]C\Phi_1\Phi_2\varphi_3P_2 \quad (A.8)$$

Since t_i^* is greater than $E(t_i \mid t_i < t_i^*)$, both bracketed terms in Expression (A.8) are positive. Since C, Φ_1, Φ_2, φ_2, φ_3, P_1, and P_2 are all positive, Expression (A.8) is positive, implying that Expression (A.1) is greater than Expression (A.2) and that Expression (13) is greater than Expression (15).

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