TRAJECTORY TRACKING WITH SMC TECHNIQUE UNDER LOAD DISTURBANCE: EXPERIMENTAL RESULTS.

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Abstract This paper deals with the application of a sliding mode control (SMC) in tracking systems with trajectory generation. The possibilities of using this kind of VSS strategy on tracking control with reference trajectory generation under load disturbance and variable inertia is tested. The practical design of such robust sliding mode controller is discussed. Control law is obtained by using the methodology of balance condition. In order to achieve a tracking with quaranteed precision, smoothing of control function in constant and variable boundary layer are compared. Laboratory setup for experimental verification are realised. It consists of host system (based on MC 68332) and two mechanically coupled synchronous motor permanent magnet (SMPM), where one motor is load for another one. Economical loading is used, connecting the DC Busbars of both converters together so only a minimum energy for losses are consummated.

Keywords. Sliding mode control, robust control, balance condition, tracking system, trajectory generation, economical loading.

INTRODUCTION

The sliding mode is a special case of Variable Structure Systems (VSS) and keeps invariant trajectory of moving under different plants uncertainties. It is especially suited for systems where robustness is a crucial performance requirement. Many practical applications [4], [5], [6] confirmed that the robust nature is quaranteed by sliding mode. This nature is achieved with control algorithm very simple and easy to implement in real time computer control systems. The main drawback of sliding mode is that the resulting control input is discontinuous on the switching surface and, consequently, the control input chatters at a theoretically infinite frequency. Chattering is highly undesirable, since it involves extremely high control activity, and furthermore may excite high-frequency dynamics neglected in the course of modelling. To overcome this problem, the discontinuous function is replaced by a proper control function which consists of continuos part, i.e. equivalent control [1], [3], [7], and discontinuous part (relay type component). In addition, discontinuous part in control input is replaced by continuos one in thin boundary layer. According to balance condition, boundary layer thickness can be made time-varying [2], [3], [4], [6]. In that case one can specify the best attainable tracking performance, given the desired control bandwidth and the extent of load variations and parameter uncertainty.

DESIGN OF SLIDING MODE CONTROLLER

For the system in Fig. 1. we find that is:

$$\varepsilon_d - \varepsilon = x_1 \ , \quad -k_v \dot{\varepsilon} / k_\varepsilon = x_2 ,$$

$$k_{DA}\, k_{zi}\, k_m\, u(t) - m_t(t) = -\dot{x}_2\, J / k_\omega \ , \qquad (1)$$

and eliminating x_2 we can show that is:

$$\ddot{\varepsilon} = \frac{k_{DA}\, k_{zi}\, k_m\, k_\varepsilon}{J} u(t) - \frac{k_\varepsilon}{J} m_t(t) \ , \qquad (2)$$

where ε is actual position, $u(t)$ control input, $m_t(t)$ load torque, $G_{zi}=k_{zi}/(1+pT_{zi}) \approx k_{zi}$ transfer function $i_q(p)/i_q^*(p)$, k_ε coefficient in position feedback loop, k_{DA} D/A converter constant, k_m torque constant and J inertia of the drive. Taking that $k_1=k_{DA}\, k_{zi}\, k_m\, k_\varepsilon /J$ and $k_2=k_\varepsilon/J$, we have from (1) and (2):

$$\ddot{\varepsilon} = k_1\, u(t) - k_2 m_t(t). \qquad (3)$$

With $\tilde{\varepsilon} = \varepsilon - \varepsilon_d$ and $\dot{\tilde{\varepsilon}} = \dot{\varepsilon} - \dot{\varepsilon}_d$ sliding function is defined as [2]:

$$s=\left(\frac{d}{dt}+\lambda\right)\tilde{\varepsilon} = \dot{\tilde{\varepsilon}} + \lambda\tilde{\varepsilon} = \dot{\varepsilon} - \dot{\varepsilon}_d + \lambda\tilde{\varepsilon} \ , \qquad (4)$$

where $\tilde{\varepsilon}$, ε_d are error position and desired position, $\dot{\tilde{\varepsilon}}$, $\dot{\varepsilon}_d$ are error speed and desired speed. For continuos part of control input based on equivalent control (u_{eq}, s=0), using (3) and (4), we find the best approximation $\hat{u}$ for equivalent control:

$$\hat{u}=\left(\hat{k}_2\, \hat{m}_t + \ddot{\varepsilon}_d - \lambda\dot{\tilde{\varepsilon}}\right) , \qquad (5)$$

where $\hat{u}, \hat{k}_1, \hat{k}_2, \hat{m}_t$ are estimated values of continuos part (equivalent control) u_{eq}, coefficients k_1, k_2 and load torque m_t respectively. Then, we find the total control input as:

