Low Sensitivity, Low Power BP Active Filter Structures with Biquads using Impedance Tapering

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***Abstract* –This paper presents the realization of narrow high-order band-pass (BP) active resistance-capacitance (*RC*) filters. It was shown that such filters have large sensitivities to component tolerances, and if they were built using structures with multiple feedbacks, theirs sensitivities are substantially decreased. Furthermore, it was demonstrated that sensitivities of inner 2nd-order blocks ("biquads") building the structure can be reduced by the "impedance tapering" design procedure. Using it we provide an additional decrease of BP filter structures sensitivities. As an illustration of the efficiency of the proposed design procedure, the sensitivity analysis, using Schoeffler's sensitivity measure, is performed for an 8th-order Chebyshev narrow BP filter. The filter was realised as: cascade (CAS), follow-the-leader-feedback (FLF), cascaded "biquarts" (CBQ) and leap-frog (LF) structure, using impedance tapered sub-circuits.**

I. INTRODUCTION

In [1] and [2] it was shown that the sensitivity of filters transfer function magnitude to passive components of the filter, such as resistors and capacitors, is proportional to filters pole-Q factors. Therefore, high sensitivity to filter components is a very important problem in the realization of a narrow band-pass filter, because they have very high pole Q-factor values. It is well know that by increasing the filter order, the pole-Q factors increase even more.

Active-*RC* high-order BP filters with narrow pass-band are usually realized by 2nd-order blocks (biquads), which are mutually interconnected. The manner in which this connection is accomplished is denoted as a filter structure, making the resulting BP filter more or less sensible to component variations. In [3] it was shown that sensitivities of active-*RC* filters with multiple feedbacks are reduced. We investigate sensitivities of various narrow BP active filter structures having the same transfer function.

In this paper we compare properties of BP filter structures having the inner 2nd-order BP blocks realized using general purpose (GP) sections, with three operational amplifiers (opamps), to those having single-opamp "biquads". The latter 2nd-order blocks are simpler, since they have less opamps and therefore reduced power consumption, but have larger sensitivities than the former. However, single-opamp "biquads" can be designed using recently introduced design technique in [1] and [2] called "impedance tapering", which reduces theirs sensitivities to component tolerances of the circuit and thus decreases this unwanted property.

In the design of low-sensitivity, 2nd-order class-4 Sallen and Key active-*RC* allpole filters, using "impedance tapering" [1] and [2], L-sections of the *RC* ladder network are successively impedance scaled upwards, from the driving source to the positive amplifier input, providing the significantly decreased sensitivity of the filter characteristics to component tolerances. Thus, the sensitivity to passive components of multiple feedback BP filter can be additionally decreased if it were built using "impedance tapered" biquads.

II. SENSITIVITY OF ACTIVE FILTERS

A. Definition of sensitivity

Component values in electrical circuits can deviate from their nominal values due to ageing, temperature, tolerances, etc. Sensitivity analysis gives the information on network function changes caused by small deviations of component values [3].

Given the network function $F(s, x_1, \dots, x_n)$, where s is complex variable and x_k (k=1, …,n) are real parameters of the filter, the relative function deviation $\Delta F/F$ due to single parameter value relative deviation $\Delta x_k/x_k$, in the first approximation, is given by:

$$\frac{\Delta F}{F} \cong S_{x_k}^{F} \cdot \frac{\Delta x_k}{x_k}\,, \tag{1}$$

where $S_{x_k}^{F}$ represents the relative sensitivity of a function F to a single parameter x_k, and equals to

$$S_{x_k}^{F} = \frac{x_k}{F}\frac{dF}{dx_k}\,. \tag{2}$$

If several components deviate from the nominal value, a criterion for assessing function deviation due to change of many parameters must be used. Let $\Delta x_k/x_k$ be independent normal random variables with zero means and identical standard deviations equal to σ_x. The squared standard deviation σ^2_F of relative function change $\Delta F/F$ is given by:

$$\sigma_F^2 = \sigma_x^2 \sum_{k=1}^{n}\left[S_{x_k}^{|F(j\omega)|}\right]^2 = \sigma_x^2 S_2\,, \tag{3}$$

where S_2 is so called Schoeffler sensitivity defined as

$$S_2 = \sum_{k=1}^{n}\left[S_{x_k}^{|F(j\omega)|}\right]^2\,. \tag{4}$$

The Schoeffler sensitivity is a reliable measure for estimation of different circuits from the sensitivity point, and it is used in this paper.

