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Conference paper
Extended abstract

2-D biped walking system

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SUMMARY. – The study of human locomotion presents both theoretical and practical interest from the biomechanical and robotics point of view. This paper deals with mathematical modelling of biped gait in the sagital plane, neglecting at the moment motion in the frontal plane. A multi-link system made up of one trunk and two legs will be considered (Figure 1). The Lagrangian for the 5-link 2-D biped system was carried out and the equations of motion for the system are given. The Lagrange formalism is then written as state space representation and a block diagram scheme is proposed. Such a model helps us to analyse the effect of shock generated by the contact of feet with the ground. The state space representation of Langrange equations is originally described by C. Vibet (3). *His ideas are mainly based on observing the treatment of initial states in nonlinear systems, and are applied to the model of human locomotion.*

Keywords: Lagrangian system, gait, dynamics

A short mathematical description:

Generalized coordinates. $\theta_1, \theta_2 \ldots \theta_5$

Lagrange equation: $\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = \Gamma$

State space representation: $\dot{\vec{x}} = F(\vec{x}) + B(\vec{x})u(t)$

where:

$$F(\vec{x}) = \text{col}\left(\frac{d\theta}{dt}, \frac{\partial L}{\partial \theta}\right) ; B(\vec{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} ; u(t) = \text{col}(0, \Gamma)$$

INTRODUCTION

The study of human locomotion presents both theoretical and practical interest from biomechanical and robotics points of view. Different methods and techniques must be applied in the analysis of human locomotion in order to obtain a qualitative estimation.

Previous models of the dynamic case suffered from various constraints *(1, 5)* and such models led to unavoidable discrepancies with experimental results. The analysis of the shock generated by the contact of feet with the ground was not included in previous modelling, partly due to the complexity of the models. The suggested Lagrangian model takes into account abrupt velocity change and impulse action on forces and torques, and also allows an easy and direct application of the decoupling method.

MATHEMATICAL MODEL

A multi-link system made up of one trunk, two upper legs, two lower legs and two feet will be considered. That is a five links system as shown in Figure 1. The position of each link j = 1, 2,...5 is de-

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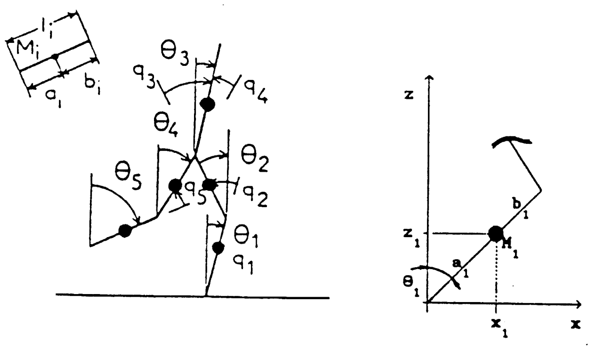

FIGURE 1. *Schematic representation of simplified five-link 2-D walking biped.*

termined by the angle with respect to the vertical of the segment j, where Mj are the weights, Ij the moments of inertia around the center of the mass. Respectively, aj, bj, lj are distances between the bottom and the center of mass of each link j, between the top and the center of mass of each link j and the total length of link j. The center of gravity Gj of each link j will have the following coordinates:

$$G_1 \quad x_1 = a_1\sin\theta_1, \quad z_1 = a_1\cos\theta_1 \qquad (1)$$

$$G_2 \quad x_2 = l_1\sin\theta_1 + a_2\sin\theta_2, \quad z_2 = l_1\cos\theta_1 + a_2\cos\theta_2 \qquad (2)$$

$$G_3 \quad x_3 = l_1\sin\theta_1 + a_2\sin\theta_2 + a_3\sin\vartheta_3, \quad z_3 = l_1\cos\theta_1 + a_2\cos\theta_2 + a_3\sin\theta_3 \qquad (3)$$

$$G_4 \quad x_4 = l_1\sin\theta_1 + l_2\sin\theta_2 - b_4\sin\theta_4, \quad z_4 = l_1\cos\theta_1 + l_2\cos\theta_2 - b_4\cos\theta_4 \qquad (4)$$

$$G_5 \quad x_5 = l_1\sin\theta_1 + l_2\sin\theta_2 - b_5\sin\theta_5, \quad z_5 = l_1\cos\theta_1 + l_2\cos\theta_2 - b_5\cos\theta_5 \qquad (5)$$

