Real Amplifier Influence on Impedance Tapered 2^{nd}-Order Filters

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***Abstract*-- In this paper, an influence of real parameters of operational amplifier to the impedance tapered filter transfer function is analyzed. As an example, a second-order low-pass (LP) and high-pass (HP) Sallen and Key filters with Butterworth and Chebyshev transfer function approximations are considered. Design tables with normalized element values for both cases are given. The filter elements were calculated using three different design criteria: equal resistors, capacitors and impedance tapered elements. Filter transfer functions are obtained using ideal and real operational amplifier. Real model of common operational amplifier is presented. Active sensitivities for all presented filters are shown. The results obtained by the analyses show significant influence of real amplifier in the frequency ranges higher than 100kHz. Reduction of active sensitivities is obtained using impedance tapering, which fortunately, reduces the passive sensitivities as well.**

***Index Terms*—Active-*RC* filters, active sensitivity, passive sensitivity, impedance tapering.**

I. INTRODUCTION

RECENTLY, a new design procedure for low-sensitivity active-*RC* allpole filters has been published [1], [2]. This procedure uses so-called "impedance tapering" in order to reduce the sensitivities of filter's transfer function magnitude to the passive component variations. The active element, i.e. the operational amplifier (OpAmp) in the filter is considered as ideal. However, that assumption is not valid, if the filter is designed for higher frequency bands. Overall filter's sensitivity performance is defined both with sensitivity to passive elements variations and the influence of non-ideal active elements, which can be described by sensitivity to active element. In this paper we investigate the active sensitivity of the various filter design approaches.

Three different design approaches are used in designing of LP and HP 2^{nd}-order Sallen and Key (SAK) filters [3]. Those filters can be used as simple filters or as building blocks in higher-order filters. The first filter example is designed starting with equal resistors, the second with equal capacitors [4], while the third filter design example is based on previously introduced impedance tapering technique.

II. SECOND-ORDER FILTERS

Consider 2^{nd}-order single-amplifier LP and HP SAK filters shown in Figures 1 and 2, respectively.

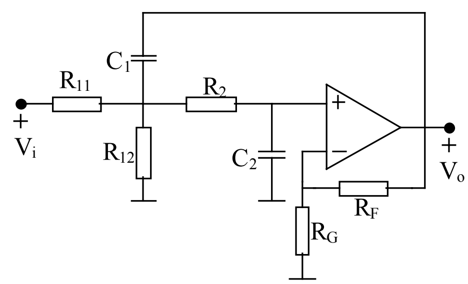

Fig. 1. The SAK 2nd-order LP filter circuit.

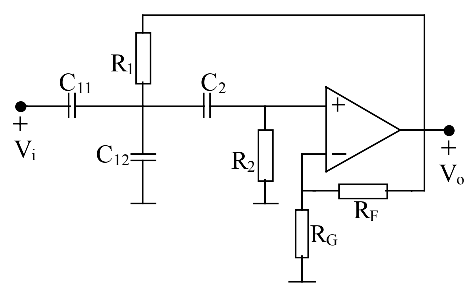

Fig. 2. The SAK 2^{nd}-order HP filter circuit.

The ideal OpAmp has infinite gain *A* and infinite gain-bandwidth (GB) product. On the other hand, real operational amplifier, such as for example TL081, has finite gain $A \approx 10^6$ and GB≈3MHz. This means at the frequency near 100kHz the gain *A* is 30 what is very low. This significantly influences the overall filter's magnitude, producing new filter's frequency response. In fact, using real amplifier, a new pole is added. Besides finite GB product, real OpAmp has finite input and output impedances. The single-pole model, which describes the real OpAmp, in the first approximation, is shown in Figure 3.

In what follows, we consider the transfer function's magnitude change due to active element non-ideality. To accomplish that task we define a difference between amplitude characteristic of the filter having an ideal OpAmp and amplitude characteristic obtained using real OpAmp. It is already known that impedance tapered filters have the reduced passive sensitivity. The question, which arises now is: will the filters with reduced passive sensitivities have reduced active sensitivities, as well?

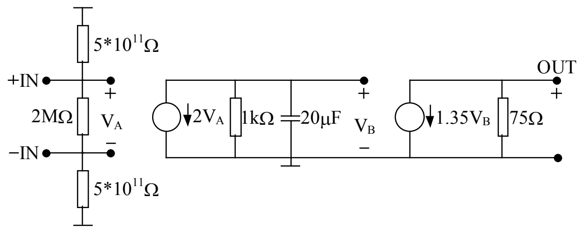

Fig. 3. One pole model of real operational amplifier.