B. Low sensitive active filter structures

Four mostly used filter structures, i.e. cascade (CAS), follow-the-leader-feedback (FLF), cascade of biquartic sections (CBQ), and leap-frog (LF) are shown in Fig. 1.

Second-order filter blocks used for the realization of structures in Fig. 1 have the BP transfer function given by:

$$T_i(s) = \frac{N_i(s)}{D_i(s)} = \frac{k_i a_{1i} s}{s^2 + a_{1i}s + a_{0i}} = \frac{k_i s(\omega_{pi}/q_{pi})}{s^2 + s(\omega_{pi}/q_{pi}) + \omega_{pi}^2}\,, \tag{5}$$

where a_{1i} and a_{0i} represent transfer function coefficients. Instead, we can use transfer function pole frequencies ω_{pi}

and pole Q-factors, q_{pi} (i=1,…, N/2; for an even filter order N). We suppose that we design even N^{th}-order BP filters.

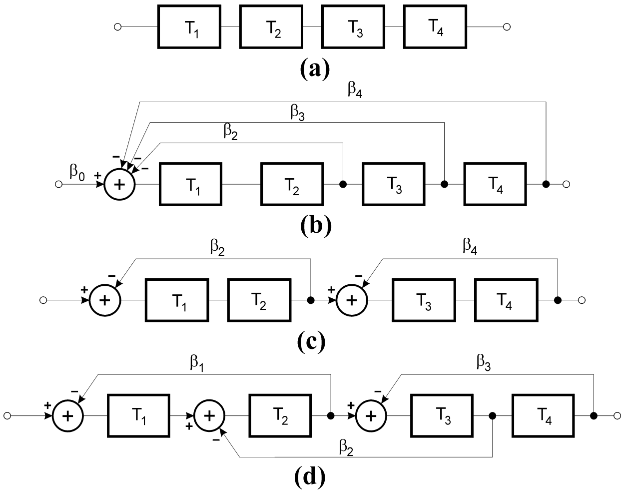

Fig. 1 **(a)** CAS; **(b)** FLF; **(c)** CBQ and **(d)** LF structure

In what follows we show that the sensitivity to passive components of the transfer function $F(s)$ of an active-RC filter structure as in Fig. 1 depends on the structure of the filter, and on the realization of 2^{nd}-order building blocks T_i(s). Consider the filter transfer function $F(s)$ given by:

$$F=F(T_i), \tag{6}$$

where

$$T_i=(s, x_{i1}, x_{i2}, \ldots, x_{ik}). \tag{7}$$

Arguments x_{ij} (j=1,…, k) represent passive components, of 2^{nd}-order filter blocks T_i(s). The sensitivity of the filter transfer function $F(s)$ to the component x_{ij} variation is then given by:

$$S_{x_{ij}}^{F}=S_{T_i}^{F}\cdot S_{x_{ij}}^{T_i}\,. \tag{8}$$

In (8) the first factor $S_{T_i}^{F}$ is called *structure-to-block sensitivity* and it depends on the structure of the filter, while the second factor $S_{x_{ij}}^{T_i}$ depends exclusively on the way in which the i^{th} block was realized. Note that filters having identical transfer functions and the same blocks, which are realized by different structures, will not have the same sensitivities.

Furthermore, the second factor in (8) can be represented by:

$$S_{x_{ij}}^{T_i}=S_{a_k}^{T_i}\cdot S_{x_{ij}}^{a_k}\,, \tag{9}$$

where a_k (k=0, 1, 2) represent transfer function coefficients in (5). It is shown in [1] that the first factor in (9) is called *coefficient sensitivity* $S_{a_k}^{T_i}$ and it depends exclusively on the generic filter type (Butterworth, Chebyshev, etc.), i.e. on the pole-Q factor values and can't be influenced. On the other hand, second factor in (9) represents *coefficient-to-component sensitivity* $S_{x_{ij}}^{a_k}$ and depends on the way in which some filters were designed. It was shown in [1] and [2] that by using "impedance tapering" design techniques if the 2^{nd}-order blocks $T_i(s)$ are realized by single-opamp "biquads", the *coefficient-to-component sensitivities* can be reduced, and thus the sensitivity of the structure $S_{x_{ij}}^{F}$ to its every passive component x_{ij}, given by (8), can be additionally decreased.