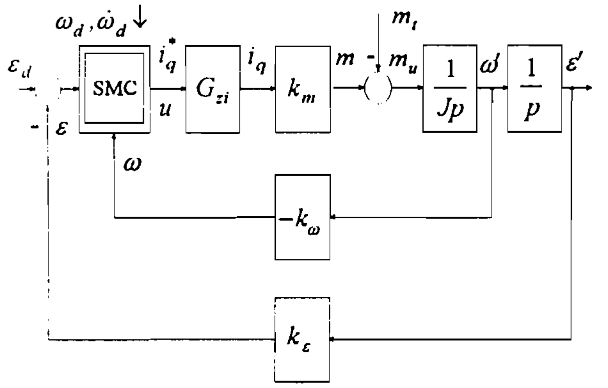

Fig.1. The simplified model of SMPM with SMC

$$u = \frac{1}{\hat{k}_1}\left(\hat{u} - k\, sat\frac{s}{\Phi}\right) \quad , \tag{6}$$

where Φ is boundary layer thickness where control function is linearized, and function $sat(s)$ is defined as:

$$sat\frac{s}{\Phi} = \begin{cases} sgn\frac{s}{\Phi} \quad za \left|s\right| > \Phi \; , \\ \frac{s}{\Phi} \quad za \left|s\right| < \Phi \; . \end{cases} \tag{7}$$

The coefficient k in (6) is found consider Lyapunov function as condition for sliding mode existence. For constant boundary layer thickness the value of k which quarantees that the state trajectories be always directed towards the sliding surface S is:

$$k \geq \beta\,(\,M + \eta\,) + (\,\beta - 1\,)\left|\,\hat{u}\,\right| \, , \tag{8}$$

where η is positive constant. If load torque is estimated with m_t, coefficient M has to be chosen to satisfy $\left|k_2\, m_t\,(t) - \hat{k}_2 \hat{m}_t(t)\right| \leq M$. Conversely, without load estimation we specify $\left|k_2 m_t(t)\right| \leq M$. Parameter uncertainty is defined with:

$$\beta = \sqrt{\frac{k_{1max}}{k_{1min}}} = \sqrt{\frac{J_{max}}{J_{min}}} = \left(\frac{\hat{k}_1}{k_1}\right)_{max} \quad , \tag{9}$$

so that:

$$\frac{1}{\beta} \leq \frac{\hat{k}_1}{k_1} = \frac{\hat{J}}{J} \leq \beta \quad \text{and}$$

$$\hat{k}_1 = \sqrt{k_{1max} k_{1min}} = \sqrt{J_{max}\, J_{min}} \; . \tag{10}$$

Equation (9) and (10) show that we take in account only uncertainty of J while other parameter uncertainties in k_l are neglected. For variable boundary layer coefficient k in (6) has to be changed in $\bar{k}$ and defined as [2], [3], [6], [8]:

$$\bar{k} = k(\Theta, t) - k\left(\Theta_d\,, t\right) + \lambda\Phi / \beta \quad , \tag{11}$$

where $k(\Theta,\, t)$ and $k(\Theta_d,\, t)$ are coefficients expressed in depends of actual and desired state respectively and $\Theta = \left[\varepsilon,\, \dot{\varepsilon},\, \ddot{\varepsilon}\right]^T$ and $\Theta_d = \left[\varepsilon_d,\, \dot{\varepsilon}_d,\, \ddot{\varepsilon}_d\right]^T$.. Dynamics of Φ is described as follows:

$$for \quad k(\Theta, t) < \frac{\lambda\Phi}{\beta} \;\Rightarrow\; \dot{\Phi} + \frac{\lambda\Phi}{\beta^2} = \frac{k(\Theta, t)}{\beta} \, , \tag{12}$$