The Lagrangian *(4)* of an n-link system is given by difference L = T -V, where potential energy is expressed as:

$$V = \sum_{j=1}^{5} v_j \qquad \text{with } v_j = M_j g z_j \qquad (6)$$

and kinetic energy is given by:

$$T = \sum_{j=1}^{5} T_j \qquad \text{with } T_j = \frac{1}{2}\,[I_j\dot{\theta}_j^2 + M_j(\dot{x}_j^2 + \dot{z}_j^2)] \qquad (7)$$

For future use Lagrangian will be written in the vector-matrix form:

$$L = \frac{1}{2}\,[\dot{\theta}^T A(\theta)\dot{\theta} + N(\theta)];\ \theta^T = [\theta_1\,\theta_2\,\theta_3\,\theta_4\,\theta_5];\ \dot{\theta} = \frac{d\theta}{dt} \qquad (8)$$

where A (Θ) is the inertia matrix of the system. The N (Θ) is a continuous function of vector (Θ). The equations of motion for the considered system are given by:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = \Gamma \qquad (9)$$

where $\Gamma = \mathrm{col}\,(\Gamma_1, \Gamma_2,.. \Gamma_5)$ is a generalized torque vector applied to joints.
The left side of the equation (9) by long procedure after arranging gives the general form:

$$I\theta + B\dot{\theta}^2 - M_G = \Gamma \qquad (10)$$

The active moments Γ are the sum of muscle moments Γ_M and produced moments of ground reaction forces Γ_R [1]. It was shown that A (θ) ≈I(θ). The active moment in the particular links can be achieved from equation:

$$\Gamma_M = I\theta + B\dot{\theta}^2 + M_G - \Gamma_R \qquad (11)$$

The moments Γ_R resulting from ground reaction forces can be given in matrix form:

$$\Gamma_R = RF \qquad (12)$$

where:

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & 0 & 0 \\ R_{21} & R_{22} & R_{23} & 0 & 0 \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{36} \\ 0 & 0 & R_{43} & R_{44} & R_{45} \\ 0 & 0 & R_{53} & R_{54} & R_{55} \end{bmatrix};\ \mathbf{F}^T = [F_{H1}\,F_{V1}\,1\,F_{H2}\,F_{V2}] \qquad (13)$$

I is the matrix of moments of inertia while the elements **B** are originated by centripetal forces.

$$\mathbf{I} = \begin{bmatrix} I_{11} & I_{12} & I_{13} & I_{14} & I_{15} \\ I_{21} & I_{22} & I_{23} & I_{24} & I_{25} \\ I_{31} & I_{32} & I_{33} & 0 & 0 \\ L_{41} & L_{42} & 0 & L_{44} & L_{45} \\ I_{51} & I_{52} & 0 & I_{54} & I_{56} \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} 0 & B_{12} & B_{13} & B_{14} & B_{15} \\ B_{21} & 0 & B_{23} & B_{24} & B_{25} \\ B_{31} & B_{32} & 0 & 0 & 0 \\ B_{41} & B_{42} & 0 & 0 & B_{45} \\ B_{51} & B_{52} & 0 & B_{54} & 0 \end{bmatrix} \qquad (14)$$