Consider sensitivities of filter structures as presented in Fig. 1.

a) <u>CAS Structure:</u> Transfer function of the active filter is given by:

$$F(s)=\prod_{i=1}^{N/2}T_i(s)\,. \tag{10}$$

Using rules for calculating sensitivities, which are given in literature (see [3]), the sensitivity of the transfer function $F(s)$ to the variation of any $T_i(s)$ is given by:

$$S_{T_i}^{F}=1\,, \tag{11}$$

where i=1, …, N/2. The structure-to-block sensitivity for the CAS structure is equal to unity and can't be influenced. Thus, the sensitivity of the transfer function $F(s)$ to any element x_{ij} in i^{th} 2^{nd}-order block is given by:

$$S_{x_{ij}}^{F}=S_{x_{ij}}^{T_i}\,. \tag{12}$$

Consequently, the sensitivity optimisation of the CAS structure reduces to the optimisation of the sensitivities of each block in the structure.

b) <u>FLF Structure:</u> Transfer function of the active filter is given by:

$$F(s)=\left[\beta_0\prod_{i=1}^{N/2}T_i(s)\right]\cdot\left[1+\sum_{i=1}^{N/2}\beta_j\prod_{i=1}^{j}T_i(s)\right]^{-1}, \tag{13}$$

where β_i (i=1,…, N/2) are negative feedback coefficients and β_0 is the input gain. After some calculation the sensitivity of the transfer function $F(s)$ to the variation of any building-block transfer function $T_i(s)$ is given by:

$$S_{T_k}^{F}=\left[1+\sum_{i=1}^{k-1}\beta_i\prod_{j=1}^{j}T_j(s)\right]\cdot\left[1+\sum_{i=1}^{N/2}\beta_i\prod_{j=1}^{j}T_j(s)\right]^{-1}, \tag{14}$$

where k=1,…, N/2. From (14) follows the sensitivity of the magnitude $F(\omega)=|F(j\omega)|$ to the magnitude of the k^{th} 2^{nd}-order filter block $T_k(\omega)=|T_k(j\omega)|$, given by:

$$f_k(\omega)=S_{T_k(\omega)}^{F(\omega)}=\mathrm{Re}\left[S_{T_k(s)}^{F(T_k,s)}\Big|_{s=j\omega}\right]. \tag{15}$$

The frequency-dependent functions $f_k(\omega)$ in (15) are lower than unity inside the pass-band of the filter. This reduction in sensitivity came as a consequence of negative feedbacks. Obviously, because the sensitivity in (15) is lower than unity, the FLF filter structure is less sensitive than the CAS structure, which has the sensitivity given by (11) equal to unity for the whole frequency range.

Furthermore, from (8) it is obvious that an additional sensitivity reduction of the FLF structure can be accomplished by the reduction of sensitivities $S_{x_{ij}}^{T_i}$ of each 2^{nd}-order block $T_i(s)$.

c) <u>CBQ Structure:</u> This is a cascade of minimal FLF structures. The minimal FLF has only two blocks inside feedback loop and it is called "biquart". Transfer function of the active filter structure is given by:

$$F(s)=\prod_{i=1}^{N/4}F_{4i};\ \tfrac{N}{2}\text{ even},\ F(s)=T_N\prod_{i=1}^{(N-2)/4}F_{4i};\ \tfrac{N}{2}\text{ odd}\,, \tag{16}$$

where

$$F_{4i}(s)=T_{i1}(s)T_{i2}(s)\cdot\left[1+\beta_i T_{i1}(s)T_{i2}(s)\right]^{-1}, \tag{17}$$

represents the transfer function of an i^{th} "biquart" section.