$$for \quad k(\Theta, t) > \frac{\lambda\Phi}{\beta} \;\Rightarrow\; \dot{\Phi} + \lambda\Phi = \beta\, k(\Theta, t) \, , \tag{13}$$

and λ is chosen taking into account the frequency range of unmodelled high frequency dynamics. With desired control bandwidth and specified extent of uncertainty (load disturbance and parameter uncertainty), one can take the best tracking performance according to balance condition, [2]:

$$\lambda^2\, \tilde{\varepsilon}_{max} = \beta\, k\left(\Theta_d, t\right) \, , \tag{14}$$

where $\tilde{\varepsilon}_{max} = \Phi/\lambda$ is maximal position error.

SIMULATION RESULTS

Using software package MATRIXx-System Build/WS simulation model of trajectory tracking control system with SMPM, based on simplified mathematiçal model, is realised. All parameters needed for simulation model are identificated in [8]. In the case when motor M2 is not supplied from his amplifier (No.2), motor M1 is unloaded. But, increasing the speed of the drive, the losses in motor M2 becomes significant and they make a load for main drive even motor M2 is not electrically connected. This additional load is accounted in simulation model. For testing sliding mode control algorithms, desired trajectory in Fig.2. are chosen. Control algorithm is based on relations (6). to (13). According to relation (7) control function

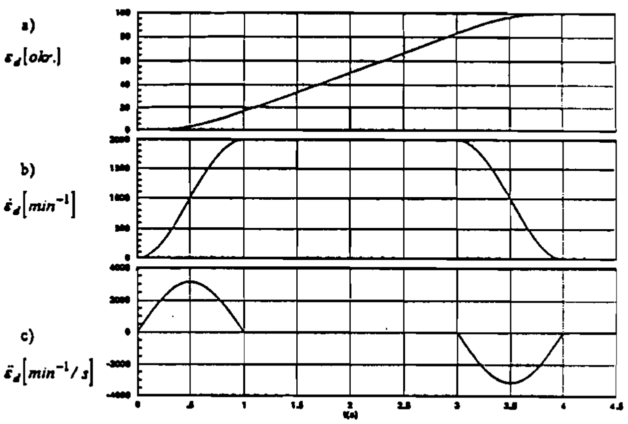

Fig.2. Reference (desired) trajectory, position a), speed b), acceleration c).

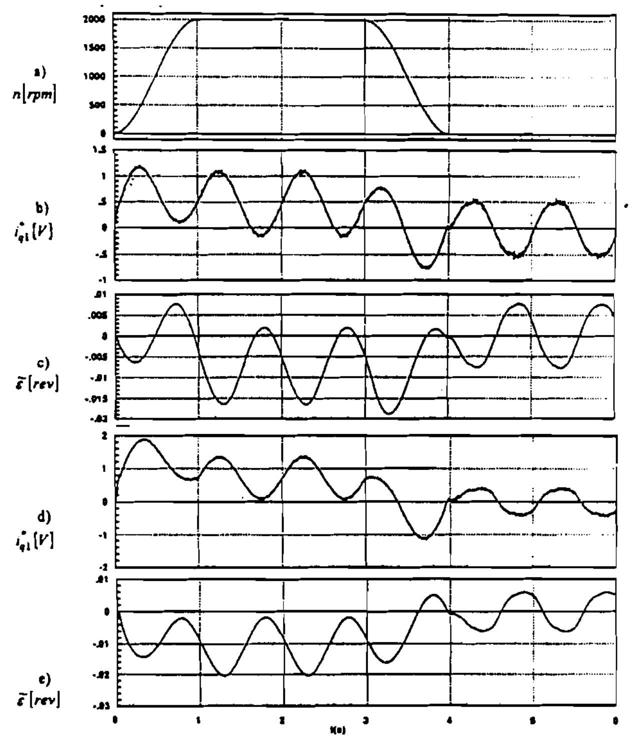

Fig.3. Speed a), control input b) and position error c) for J=7.5kgcm^2 ; control input d) and position error e) for J=16kgcm^2. Sinusoidal load, max. value 0.1M$_n$ of main drive