III. EXAMPLE

In the following example the LP and HP filters are realized using Butterworth (Bu) and Chebyshev with 0.5dB pass-band ripple and –3dB cut-off frequency (Ch) approximations. Amplifier feedback gain K is equal to 2 and 1. The filters have unity gain in the pass-band. Therefore we have a voltage attenuator at the filters input, i.e. input resistor or capacitor is split into two elements. Thus in the LP case, input resistor values are given by:

$$R_1 = \frac{R_{11}R_{12}}{R_{11}+R_{12}}; \tag{1}$$

$$\frac{R_{12}}{R_{11}+R_{12}} = \frac{1}{K}; \tag{2}$$

$$K = 1 + \frac{R_F}{R_G}. \tag{3}$$

Normalized element values are given in Tables 1 and 2. For LP filters tapering is done on capacitors, and for HP filters tapering is done on resistors. As shown in [1] and [2], those are the solutions, providing the reduced passive sensitivities for LP and HP filters, respectively. The other combinations use equal-*C* and equal-*R* filters.

Denormalization is performed for the frequency 100kHz and with resistor value R_0=2842.05Ω, which gives C_0=560pF. Simulation of the filter's transfer function magnitude $\alpha(\omega)=20\log|T(j\omega)|$[dB] was performed using MATLAB. Transfer function magnitudes obtained for filters with an ideal OpAmp (denoted by I) and real model of OpAmp are shown in Figures 4 and 5.

TABLE I

SAK LP 2ND-ORDER FILTER, NORMALIZED ELEMENTS.

Filter Type		R_1	R_2	R_F	R_G	C_1	C_2
Eq. R	Bu	1	1	1	1	0.87403	1.14412
	Ch	1	1	1	1	1.07133	1.18902
Eq. C	Bu	0.70711	1.41431	1	1	1	1
K=2	Ch	0.97483	1.30672	1	1	1	1
Tap. C	Bu	1.41421	1.41421	0	∞	1	0.5
K=1	Ch	1.94966	1.94966	0	∞	1	0.3351

TABLE II

SAK HP 2ND-ORDER FILTER, NORMALIZED ELEMENTS.

Filter Type		R_1	R_2	R_F	R_G	C_1	C_2
Eq. R	Bu	1	1	1	1	1.41431	0.70711
K=2	Ch	1	1	1	1	1.02582	0.76528
Eq. C	Bu	1.14412	0.87403	1	1	1	1
K=2	Ch	0.93342	0.84103	1	1	1	1
Tap. R	Bu	1	2	0	∞	0.70711	0.70711
K=1	Ch	1	2.98418	0	∞	0.51291	0.51291

Equal-*R* design is denoted by R, equal-*C* design by C and the magnitude obtained from the design, which uses impedance tapering is marked with T.

In simulation of the magnitude response all passive components take their nominal values. To investigate active sensitivity we define a figure-of-merit, which is the difference of transfer function magnitudes defined by:

$$\Delta\alpha(\omega) = |\alpha_i(\omega) - \alpha_r(\omega)| \ \ [\text{dB}], \tag{4}$$

where with $\alpha_i(\omega)$ is marked transfer function magnitude with ideal and with $\alpha_r(\omega)$ a transfer function magnitude obtained with real OpAmp. Magnitude differences $\Delta\alpha(\omega)$ for the filters in Tables 1 and 2 are presented in Figures 6 and 7.

Observing Figures 6 and 7, we can note a significant deflection of real cases from ideal in the frequency band higher than few hundreds kilohertz. Furthermore, Chebyshev approximations are more sensitive to active component influence than Butterworth approximation, due to their higher pole-Q values. This is also true for the passive sensitivities.

Fortunately, from Figures 6 and 7, it follows that the tapered filter design has reduced active as well as passive sensitivities (see [1] and [2]). That means, the design using impedance tapering provides us with low-sensitivity filters in both aspects.

Furthermore, numerical results show higher influence of real operational amplifier on filter's transfer function for frequencies higher than 100kHz. For lower frequencies than this cut-off frequency, reduction of passive sensitivity is of greater interest.

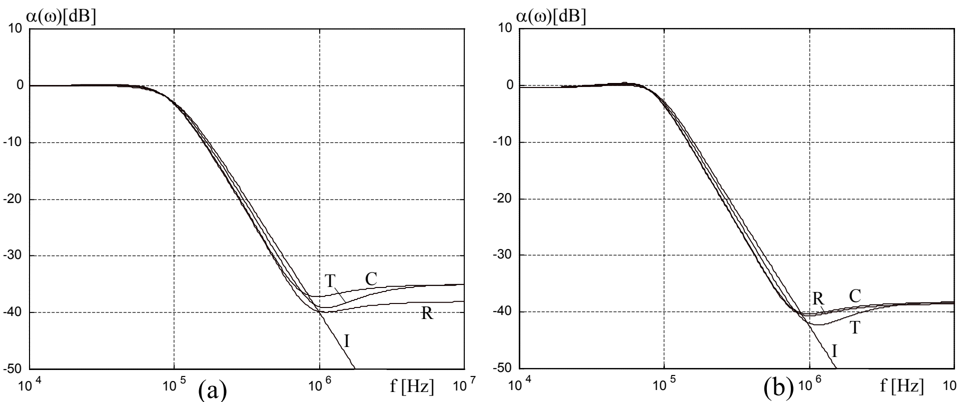

Fig. 4. Transfer functions of 2nd-order LP filters. (a) Butterworth. (b) Chebyshev.