Sensitivity of the transfer function $F(s)$ to any element x_{jim} in m^{th} block of i^{th} "biquart" section is given by:

$$S_{x_{jim}}^{F}=S_{F_{4i}}^{F}\cdot S_{T_{im}}^{F_{4i}}\cdot S_{x_{jim}}^{T_{im}}\,. \tag{18}$$

Note that "biquarts" are cascaded, therefore $S_{F_{4i}}^{F}=1$ and the sensitivity optimisation of the cascaded-biquarts reduces to the optimisation of one biquart section. In [3] and [4] it was shown that if two 2^{nd}-order BP blocks inside a biquart section are identical, then the sensitivity of the biquart to each of its blocks $S_{T_{im}}^{F_{4i}}$ is minimal. Furthermore, since a biquart section is a special case of FLF section it can be desensitised in the same way.

d) LF Structure: Transfer function of the active filter is given by:

$$F(s)=\frac{1}{K(1,n)}\,, \tag{19}$$

where $K(1,n)$ represents continuants defined by:

$$K(i,n)=\frac{1}{T_i(s)}K(i+1,n)+\beta_i K(i+2,n);\ K(n,n)=\frac{1}{T_n(s)} \tag{20}$$

and $K(n+1,n)=1$; $i=1,\dots,n$; $n=N/2$. The sensitivity of the transfer function $F(s)$ to the variation of any building-block transfer function $T_i(s)$ is given by:

$$S_{T_i}^{F}=\frac{1}{1+\chi}\,,$$

where

$$\chi=T_i(s)\frac{\beta_{i-1}K(1,i-2)K(i+1,n)+\beta_i K(1,i-1)K(i+2,n)}{K(1,i-1)K(i+1,n)}\,. \tag{21}$$

Observing (21) note the proportionality of the χ to the transfer function of the i^{th} block $T_i(s)$, and that there is no $T_i(s)$ in the numerator and denominator of χ. That means, at frequencies where $T_i(s)\to 0$, the sensitivity $S_{T_i}^{F}$ is equal to unity, while at frequencies where $T_i(s)\to\infty$, $S_{T_i}^{F}$ is equal to zero. Since $T_i(s)$ are 2^{nd}-order BP blocks with infinite pole Q-factors, they take infinite values at pole frequencies, which are inside pass-band of the filter. This is the reason of very low sensitivity of LF filters [3].

III. EXAMPLE

As an illustration consider an 8^{th}-order Chebyshev narrow band BP filter with normalized bandwidth $B=0.1$, center frequency $\omega_0=1$, and reflection coefficient $\rho=10\%$, which corresponds to the pass-band ripple $R_p=0.044$dB. The filter is realized by structures in Fig. 1. Its transfer function magnitude $\alpha(\omega) = 20\log|F(j\omega)|$ dB is shown in Fig. 2(a) and (b), in large and small magnification, respectively.

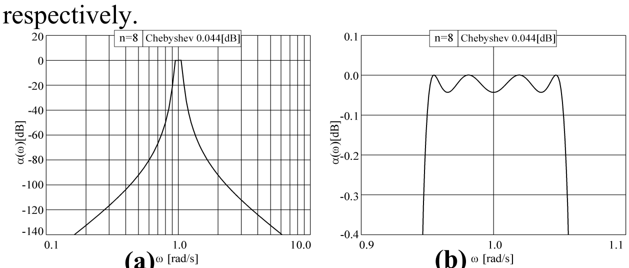

Fig. 2. Magnitude of narrow Chebyshev BP-filter used in the example. **(a)** Normal. **(b)** Magnified.

Normalized transfer function parameters of structures and theirs 2^{nd}-order blocks $T_i(s)$ are given in Table I. The gain values k_i and feedback values β_i ($i=1, \dots, 4$) in Table I are optimised by the method of D. J. Perry [5], to provide maximum dynamic range of the filter. The magnitude at the output of every block is equated to the maximally allowed level for a given input signal [3].

In what follows we present realizations of 2^{nd}-order sub-circuits $T_i(s)$ and summing devices of structures in Fig. 1.

Table I Parameters of structures in Fig. 1.

	i	q_p	ω_p	k	β
CAS	1	13.202	0.9755	1.0000	
	2	13.202	1.0251	1.4274	
	3	31.919	0.9420	2.1733	
	4	31.919	1.0615	7.1188	
FLF $\beta_0=0.4582$	1	18.665	1.0000	2.5933	0.0000
	2	18.665	1.0000	2.4669	0.8571
	3	18.665	1.0000	2.3572	0.2263
	4	18.665	1.0000	2.1840	0.1601
CBQ	1	13.198	1.0000	1.4482	
	2	13.198	1.0000	1.0363	0.2843
	3	31.862	1.0000	3.8230	
	4	31.862	1.0000	3.4848	1.0880
LF	1	9.357	1.0000	1.2085	0.8116
	2	∞	1.0000	0.07892*	0.7508
	3	∞	1.0000	0.08267*	0.6810
	4	9.307	1.0000	1.3737	

*Instead of k the value of $k\omega_p/q_p$ is given

1.) 2^{nd}-order transfer functions in (5) can be realized by a general purpose (GP) section [6] shown in Fig. 3.