Matrix of moments of gravitation forces is:

$$M_G^T = [M_{G1}\,M_{G2}\,M_{G3}\,M_{G4}\,M_{G5}] \qquad (15)$$

STATE SPACE REPRESENTATION

With the aim of giving a complete definition of the solution, the behavior of multibody systems is usually represented by a set of differential equations supplemented by the values of initial conditions. In case of collision the change in the momentum have to be included *(2)*. The Lagrangian formalism will be extended to include these effects. Collision occuring at time t_c during a very short interval δt_c results in the momentum change and is given by:

$$\delta P_c = \int_{t_c-\delta\frac{t}{2}}^{t_c+\delta\frac{t}{2}} \Gamma(t)dt = \frac{\partial L}{\partial \dot{\theta}}\Bigg|_{t_c-\delta\frac{t}{2}}^{t_c+\delta\frac{t}{2}} \qquad (16)$$

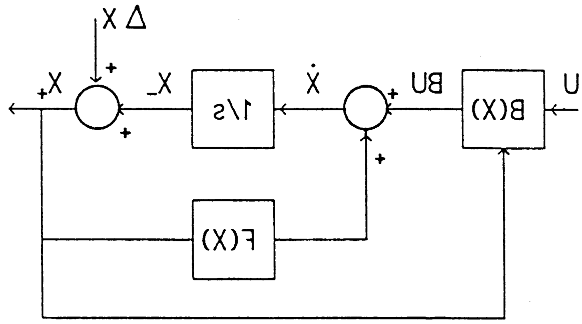

FIGURE 2. *Block-diagram representation of the Lagrange formalism including the initial and pulse conditions.*

To introduce consistently this momentum change into the dynamics, the following state variables are introduced $x = col(\theta, P)$ instead of usual $x_0 = col(\theta, \dot{\theta})$. The generalized momentum P is defined by:

$$P = \frac{\partial L}{\partial \dot{\theta}} (\theta, \dot{\theta}, t) \tag{17}$$

In the condition the system dynamics takes the following classical form:

$$\dot{\vec{x}} = F(\vec{x}) + B(\vec{x})u(t) \tag{18}$$

where

$$F(\vec{x}) = col\left(\frac{d\theta}{dt}, \frac{\partial L}{\partial \theta}\right); B(\vec{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; u(t) = col(0, \Gamma) \tag{19}$$

The block diagram representation of reformulated Langrange equations system exhibiting the impulse effects is given in Figrue 2. By Δx impuls effects are introduced into the system.

CONCLUSION

For simulation studies a set of differential equations of Lagrange type supplemented by the values of reaction forces is given in order to give the complete definition of the muscle moments. Also, a general approache extending Langrange formalism has been developed in which the problem of the consequences of an impulse momentum chanbge resulting from a shock in a multibody system can be analysed and the block diagram representation can be built up.

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APPENDIX

MATRIX ELEMENTS I

```
I(1,1)=In1+m1*a1^2+M2*l1^2+M3*l1^2+M4*l1^2+M5*l1^2
I(1,2)=(m2*l1*a2+M3*l1*l2+M4*l1*l2+M5*l1*l2)*cos(θ1-θ2)
I(1,3)=M3*l1*a3*cos(θ1-θ3);
I(1,4)=-(M4*l1*b4+M5*l1*l4)*cos(θ1-θ4);
I(1,5)=-M5*l1*b5*cos(θ1-θ5);
I(2,1)=(M2*l1*a2+M3*l1*l2+M4*l1*l2+M5*l1*l2)*cos(θ1-θ2);
I(2,2)=In2+M2*a2^2+M3*l2^2+M4*l2^2+M5*l2^2;
I(2,3)=M3*l2*a3*cos(θ2-θ3);
I(2,4)=-(M4*l2*b4+M5*l2*l4)*cos(θ2-θ4);
I(2,5)=-M5*l2*b5*cos(θ2-θ5);
I(3,1)=M3*l1*a3*cos(θ1-θ3);
I(3,2)=M3*l2*a3*cos(θ2-θ3);
I(3,3)=In3+M3*a3^2;
I(3,4)=0;        I(3,5)=0;
I(4,1)=-(M4*l1*b4+M5*l1*l4)*cos(θ1-θ4);
I(4,2)=-(M4*l2*b4+M5*l2*l4)*cos(θ2-θ4);
I(4,3)=0;
I(4,4)=In4+M4*b4^2+M5*l4^2;
I(4,5)=M5*l4*b5*cos(θ4-θ5);
I(5,1)=-M5*l1*b5*cos(θ1-θ5);
I(5,2)=-M5*l2*b5*cos(θ2-θ5);
I(5,3)=0;
I(5,4)=M5*l4*b5*cos(θ4-θ5);
I(5,5)=In5+M5*b5^2;
```

