EXPERIMENTAL RESULTS OF TRAJECTORY TRACKING SYSTEM WITH SMC TECHNIQUE UNDER SINUSOIDAL AND STEP LOAD DISTURBANCE

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Abstract

Practical design of a sliding mode control (SMC) in tracking systems with trajectory generation is presented. Sliding mode controller, based on balance condition, is designed under load disturbance and variable inertia drive. The results are compared for control function smoothed in constant and variable boundary layer. Laboratory setup for experimental verification consists of host system (based on MC 68332) and two drive controllers with mechanically coupled synchronous motor permanent magnet (SMPM), where one motor is load for another one.

1. Introduction

For many applications where robustness is a crucial performance requirement, sliding mode is a practical solution [4], [5]. SMC is a special case of Variable Structure Systems (VSS) where trajectory of moving is invariant under different plants uncertainties. This is achieved with control algorithm very simple and easy to implement in real time computer control systems. Control input is discontinuous on the switching surface and, consequently, chatters at a theoretically infinite frequency. Chattering is highly undesirable, since it involves extremely high control activity, and furthermore may excite high-frequency dynamics neglected in the course of modeling. To overcome this problem, the discontinuous function is replaced by a proper control function which consists of continuos part, i.e. equivalent control and discontinuous part (relay type component), [1], [3]. In addition, discontinuous part in control input is replaced by continuos one in thin boundary layer. According to balance condition, boundary layer thickness can be made time-varying [2], [3], [4]. In that case one can specify the best attainable tracking performance, given the desired control bandwidth and the extent of load variations and parameter uncertainty.

2. Design of sliding mode controller

From tracking system's model in Fig.1. we find that is:

$$\varepsilon_d - \varepsilon = x_1 \;, \quad -k_v \dot{\varepsilon} / k_\varepsilon = x_2 ,$$

$$k_{DA}\, k_{zi}\, k_m\, u(t) - m_t(t) = -\dot{x}_2\, J / k_\omega \;, \tag{1}$$

and eliminating x_2 we can show that is:

$$\ddot{\varepsilon} = \frac{k_{DA}\, k_{zi}\, k_m\, k_\varepsilon}{J} u(t) - \frac{k_\varepsilon}{J} m_t(t) \;, \tag{2}$$

where ε is actual position, $u(t)$ control input, $m_t(t)$ load torque, $G_{zi}=k_{zi}/(1+pT_{zq}) \approx k_{zi}$ transfer function $i_q(p)/i_q^*(p)$, k_ε coefficient in position feedback loop, k_{DA} D/A converter constant, k_m torque constant and J inertia of the drive. Taking that $k_1=k_{DA}\, k_{zi}\, k_m\, k_\varepsilon/J$ and $k_2=k_\varepsilon/J$, we have from (1) and (2):

$$\ddot{\varepsilon} = k_1 u(t) - k_2 m_t(t). \tag{3}$$

With $\tilde{\varepsilon} = \varepsilon - \varepsilon_d$ and $\dot{\tilde{\varepsilon}} = \dot{\varepsilon} - \dot{\varepsilon}_d$ sliding function is defined as [2]:

$$s = \left(\frac{d}{dt} + \lambda\right)\tilde{\varepsilon} = \dot{\tilde{\varepsilon}} + \lambda \tilde{\varepsilon} = \dot{\varepsilon} - \dot{\varepsilon}_d + \lambda \tilde{\varepsilon} \;, \tag{4}$$

where $\tilde{\varepsilon}$, ε_d are error position and desired position, $\dot{\tilde{\varepsilon}}$, $\dot{\varepsilon}_d$ are error speed and desired speed. For continuos part of control input based on equivalent control (u_{eq}, s=0), using (3) and (4), we find the best approximation $\hat{u}$ for equivalent control:

$$\hat{u} = \left(\hat{k}_2\, \hat{m}_t + \ddot{\varepsilon}_d - \lambda \dot{\tilde{\varepsilon}}\right) , \tag{5}$$

where $\hat{u}, \hat{k}_1, \hat{k}_2, \hat{m}_t$ are estimated values of continuos part (equivalent control) u_{eq}, coefficients k_1, k_2 and load torque m_t respectively. Then, we find the total control input as:

