Bifurcation diagrams of the buck converter

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Abstract – The steady-state responses of the buck converter are determined by means of computer simulation and presented in a form of one-parameter bifurcation diagrams and two-parameter bifurcation diagrams. It is shown that analysed converter can be driven into chaos through a process of period doublings for each of the three chosen bifurcation parameters: the input voltage, the load inductance and the period of the external sawtooth ramp voltage. The results of simulation agree well with those obtained by measurements carried out on a physical model of the circuit.

Keywords: buck converter, chaos, period-doubling, bifurcation diagram, bifurcation parameters.

I. INTRODUCTION

Dc converters are time-varying nonlinear dynamical systems exhibiting several periodic steady state responses as well as chaotic behaviour [1-3]. The correct design of the dc converters assumes that all possible steady state responses and their dependence on variation of converter parameters are known in advance. In this way it is possible to avoid occurrence of some undesirable properties during service like increased output ripple or audible sound.

In the paper the buck converter studied by Chakrabarty et al is reexamined [4]. A special attention is paid to the one-and two-parameter bifurcation diagrams. Knowledge of these diagrams is important to the converter designer as it helps him, at the design stage, to choose converter parameters well inside the desired zone of operation and allows possible undesirable properties to be avoided.

All bifurcation diagrams are obtained by computer simulation. To evaluate the validity of the converter mathematical model the corresponding physical model of the buck converter has been built. For a fixed set of converter parameters and varying the input voltage only the zones of operation predicted by computer simulation are compared with the zones of operation obtained by measurements [5].

II. STATE EQUATIONS

The functional diagram of the buck converter is shown in Fig.1. The continuous conduction mode of operation is assumed. Thus the buck converter can be thought of as a piece-wise linear time-varying system which passes in a period of operation T through two different configurations.

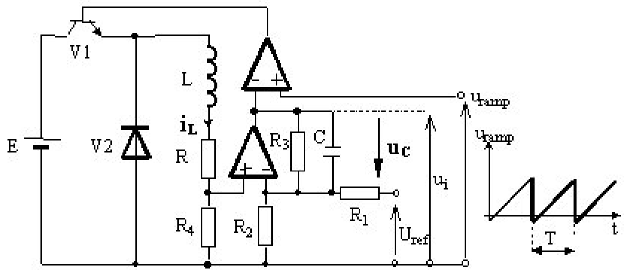

Fig.1. Functional diagram of the buck converter

When $u_{\text{ramp}} \geq u_i$, the controlled switch V_1 is in the ON-state and the state equations are:

$$\begin{aligned} \frac{du_C}{dt} &= \frac{U_{ref}}{CR_1} - i_L \frac{R_4}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \frac{u_C}{CR_3} \\ \frac{di_L}{dt} &= \frac{E}{L} - \frac{R+R_4}{L} i_L \end{aligned} \qquad (1)$$

where the inductor current i_L and feedback capacitor voltage u_C are state variables.

When $u_{\text{ramp}} < u_i$, the controlled switch V_1 is in the OFF-state and the state equations are:

$$\begin{aligned} \frac{du_C}{dt} &= \frac{U_{ref}}{CR_1} - i_L \frac{R_4}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \frac{u_C}{CR_3} \\ \frac{di_L}{dt} &= -\frac{R+R_4}{L} i_L \end{aligned} \qquad (2)$$

III. SIMULATION RESULTS

All simulations are carried out for a fixed set of converter parameters: R_1=1.8kΩ, R_2=220Ω, R_3=10kΩ, R_4=1Ω, R=13.8Ω, C=25nF, U_{ref}=5V. Three parameters are considered as bifurcation parameters: the input voltage E, the load inductance L and the period T=1/f of the external sawtooth ramp voltage (f- switching frequency). State equations are integrated by using the fourth-order Runge-Kutta method.

One-parameter bifurcation diagrams

Simulation results may be presented as steady-state waveforms of the state variables, as Poincaré maps, as trajectories in the phase plane or as bifurcation diagrams.