is linearized in variable boundary layer $\Phi=\Phi(t)$. Because the inertia of a load in laboratory setup are invariable. it is much convenient to analyse the influence of inertia variations on tracking accuracy through simulation process. Fig.3. shows speed, control input and position error for J=16kgcm^2 (with coupled motors M1 and M2) and the same variables for J=7.5kgcm^2 (only main drive M1). The tracking system is exposed under additional sinusoidal load disturbance, max. value 10% nominal torque of main drive. Increasing inertia load more then twice, position error practically does not change. There is no chattering phenomena in control input, thanking to smoothing of control function in variable boundary layer. Although load torque is symmetrical around zero ordinate axes for all the time (sinusoidal current reference i^*_{q2} for motor M2), position error is symmetrical around zero ordinate only when the speed is zero. For no–zero speed, position error is greater when the load torque originating from losses (friction in bearings, hysteresis losses) and sinusoidal load torque from motor M2 have the same direction. The losses in dependence of speed for both SMPM are identificated and built in simulation model.

EXPERIMENTAL RESULTS

Experimental setup for sliding mode control algorithms evaluation is realised, Fig.4. It consists of two mechanically coupled SMPM with two identical amplifiers (voltage and frequency converters) and host system where sliding mode control algorithms are realised. Rated data for main drive (M1) are M_0=6Nm, I_0=21A, P=2.82kW, n=6000rpm, and for load (M2)

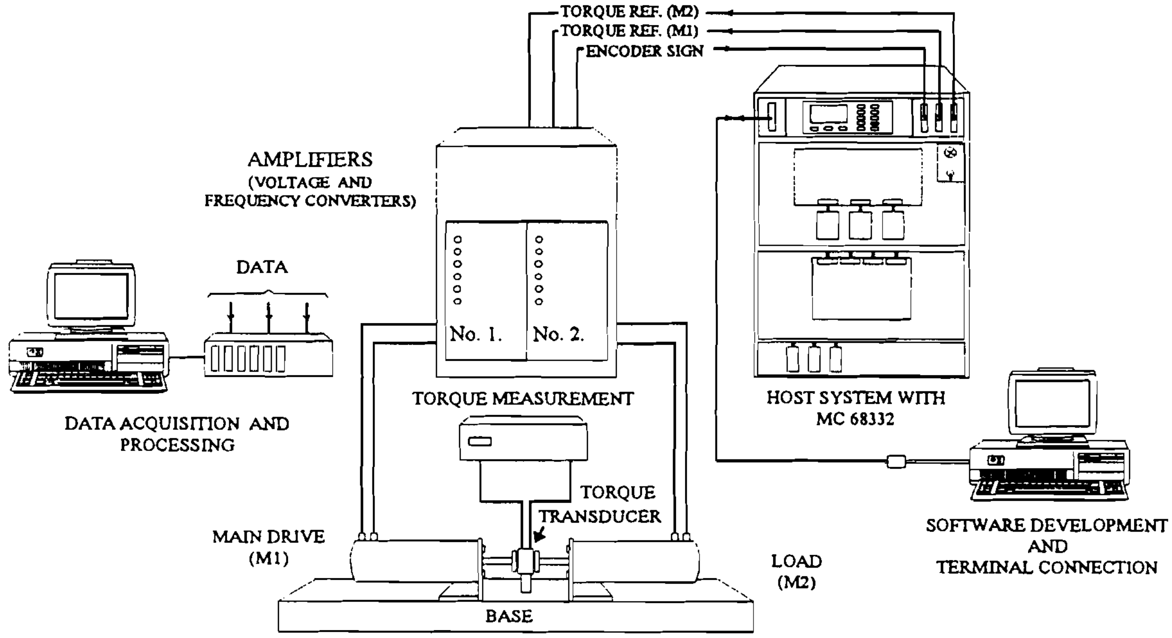

Fig.4. Laboratory setup for experimental verification.