$$u = \frac{1}{\hat{k}_1}\left(\hat{u} - k\, sat\frac{s}{\Phi}\right) \;, \tag{6}$$

where Φ is boundary layer thickness where control function is linearized, and function $sat(s)$ is defined as:

$$sat\frac{s}{\Phi} = \begin{cases} sgn\dfrac{s}{\Phi} & za\ |s| > \Phi \;, \\ \dfrac{s}{\Phi} & za\ |s| < \Phi \;. \end{cases} \tag{7}$$

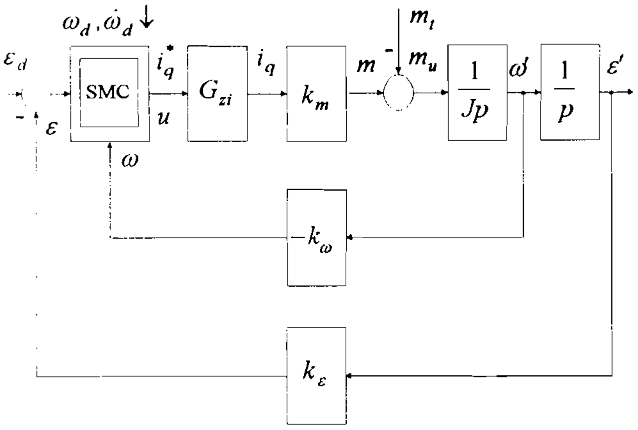

Fig.1. The simplified model of SMPM with SMC

The coefficient k in (6) is found consider Lyapunov function as condition for sliding mode existence. For constant boundary layer thickness the value of k which quarantees that the state trajectories be always directed towards the sliding surface S is:

$$k \ge \beta (M+\eta)+(\beta -1)|\hat{u}| , \tag{8}$$

where η is positive constant. If load torque is estimated with $\hat{m}_t$, coefficient M has to be chosen to satisfy $|k_2 m_t(t) - \hat{k}_2 \hat{m}_t(t)| \le M$. Conversely, without load estimation we specify $|k_2 m_t(t)| \le M$. Parameter uncertainty is defined with:

$$\beta = \sqrt{\frac{k_{1max}}{k_{1min}}} = \sqrt{\frac{J_{max}}{J_{min}}} = \left(\frac{\hat{k}_1}{k_1}\right)_{max} , \tag{9}$$

so that:

$$\frac{1}{\beta} \le \frac{\hat{k}_1}{k_1} = \frac{\hat{J}}{J} \le \beta \quad , \quad \hat{k}_1 = \sqrt{k_{1max} k_{1min}} = \sqrt{J_{max} J_{min}} \tag{10}$$

Equation (9) and (10) show that we take in account only uncertainty of J while other parameter uncertainties in k_1 are neglected. For variable boundary layer coefficient k in (6) has to be changed in $\bar{k}$ and defined as [2], [3], [6]:

$$\bar{k} = k(\Theta, t) - k(\Theta_d, t) + \lambda\Phi / \beta , \tag{11}$$

where $k(\Theta, t)$ and $k(\Theta_d, t)$ are coefficients expressed in depends of actual and desired state respectively and:

$$\Theta = [\varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}]^T , \ \Theta_d = [\varepsilon_d, \dot{\varepsilon}_d, \ddot{\varepsilon}_d]^T . \tag{12}$$

Parameter λ is chosen taking into account the frequency range of unmodeled high frequency dynamics. With desired control bandwidth and specified extent of uncertainty (load disturbance and parameter uncertainty), one can take the best tracking performance according to balance condition, [2]:

$$\lambda^2 \tilde{\varepsilon}_{max} = \beta \, k(\Theta_d, t) , \tag{13}$$

where $\tilde{\varepsilon}_{max} = \Phi/\lambda$ is maximal tracking error.

