# Optimization of PM Brushless DC Motor Drive

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*Abstract* – **Traditional approaches to controller designs that guarantee fast compensation of load torque and reference variations result in design iterations and most of the time in poorer response for load torque variations. In this paper a reference model for desired drive behavior generation and optimization methods has been applied to achieve controller integral time constant lower than the maximum time constant of the PM brushless DC motor drive. Presented simulation results show that using reference model for desired drive behavior generation, it is possible to determine optimal controller parameters for faster (10 time) and better (2 time) load torque compensation than in the case of traditional design of speed controller parameters. Response due to reference input with constrained overshoot has been achieved using a filter in the servosystem input. Thereby, the proposed method demonstrates the design of a speed controller that is optimal for both load torque and reference variations and its verification with simulation are accomplished for a permanent magnet brushless dc motor drive.**

## 1 Introduction

There are a large number of techniques for synthesizing the controllers [1, 2, 4]: frequency methods, root locus, state variables, experimental methods and optimization methods. Frequency domain techniques are widely used in the controller design (symmetric optimum and maximum time constant compensation). For torque disturbance, it is found that only the symmetric optimum method provides faster and better compensation of the speed variations induced by the disturbance [3].
Using integral error criteria and optimization methods [4] it is possible to achieve desired overshoot of the servosystems. But optimal value of the controller integral time constant in that case is grater than maximum servosystem time constant. This is not optimal for load torque compensation.

In this paper a reference model for desired drive behavior generation and optimization methods has been applied to achieve integral time constant of the permanent magnet brushless direct current (PMBDC) motor drive controller that is lower than maximum drive time constant. Desired overshoot response for a change in reference value has been achieved using a filter in the drive input.

## 2 Optimization Criteria

For design of controller parameters, it is possible to use different optimization methods (gradient, simplex, Hooke-Jeves). Matlab uses gradient and simplex methods [4]. Optimization criteria could be: integral and transient response indicies.

Using standard integral criteria (ISE, ITSE, IAE, ITAE) of the error in reference to ideal response results in optimal solution with percentage overshoot around 20% and controller integral time constant greater than maximum servosystem time constant. This solution is not optimal for load torque compensation. For better compensation of load torque's influence the second order reference model for desired drive behavior generation and optimization methods have been applied to optimize PI speed controller parameters to achieve integral time constant of the PMBDC motor drive controller lower than maximum drive time constant.

Consider the second order reference model given by:

$$G_{M2}(s) = \frac{\omega_M(s)}{\omega_r^*(s)} = \frac{K_M}{1 + 2\zeta T_M s + T_M^2 s^2}. \tag{1}$$

The percent maximum response overshoot $M_p$ is determined by relative damping coefficient ς:

$$M_p = 100 e^{\pi\zeta/(1+\zeta^2)^{1/2}}. \tag{2}$$

The relation for the time at which maximum overshoot occurs $t_p$ has the form:

$$t_p = \pi/\omega_p = \pi/\omega_n(1-\zeta^2)^{1/2} \;\; ; \;\; \omega_n = 1/T_M. \tag{3}$$

For desired overshoot $M_p$ and time $t_p$ from relations (2) and (3) it is possible to calculate $T_M$ and ς.

Integral of square error has been applied for controller parameter optimization:

$$I_1 = \int \delta_{mr}^2(t)dt, \tag{4}$$

where $\delta_{mr}(t)$ is an error between the speed feedback signals of the second order reference model $\omega_{Mmr}$ and PMBDCM drive $\omega_{mr}$:

$$\delta_{mr}(t) = \omega_{Mmr}(t) - \omega_{mr}(t).$$

## 3 Model of the Permanent Magnet Brushless DC Motor Drive

This model is based on the PMBDCM drive discussed, derived and given in [5, 6]. For the sake of easy reference, the model is derived in brief and given in the following. During two phase conduction, the entire DC voltage is applied to the two phases having an impedance of:

$$Z = 2\{R_s + p(L - M)\} = R_a + pL_a, \tag{5}$$

where

$$R_a = 2R_s, \tag{6}$$

$$L_a = 2(L - M), \tag{7}$$

where $R_s$ is the stator resistance per phase and $L$ is the self inductance per phase and $M$ is the mutual inductance per phase and $p$ is the derivative operator $d/dt$.