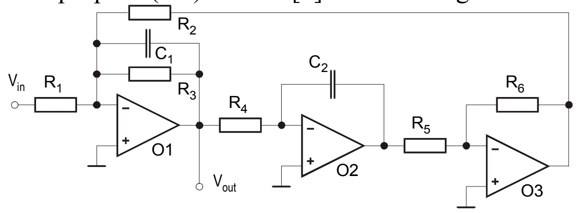

Fig. 3. GP-2 section.

Definitions of parameters for the section in Fig. 3 are:

$$k=\frac{R_3}{R_1};\ \omega_p^2=\frac{R_6}{R_2R_4R_5C_1C_2};\ q_p=R_3C_1\omega_p\,. \tag{22}$$

To design the 2^{nd}-order GP-2 filter section with given ω_p, q_p and k we accomplish the following step-by-step procedure:

i) Select C_1, C_2, R_4, R_5 and R_6 and calculate R_1, R_2, and R_3:

$R_2=R_6/(\omega_p^2\ R_4\ R_5\ C_1\ C_2)$

ii-a) If $q_p<<\infty$ we calculate R_1 and R_3 by:

$R_1=R_3/k$, $R_3=q_p/(C_1\ \omega_p)$.

ii-b) If $q_p\to\infty$ instead of k the value of $k^*=k\omega_p/q_p$ is given.

We have $R_3=\infty$ and $R_1=(k^*C_1)^{-1}$.

2.) 2^{nd}-order transfer functions $T_i(s)$ in (5) can be realised by a single-opamp BP active-*RC* filter shown in Fig. 4. [2].

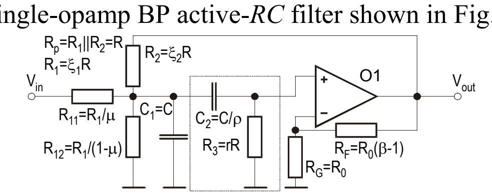

Fig. 4. BP active-*RC* class-4 allpole filter, with single-opamp (impedance scaling factors are r and ρ).

Definitions of parameters for that class-4 allpole filter are:

$$N(s)=\beta\frac{1}{R_1C_1};\ \beta=1+\frac{R_F}{R_G};\ a_0=\omega_p^2=\frac{R_1+R_2}{R_1R_2R_3C_1C_2}; \tag{23a}$$

$$a_1=\frac{\omega_p}{q_p}=\frac{R_1R_2C_1+(R_1R_2+R_1R_3+R_2R_3)C_2-\beta R_1R_3C_2}{R_1R_2R_3C_1C_2}\,. \tag{23b}$$

To design the 2^{nd}-order BP filter in Fig. 4 with given ω_p, q_p and k we accomplish the following step-by-step procedure:

i) Select r, ρ and ξ_1 and calculate ω_0 and β:

$$\omega_0=(RC)^{-1}=\omega_p\sqrt{r/\rho}\,;\ \beta=\xi_1[1+(1+\rho)/r-1/q_p\cdot\sqrt{\rho/r}]\,.$$

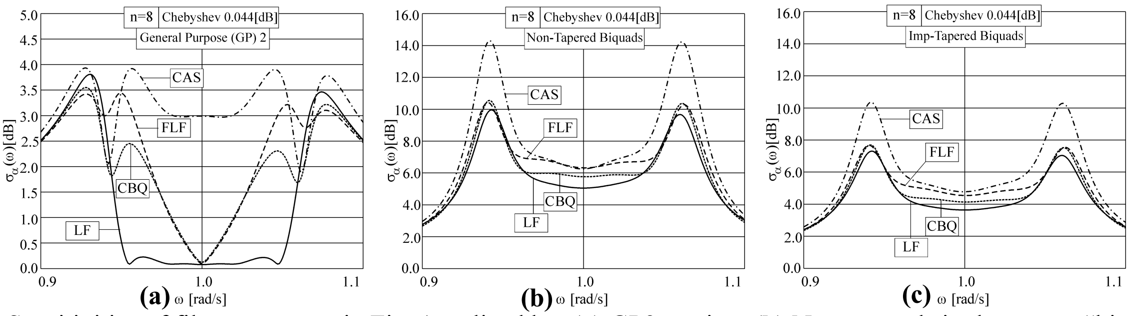

Fig. 5. Sensitivities of filter structures in Fig. 1 realized by: **(a)** GP2 section. **(b)** Non tapered single-opamp "biquads". **(c)** Ideally impedance-tapered single-opamp "biquads".