4. Experimental results

Experimental setup consists of two mechanically coupled SMPM with two identical amplifiers (drive controllers) and host system where sliding mode control algorithms are realized, Fig.2. Rated data for main drive (M1) are M_0=6Nm, I_0=21A, P=2.82kW, n=6000rpm, and for load (M2) M_0=3.8Nm, I_0=6.5A, P=1.2kW, n=4000rpm.

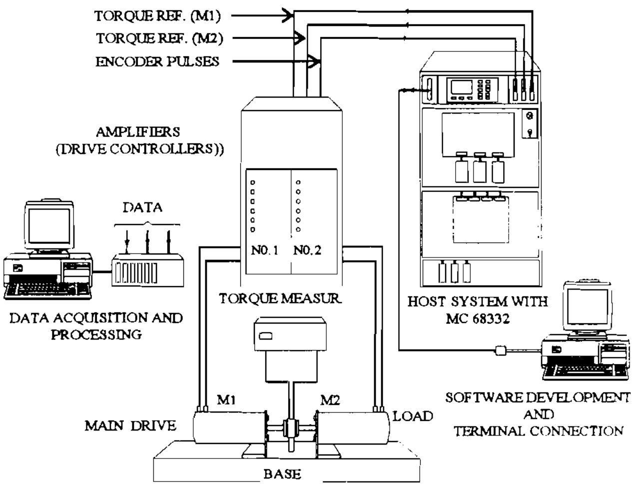

Fig.2. Laboratory setup for experimental verification.

Drive controllers are current (torque) controlled, i.e. the outer controlled loop is current loop. Host system generates the current reference i_{q1}^* for the main (M1) drive (control function from sliding mode algorithm) and load torque reference (i_{q2}^*) for drive M2. This reference is programmable at will. Economical loading is used, connecting the DC link of both converters together so only a minimum energy for losses are consummated. The main part of energy is returned in DC link.

Host system is based on microcontroller MC 68332. It's equipped with two basic 12-bit D/A converters and up to four expanded input/output units. Each of expanded units can have one of two programmable option: 2x12-bit D/A channels or 4x8-bit D/A channels. Furthermore, hardware communication module with communication interface (with software support) enable "vertical" and "horizontal" communication (optical fibber and RS232/RS485 communication). Hardware and software design are based on modularity concept. The sliding mode algorithms are realized by means of graphically oriented computer language.

Because of very stiff mechanical coupling between two motors and very low neglected time constant T_{zi}=0.25ms, the stringent limitation of λ is sampling time (1ms), and one can take λ_{max}=250-300Hz, [6]. For testing sliding

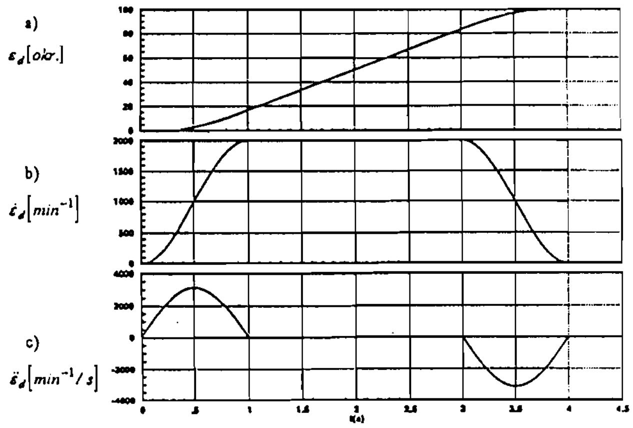

Fig.3. Reference (desired) trajectory, position a), speed b), acceleration c).

mode control algorithms, desired trajectory in Fig.3. is chosen.

Practical evaluation of balance condition (13) is presented in Fig.4. Sliding mode control algorithm with constant boundary layer is applied. For the constant extent of uncertainty (right side of equation (13)), tracking accuracy is better for higher λ. For $\lambda > \lambda_{max}$ chattering in control input is unacceptable, and causes undesirable motor's shaft vibrations. Practically same curve is obtained using algorithm with variable boundary layer, but control input chattering is smaller, [6].