The voltage equation for the stator is given by:

$$v_{is} = (R_a + pL_a)i_{as} + e_{as} - e_{cs}, \tag{8}$$

where the last two terms are the induced emfs in phases $a$ and $c$, respectively. But the induced emfs in phases $a$ and $c$ are equal and opposite during the regular operation of the drive scheme and given as:

$$e_{as} = -e_{cs} = \lambda_p \omega_m, \tag{9}$$

where $\lambda_p$ is the flux linkages per phase and $\omega_m$ is the rotor speed and which on substitution gives the stator voltage equation as:

$$\begin{aligned} v_{is} &= (R_a + pL_a)i_{as} + 2\lambda_p \omega_m = \\ &= (R_a + pL_a)i_{as} + K_b \omega_m, \end{aligned} \tag{10}$$

where the emf constant for both the phases is combined into one constant as:

$$K_b = 2\lambda_p, \ [\text{V/rad/s}] \tag{11}$$

The machine with an inner current control loop is shown in Fig. 1. Note that the electromagnetic torque for two phases combined is given by:

$$T_e = 2\lambda_p I_{as}, \text{ Nm}. \tag{12}$$

The machine contains an inner loop due to the induced emf. It is not physically seen as it is magnetically coupled. The inner current loop will cross this back emf loop creating a complexity in the development of the model. The interactions of these loops can be decoupled by suitably redrawing the block diagram. The load is assumed to be proportional to speed:

$$T_1 = B_2 \omega_m. \tag{13}$$

With that included in the feedback path, the speed to air gap torque transfer function can be evaluated as:

$$\frac{\omega_m(s)}{T_e(s)} = \frac{1/B_t}{1 + sT_m}, \tag{14}$$

where $s$ is the Laplace operator, $B_t$=$B_1$+$B_2$, $T_m$= $J/B_t$ where $B_1$ is the friction coefficient of the motor and $J$ is the inertia of the machine.

The current feedback has a low pass filter with a gain of $K_c$ and a time constant of $T_c$. The speed feedback has a similar filter with a gain of $K_\omega$ and a time constant of $T_\omega$ which is shown in Fig. 2.

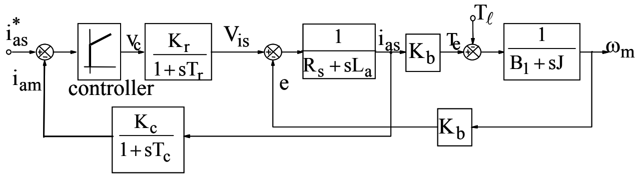


**Fig. 1:** PMBDCM with current control loop.

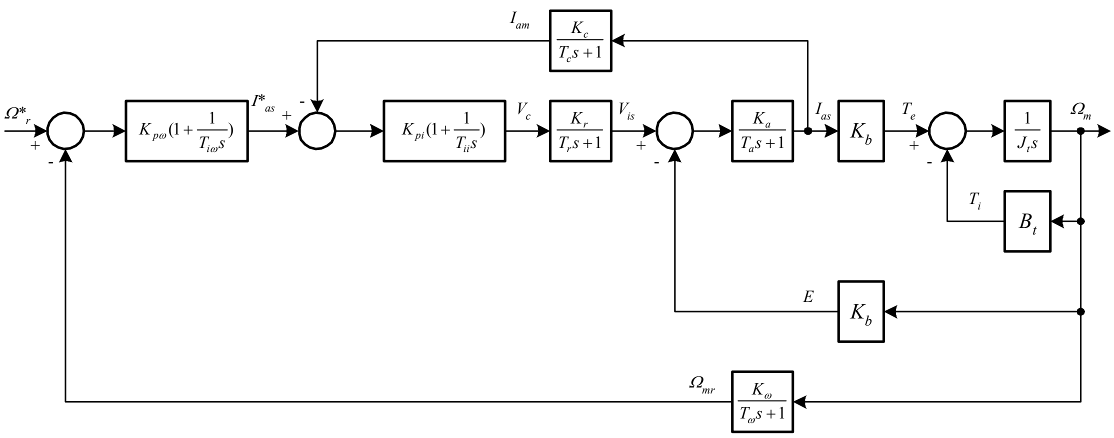


**Fig. 2:** Block diagram of cascade control of the PMBDCM drive.

Block diagram of cascade control of the PMBDCM dive is showen in Fig. 2 [5, 6].

Numerical value of the drive parameters are: $n_b$=4000 rev/min, $P_b$=373 W, $I_b$=17.35 A, $V_b$=40 V, $T_b$=0.89 Nm, $V_s$=160 V, $I_{max}$=2$I_b$=34.7 A, $T_{max}$=2$T_b$=1.78 Nm, $K_r$=16 V/V, $T_r$=50 µs, $R_a$=1.4 Ω, $L_a$=2.44 mH, $T_a$=$L_a$/$R_a$=1.743 ms, $K_a$=1/$R_a$=0.71428 A/V, $K_b$=0.051297 Vs, $B_t$=0.002125 Nm/rad/sec, $J$=0.0002 kgm$^2$, $K_m$=1/$B_t$=41.89, $T_m$=$J$/$B_t$=94.1ms, $K_c$=0.288V/A, $T_c$=0.159ms, $K_\omega$=0.02387 Vs, $T_\omega$=1ms.