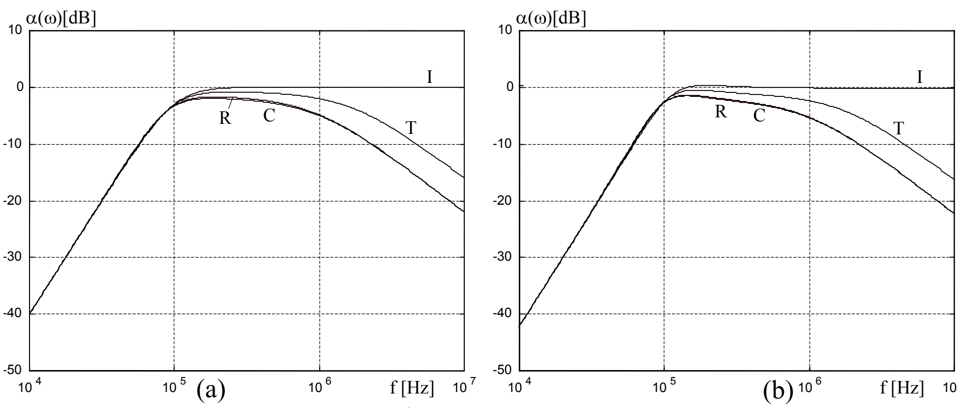

Fig. 5. Transfer functions of 2nd-order HP filters. (a) Butterworth. (b) Chebyshev.

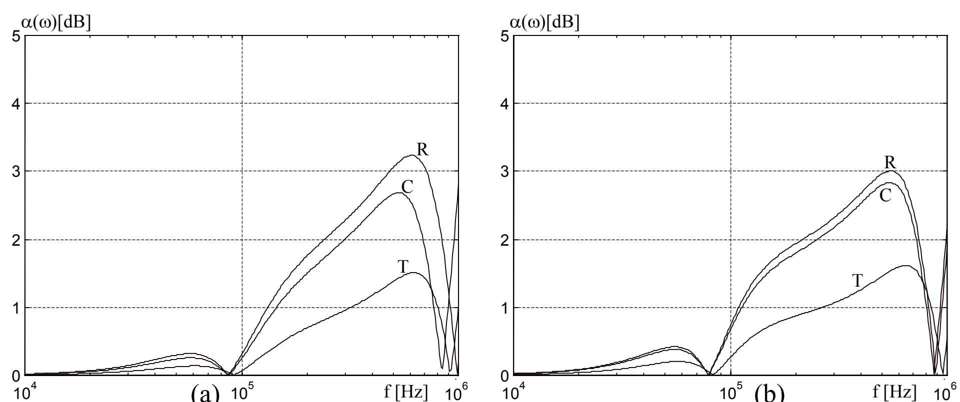

Fig. 6. Active sensitivity of 2nd-order LP filters. (a) Butterworth. (b) Chebyshev.

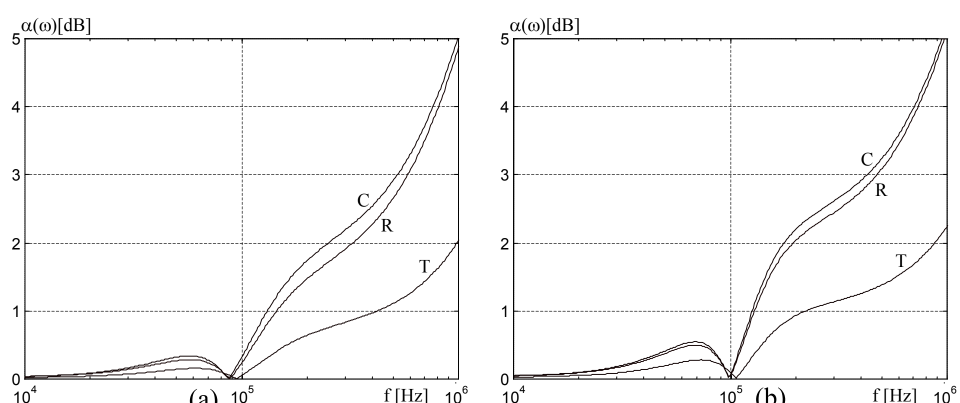

Fig. 7. Active sensitivity of 2nd-order HP filters. (a) Butterworth. (b) Chebyshev.

IV. Conclusion

In this paper it is shown that real parameters of operational amplifier have significant influence on the filter's transfer function magnitude for frequencies higher than 100kHz. Presented 2nd-order active-*RC* filters will have significantly reduced sensitivity to variation of passive elements and sensitivity to non-ideality of operational amplifier if impedance tapering is used in filter elements calculation. Filter circuits, obtained using that new design technique, show very reduced difference between theirs transfer function magnitudes simulated using ideal and real active element at higher frequencies. Both Butterworth and Chebyshev filter transfer function approximations can be improved.

References

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