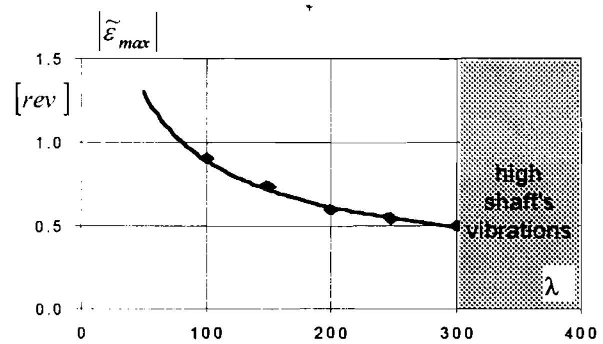

Fig.4. Tracking error as a function of λ , M=0.

In this experiment the term "no-load operation" defines the state when the current reference for drive controller M2 is zero. This doesn't mean that there is no load because the losses in SMPM M2 are always present and they increase with higher speed making a load for main SMPM (M1), Fig.5. According to this, it is not possible to compensate large tracking error using only parameter λ. Parameter M has to be added. Adding parameter M in coefficient k (see

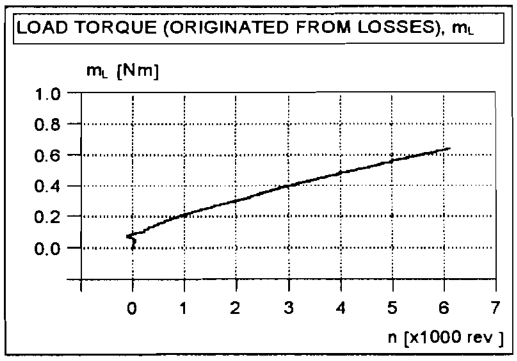

Fig.5. Load torgue (originated from losses)

expression (8)), tracking error is considerable reduced, but, for a consequence, chattering in control function is higher, Fig.6.

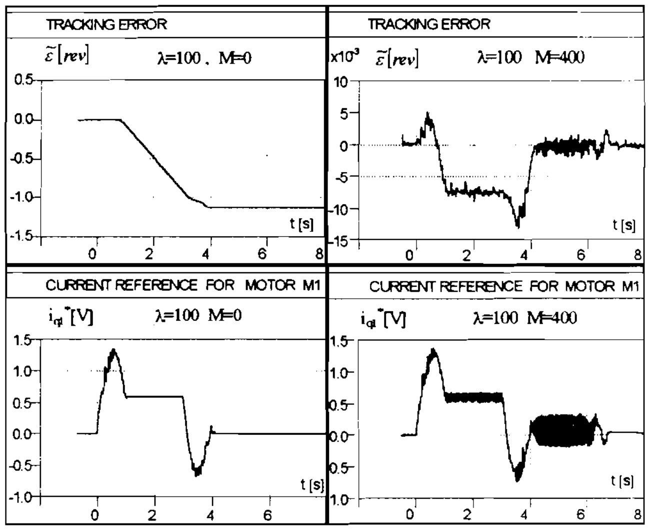

Fig. 6. Tracking error and current reference with $i_{q2}^* = 0$. λ=100, M=0 (left column) and λ=100, M=400 (right column); variable boundary layer.

Comparing constant and variable boundary layer, for same parameter λ and M, chattering in control input for variable boundary layer is smaller at the practically same tracking error, [6]. However, taking large parameters λ and M, oscillations in final position are present. It is specially case when synchronous motor has large reluctant torques (due to design features).

Experimental results with sinusoidal load are presented in Fig.7. It's obvious that the worst case concerning chattering in control input and noise due shaft's vibrations is in no-load operation. The large load disturbance act's as a damper for mechanical vibrations and enables the use of large parameter λ and M. For low values of sinusoidal load, impact of load torque originated from losses is visible in current reference (left column).