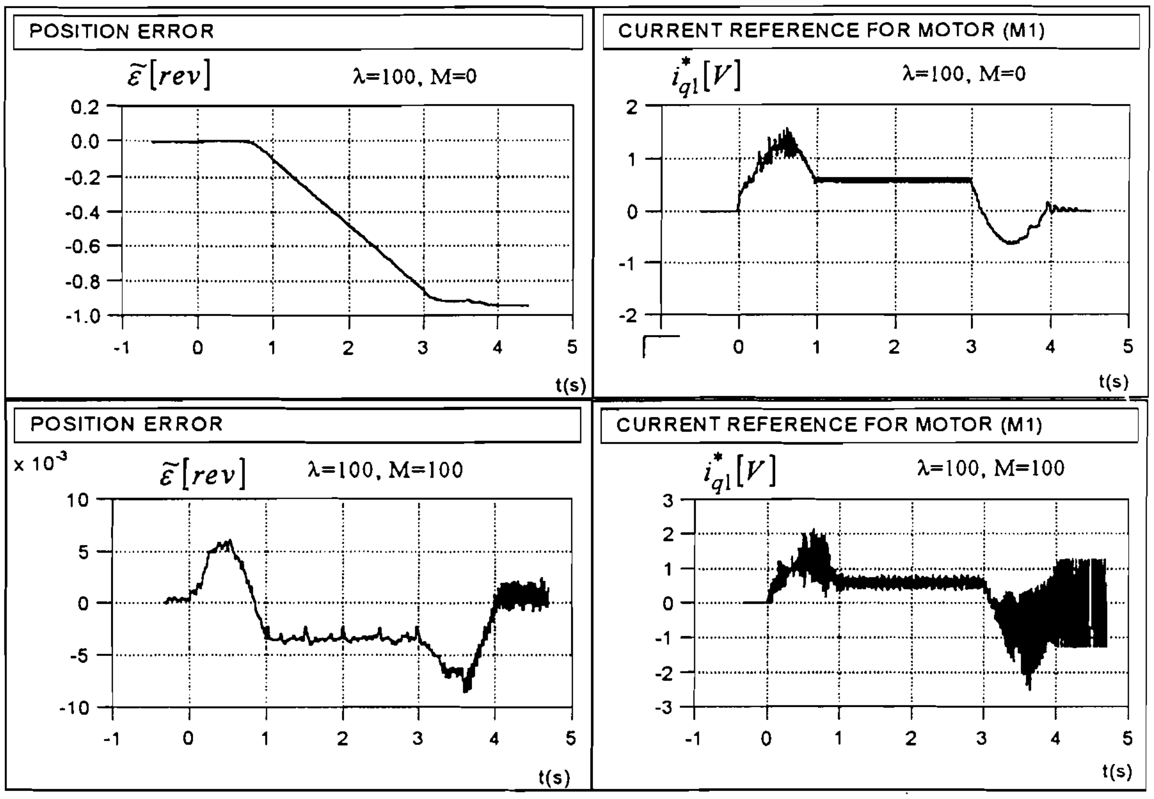

Fig. 5. Position error and current reference without load with λ=100, M=0 (upper row) and λ=100, M=100 (lower row) for constant boundary layer - experimental results.