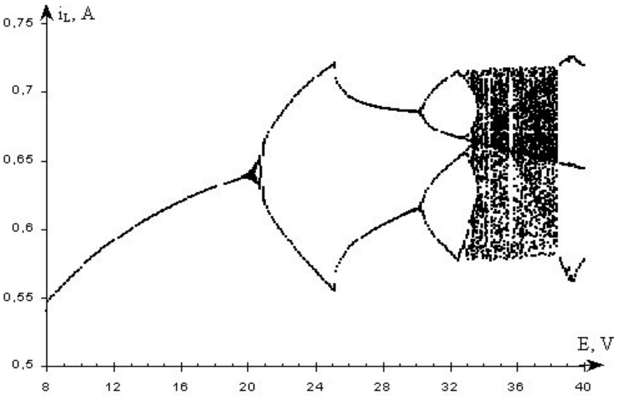

Fig.2. Bifurcation diagram with the input voltage as the bifurcation parameter (L=15 mH, T=330μs, ΔE=0.1V)

The most complete insight into behaviour of some dynamical system provide one-parameter bifurcation diagrams.

In all one-parameter bifurcation diagrams, presented here, 150 samples of inductor current $i_L(t_k)$ for each value of the parameter bifurcation parameter, taken at the instants $t_k=kT$; k=50,51,...200 are displayed. Though the buck converter is at steady state after a few periods, the safety margin of 50 periods is accepted. In this way we are sure that period-N operation of the converter will be represented by exactly N points on a bifurcation diagram. Also, 150 points dispersed randomly between the maximum and the minimum value of sampled variable i_L indicates the possible chaotic operation.

Fig.2. shows the one-parameter bifurcation diagram $[E,i_L(t_k)]$ as the input voltage E is varied from 8V to 40V, with the load inductance L and period of operation T fixed at 15 mH and 330μs respectively. It is seen that by increasing the input voltage the period-one operation bifurcates into period-two operation that bifurcates further into period-four operation.

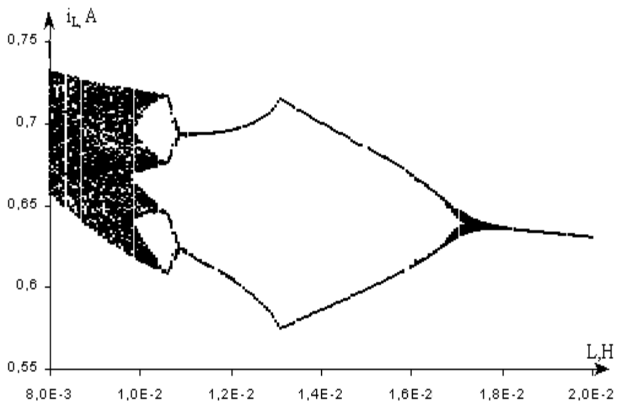

Fig.3. Bifurcation diagram with the load inductance as the bifurcation parameter (E=22V, T=330μs, ΔL=50μH)

A process of period doublings occurs which is one of the known routes to chaos [6]. Therefore we conclude that the random dispersion of points after 33V represents indeed chaotic behaviour of the converter. Also, a property characterising system which exhibit period-doubling route to chaos is obtained, that is occurrence of period-odd operation. In our case, period-three operation starts at 38,5V as it is shown in Fig.2.

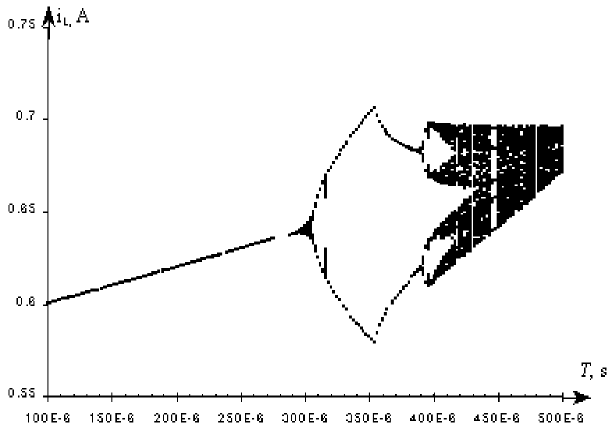

Fig.4. Bifurcation diagram with the period of external sawtooth ramp voltage as the bifurcation parameter (E=22V, L=15mH, ΔT=1μs)

Figs.3 and 4. show the one-parameter bifurcation diagram $[L,i_L(t_k)]$ and the one-parameter bifurcation diagram $[T,i_L(t_k)]$ respectively. A process of period doublings is recognised easily in the both bifurcation diagrams.