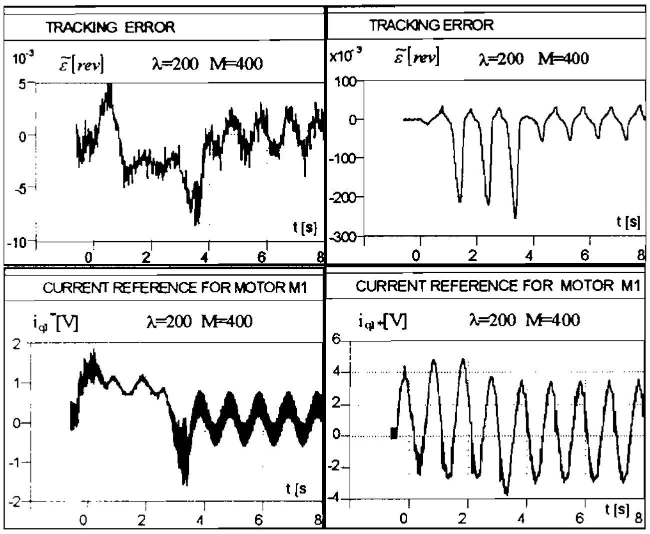

Fig.7. Experimental results with sinusoidal load; m_t=0.08$M_n$sin$t$ (left column) and m_t=0.65$M_n$sin$t$ (right column)

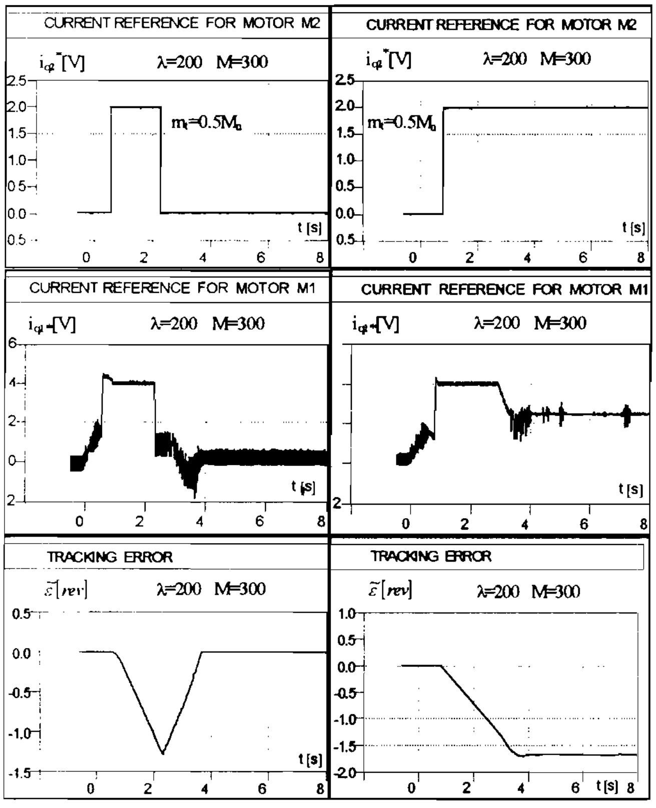

Fig.8. Experimental results with step load disturbance; with limited duration, "impulse" loading (left column), and unlimited step load duration (right column)

For sliding mode existence, disturbance function has to be limited only on infinitely derivative functions (e.g. exponentially, harmonic functions), [1], [6]. It is possible in practice to expect some "harder" disturbance, like step function. In that case, Fig.8. shows that applied control algorithm is acceptable only for low step load disturbance, and short duration step disturbance, [6]. Large step load disturbance leads to poor tracking performances.

5. Conclusion

The result of experimentally investigations trajectory tracking control system with SMPM confirmed that SMC offer a good robustness to load torque disturbance using very simple control algorithm. It is showed, for the case of no-load operation, that control algorithm which use a expression for k (8) without coefficient for load torque compensation M, can not give a satisfactory result. It is especially important concerning tracking error. Adding coefficient M, control chattering and as a consequence noise, increases. If the level of noise is impermissible, control algorithm must be modified introducing a load torque observer. This will reduce the value of discontinuity part of control function and consequently noise and torque oscillations.

Experimental results with step load disturbances show that applied control algorithm is acceptable only for low step load disturbance and short duration step disturbance. Large step load disturbance leads to poor tracking performances.

Using variable boundary layer one can get better performance of controlled system comparing with constant boundary layer algorithm. Chattering in control function decreases for practically same tracking error.

6. References

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