MATRIX ELEMENTS B

```
B(1,1)=0;
B(1,2)=(M2*l1*a2+M3*l1*l2+M4*l1*l2+M5*l1*l2)*sin(θ1-θ2);
B(1,3)=M3*l1*a3*sin(θ1-θ3);
B(1,4)=-(M4*l1*b4+M5*l1*l4)*sin(θ1-θ4);
B(1,5)=-M5*l1*b5*sin(θ1-θ5);
B(2,1)=-(M2*l1*a2+M3*l1*l2+M4*l1*l2+M5*l1*l2)*sin(θ1-θ2);
B(2,2)=0;
B(2,3)=M3*l2*a3*sin(θ2-θ3);
B(2,4)=-(M4*l2*b4+M5*l2*l4)*sin(θ2-θ4);
B(2,5)=-M5*l2*b5*sin(θ2-θ5);
B(3,1)=-M3*l1*a3*sin(θ1-θ3);
B(3,2)=-M3*l2*a3*sin(θ2-θ3);
B(3,3)=0;        B(3,4)=0;        B(3,5)=0;
B(4,1)=(M4*l1*b4+M5*l1*l4)*sin(θ1-θ4);
B(4,2)=(M4*l2*b4+M5*l2*l4)*sin(θ2-θ4);
B(4,3)=0;
B(4,4)=0;
B(4,5)=M5*l4*b5*sin(θ4-θ5);
B(5,1)=M5*l1*b5*sin(θ1-θ5);
B(5,2)=M5*l2*b5*sin(θ2-θ5);
B(5,3)=0;
B(5,4)=-M5*l4*b5*sin(θ4-θ5);
B(5,5)=0;
```

MATRIX ELEMENTS M_G

M_G (1,1) = –(M1*g*a1+M2*g*l1+M3*g*l1+M4*g*l1+M5*g*l1)*sin(Θ1);

M_G (2,1) = –(M2*g*a2+M3*g*l2+M4*g*l2+M5*g*l2)*sin(Θ2);

M_G (3,1) = – M3*g*a3*sin(Θ3);

M_G (4,1) = (M4*g*b4+M5*g*l4)*sin(Θ4);

M_G (5,1) = M5*g4*b5*sin(Θ5);

MATRIX ELEMENTS R

R (1,1) = l1*cos (Θ1);
R (1,2) = l1*sin(Θ1);
R (1,3) = Fv1;
R (1,4) = 0;
R (1,5) = 0;
R (2,1) = l1*cos(Θ1)+l2*cos(Θ2);
R (2,2) = l1*sin(Θ1)–l2*sin(Θ2);
R (2,3) = Fv1;
R (2,4) = 0;
R (2,5) = 0;
R (3,1) = l1*cos(Θ1)+l2*cos(Θ2);
R (3,2) = l1*sin(Θ1)–l2*sin(Θ2);
R (3,3) = Fv1+Fv2;
R (3,4) = l5*cos(Θ5)+l4*cos(Θ4);
R (3,5) = l5*sin(Θ5)–l4*sin(Θ4);
R (4,1) = 0;
R (4,2) = 0;
R (4,3) = Fv2;
R (4,4) = l5*cos(Θ5)+l4*cos(Θ4);
R (4,5) = l5*sin(Θ5)–l4*sin(Θ4);
R (5,1) = 0;
R (5,2) = 0;
R (5,3) = Fv2;
R (5,4) = l5*cos(Θ5);
R (5,5) = l5*sin(Θ5);