Two-parameter bifurcation diagrams

From converter designer's point of view the most important objective is to choose converter parameters in a such way that under any expected circumstances and independently on variations of parameters the desired operating regime is not changed. So, for instance, if the desired operating regime of a particular converter is the period-one operation, the designer should choose the converter parameters well inside the zone of period-one operation. But, there is a strong dependence of the one-parameter bifurcation diagram structure on other fixed converter parameters. Therefore, correct decision is hardly possible if the designer has only one-parameter bifurcation diagrams on his disposal.

By uniting two one-parameter bifurcation diagrams into one two-parameter bifurcation diagram much clearer picture about the zones of operation is obtained and the correct choice of converter parameters is facilitated. Obviously, in applications there are several parameters influencing the bifurcation behaviour of the converter and large number of two-parameter bifurcation diagrams is possible and necessary. In our study of the considered buck converter we choose three parameters as bifurcation parameters only: the input voltage E, the load inductance L and the period T of the external sawtooth ramp voltage.

Therefore three two-parameter bifurcation diagrams may be created. They are shown in Figs.5-7.

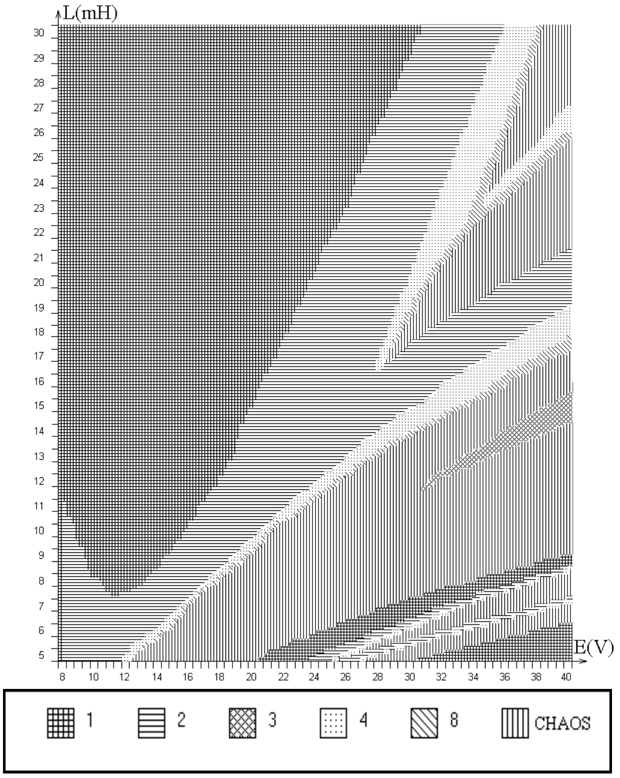

Fig.5. Input voltage vs. load inductance (*EL*) bifurcation diagram (T=330μs) representing zones of period-N operation and chaos (N=1,2,3,4,8)