## 4 Results of the PM Brushless DC Motor Drive Optimization

Current controller parameters have been adjusted on the value $K_{pi}$=1.25, $T_{ii}$=$T_a$=$L_a$/$R_a$=1.743 ms. Speed controller parameters were determined using Matlab and simplex optimization method [4, 7] for desired percent maximum response overshoot $M_p$=40% and different time at which maximum overshoot occurs $t_p$. For lower percent maximum response overshoot $M_p$ optimal speed controller parameters are inferior for good load torque compenzation (integral time constant has higher value and gain coefficient has lower value). For higher percent maximum response overshoot $M_p$ optimal gain coefficient has higher value and response to reference value is very oscillatory.

For an optimal speed controller, its parameters could be chosen from $t_p$=0.002s: $K_{p\omega}$=53.5, $T_{i\omega}$=0.0087s because $T_{i\omega}$ is relatively small and $K_{p\omega}$ is relatively high. With this value of the speed controller parameters the influence of rated load torque on a maximum speed feedback signal drop $\Delta\omega_{mr}$ and speed drop $\Delta\omega_m$ is relatively low ($\Delta\omega_{mr}$=-0.1174V=-1.174%; $\Delta\omega_m$=-5.921s$^{-1}$=-1.414%). For lower value of $t_p$ integral time constant $T_{i\omega}$ is much higher and load torque will be slowly compensated. For higher value of $t_p$ gain coefficient $K_{p\omega}$ is much lower and load torque will be inadequately compesated, that is maximum speed drop $\Delta\omega_m$ will be higher.

Speed feedback signal response $\Delta\omega_{mr}$ and current response $\Delta i_{as}$ to the step change of the nominal load torque $\Delta M_t$=0.89S($t$) and speed controller parameters determined by optimization (1. $K_{p\omega}$=53.5, $T_{i\omega}$=0.0087s) and by using Bode plot [3], that is by compensation of maximum time constant (2. $K_{p\omega}$=24.67, $T_{i\omega}$=0.0941s) has been shown on a Fig. 3. With optimal speed controller parameters the influence of a load torque is approximately 10 time faster and twice better compensated (maximum speed feedback signal drop $\Delta\omega_{mr}$ is approximately twice lower).

To achieve desired overshoot, a filter with gain coefficient $K_f$=1 and different value of a time constant $T_f$ has been added on the drive input (Fig. 4). Percent maximum response overshoot without filter has value $M_p$=51.4% and with a filter time constant $T_f$=2.1ms percent maximum response overshoot has value $M_p$=10%.

Speed feedback signal responses $\Delta\omega_{mr}$ (a) and current responses $\Delta i_{as}$ (b) to the step change of the reference value $\Delta\omega^*_r$=0.1S($t$) and speed controller parameters determined by optimization (1. $K_{p\omega}$ = 53.5, $T_{i\omega}$ = 0.0087 s) with added filter on the drive input and determined by using Bode plot, that is by compensation of maximum time constant and percent maximum response overshoot $M_p$=10% (2. $K_{p\omega}$ = 24.67, $T_{i\omega}$ = 0.0941 s) have been shown on a Fig. 5. Percent maximum speed response overshoot $M_p$ and maximum current value $i_{asm}$ in both cases have approximately the same value, but the speed of the current response is lower in the case of optimal controller parameters and filter added on the drive input.

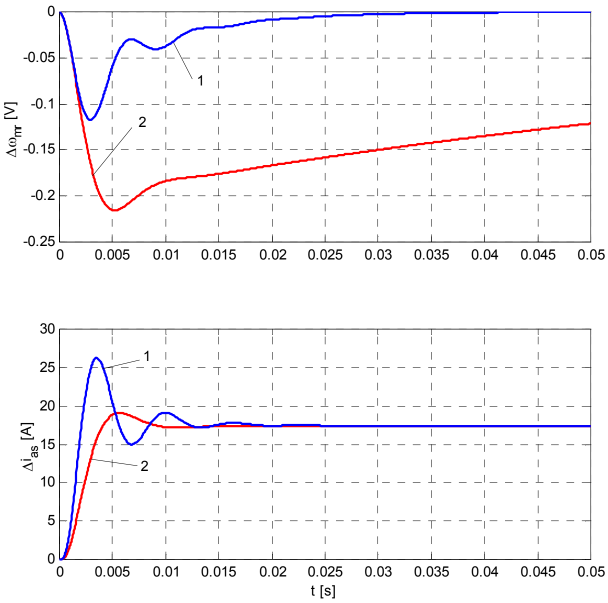


**Fig. 3**: Speed feedback signal response $\Delta\omega_{mr}$ (a) and current response $\Delta i_{as}$ (b) to the step change of the nominal load torque $\Delta M_t$=0.89S($t$); 1. $K_{p\omega}$ = 53.5, $T_{i\omega}$ = 0.0087 s - determined by optimization, 2. $K_{p\omega}$ = 24.67, $T_{i\omega}$ = 0.0941 s - determined by compensation of maximum time constant.