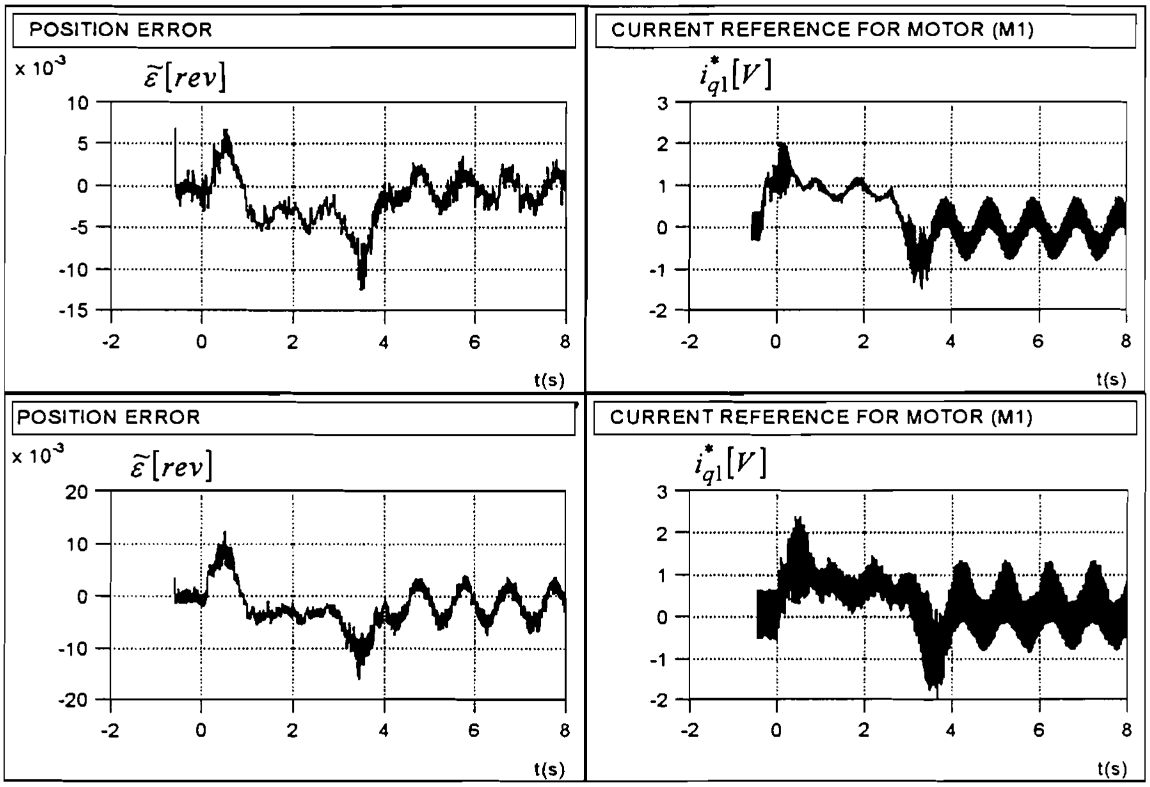

Fig. 6. Position error and current reference with sinusoidal load (max. load M_t=0.1M_n), λ=200, M=200. Variable layer (upper row) and constant layer Φ=200 (lower row) - experimental results.

M_0=3.8Nm, I_0=6.5A, P=1.2kW, n=4000rpm. Amplifiers are current (torque) controlled, i.e. the outer controlled loop is current loop. Host system generate the current reference for the main (M1) drive's amplifier, which is control function from sliding mode algorithms. The second (M2) drive's amplifier also received the current reference (load torque reference) from host system and this reference is programmable at will. Economical loading is used, connecting the DC link of both converters together so only a minimum energy for losses are consummated. The main part of energy is returned in DC link.

Host system is based on microcontroller MC 68332. It's equipped with two basic 12-bit D/A converters and up to four expanded input/output units. Each of expanded units can have one of two programmable option: 2x12-bit D/A channels or 4x8-bit D/A channels. Furthermore, hardware communication module with communication interface (with software support) enable "vertical" and "horizontal" communication (optical fibber and RS232/RS485 communication). Hardware and software design are based on modularity concept. The sliding mode algorithms are realised by means of graphically oriented computer language.

Because of very stiff mechanical coupling between two motors, and very low neglected time constant T_{zi}=0.25ms, the stringent limitation of λ is sampling time (1ms), and one can take λ_{max}=250-300Hz, [8].

It is not possible to compensate error position in steady state (in final position) in motion without load torque, using only parameter λ. Parameter M has to be added (Fig. 5). This is reasonable, because there is no "no load operation". The losses in SMPM M2 are always present and they increase with higher speed and make a load for main SMPM (M1). Adding parameter M in coefficient k (see expression (8)), position error is considerable reduced, but, for a consequence, chattering in control function us higher.

For variable boundary layer thickness $\Phi(t)$ is variable according to load torque variations making low chattering in control input comparing with constant boundary layer. For same parameter λ and M, for variable boundary layer, the chattering in control input are smaller at the practically same position error (Fig.6).

CONCLUSION

The result of experimentally investigations trajectory tracking control system with SMPM confirmed that SMC offer a good robustness to parameter variations (inertia) and load torque disturbance using very simple control algorithm. It is showed, for the case of no load operation, that control algorithm which use a expression for k (8) without coefficient for load torque compensation M, can not give a satisfactory result. It is especially important concerning position error. Adding coefficient M control chattering and, as a consequence, noise increase. If the level of noise is impermissible, control algorithm must be modified introducing a load torque observer. This will reduce the value of discontinuity part of control function and consequently noise and torque oscillations.

Using variable boundary layer one can get better performance of controlled system comparing with constant boundary layer algorithm. Chattering in control function decrease for practically same position error.

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