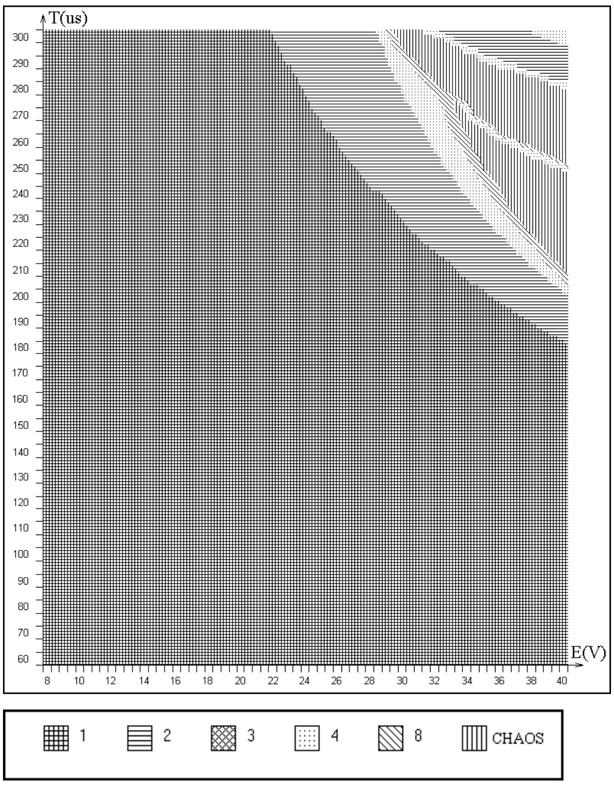

Fig.6. Input voltage vs. period of the external sawtooth ramp voltage (*ET*) bifurcation diagram (L=15mH) representing zones of period-N operation and chaos (N=1,2,3,4,8)

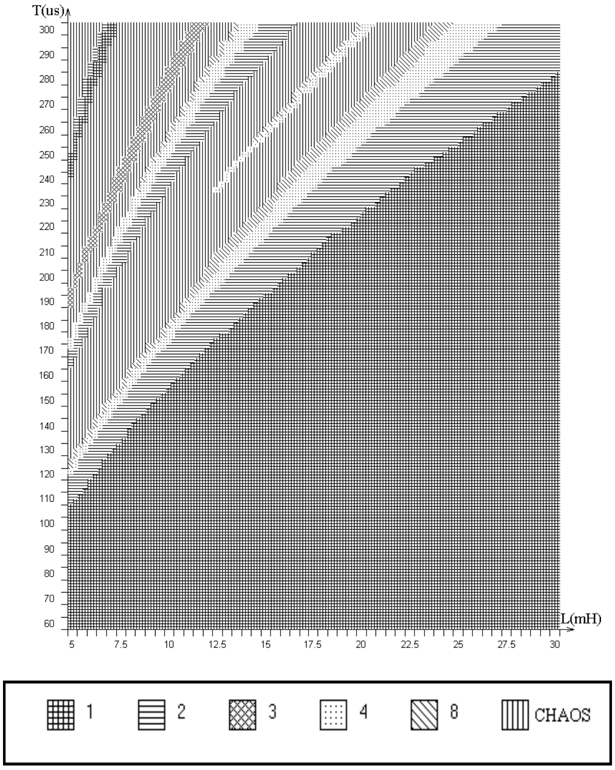

Fig.7. Load inductance vs. period of the external sawtooth ramp voltage (*LT*) bifurcation diagram (E=22V) representing zones of period-N operation and chaos (N=1,2,3,4,8)

IV. EXPERIMENTS

A physical model of the buck converter is built. The low switching frequency f=3kHz (T=330μs) is chosen to minimise the parasitic effects, making the converter closer to ideal and therefore easier to compare measurement results with simulation results.

By increasing the input voltage E from 8V to 40V gradually the bifurcation points are detected by observing at particular values of the input voltage the abrupt changes of the trajectories in the i_L-u_C phase plane. Measurement results and results obtained from the computed one-parameter bifurcation diagram $[E,i_L(t_k)]$, Fig.2, agree very well as it is shown in Table 1.

TABLE I. STEADY STATE REGIONS

Steady states	*E*,V	
	measured values	**computed values**
Period-one operation	8-20	8-19.8
Period-two operation	20-30	19.8-30.3
Period-four operation	30-33	30.3-32.77
Period-eight operation	33-33.3	32.77-33.31
Chaos	33.3-38.6	33.31-38.5
Period-three operation	38.6-40	38.5-40

V. CONCLUSIONS

A reasonable accordance between bifurcation points obtained from computed one-parameter bifurcation diagram [E,$i_L(t_k)$] and measured bifurcation points implies the validity of the chosen mathematical model of the buck converter. It can be used to predict all possible steady state responses occurring actually in applications.

Two-parameter bifurcation diagrams derived from one-parameter bifurcation diagrams give to the designer a deeper insight into the zones of operation. Thus the designer can choose the converter parameters in a such way that the desired operating point of the converter is placed at the safe distance from the zone boundaries irrespectively to the variations of converter parameters.

VI. REFERENCES

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