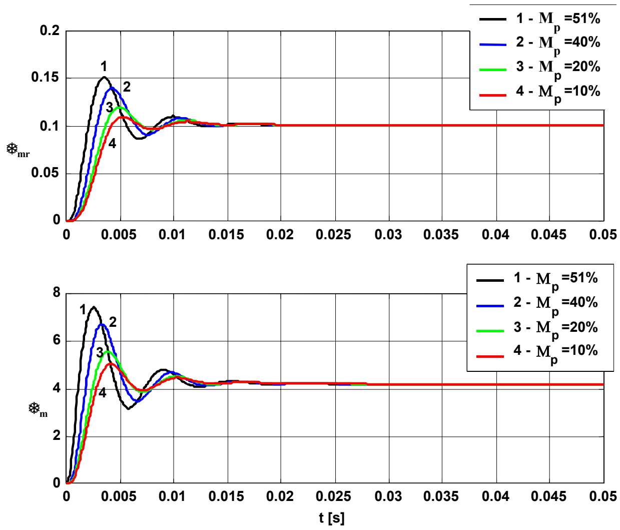


**Fig. 4**: Speed feedback signal ($T_{mr}$) and speed ($T_m$) response to the step change of the reference value $T_r^*(t)$=0.1S($t$) and controller parameters determined by optimization $T_{i\omega}$=0.0087s, $K_{p\omega}$=53.5 and filter added to the drive input:
1. $T_f$=0ms, $M_p$=51%, 2. $T_f$=0.75ms, $M_p$=40%,
3. $T_f$= 1.6ms, $M_p$=20%, 4. $T_f$=2.1ms, $M_p$=10%.

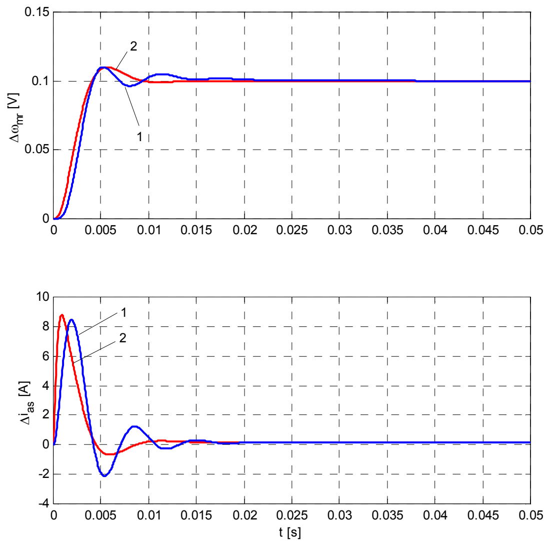


**Fig. 5:** Speed feedback signal response $\Delta\omega_{mr}$ (a) and current response $\Delta i_{as}$ (b) to the step change of the reference value $\Delta\omega^*_r$=0.1S($t$);
1. $K_{p\omega}$ = 53.5, $T_{i\omega}$ = 0.0087 s - determined by optimization and filter added to the drive input,
2. $K_{p\omega}$ = 24.67, $T_{i\omega}$ = 0.0941 s - determined by compensation of maximum time constant.

## 5 Conclusions

The key contributions of the proposed paper are summerized in the following:

(i) Using reference model for desired drive behavior generation and optimization methods for determination of a PI speed controller parameters, it is demonstrated that a speed controller is capable of compensating both reference and load torque variations and can be designed in a straightforward manner.
(ii) Using reference model for desired drive behavior generation with optimization methods, the integral time constant of the speed controller lower than maximum drive constant ($T_{i\omega}$<$T_m$=0.0941s) is derived.
(iii) It is possible to determine optimal speed cotroller parameters for faster (10 time) and better (2 time) load torque compensation than in the case of traditional design of a speed controller parameters.
(iv) Using the filter added on the drive input, the desired speed response overshoot to reference value change is achieved.

In this paper optimization of a PM brushless DC motor drive controller parameters was derived for low level input signal and linear drive model. The autors plan to investigate the influence of nonlinearities in the case of higher level input signal to motor drive behavior in the cases of optimal and traditionaly designed speed contoller parameters.

## 6 References

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