ii) Select C and compute R_1, R_2, R_3, and C_2:
$R=(\omega_0 C)^{-1}$, $R_1=\xi_1 R$, $\xi_2=\xi_1/(\xi_1-1)$, $R_2=\xi_2 R$, $R_3=rR$, $C_1=C$, $C_2=C/\rho$. Furthermore $R_{11}=R_1/\mu$; $R_{12}=R_1/(1-\mu)$ where $\mu=k/\beta$.
iii) Select R_G and calculate R_F : $R_F=R_G(\beta-1)$.
Note that in Fig. 4 the impedance scaled L section is inside dashed rectangle [2] [6].

3.) Summing devices at the input can be readily designed by the well-known scheme in Fig. 6 with one opamp.

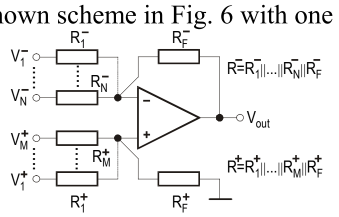

Fig. 6. General summing device configuration.

We present a general configuration of summing device having N negative inputs V_i^- (i=1,…,N), M positive inputs V_j^+ (j=1,…,M), and output V_{out}, where the voltage is given by:

$$V_{out}=-R_F^-\sum_{i=1}^{N} R_i^- V_i^- + R_F^- \frac{R^+}{R^-}\sum_{j=1}^{M} R_j^+ V_j^+ \ . \qquad (24)$$

Note that it is convenient to choose equal total resistances connected to the negative (R^-) and positive (R^+) input of the opamp, i.e. $R^+=R^-$.

A. Realization with GP-2 sections

For filter examples in Table I we realise 2nd-order sub-circuits by GP-2 sections in Fig. 3. A sensitivity analysis was performed assuming the relative changes of the inner 2nd-order sub-circuit resistors and capacitors to be uncorrelated normal random variables, with zero-means and 1% standard deviation. The standard deviation $\sigma_\alpha(\omega)$ dB (related to the Shoeffler sensitivities) of the variation of the log gain $\Delta\alpha=8.68588\Delta|F(j\omega)|/|F(j\omega)|$, with respect to passive elements, is shown in Fig. 5(a). The influences of feedback resistors to the sensitivity are not taken into consideration.

Observing the standard deviation $\sigma_\alpha(\omega)$ dB in Fig. 5(a) we conclude that structures with feedback loops has considerably decreased sensitivities, compared to the CAS structure. The LF structure gives the best results. For GP-2 section we have no design procedure, which can reduce the sensitivities of building filter blocks of 2nd-order.

B. Realization with single-opamp sections

A sensitivity analysis was performed for the filter examples in Table I, having 2nd-order single-opamp biquads in Fig. 4 as sub-circuits. In their design procedure, the values of r and ρ represent resistive and capacitive impedance scaling factors, respectively. In calculation of sub-circuit elements we have used "non-tapered" case when $\rho=r=1$, in building structures having sensitivities shown in Fig. 5(b). For an "ideally impedance tapered" biquads when $\rho=r=4$, the structure sensitivities are shown in Fig. 5(c).

Observing sensitivities in Fig. 5(c), we conclude about the significant sensitivity reduction of 8th-order BP filter structures using impedance tapered single-opamp 2nd-order sub-circuits. The sensitivities of such structures are higher but they are approaching sensitivities in Fig. 5(a) of BP filter structures, which have GP-2 sub-sections. Note the difference in the scale for $\sigma_\alpha(\omega)$. Although filters using GP-2 sections are less sensitive, from the point of power consumption the application of single-opamp filter sub-circuits is preferable because, we considerably reduced the number of opamps (in our example by 10).

IV. CONCLUSIONS

This paper presents the design of high-order narrow BP filters using multiple feedback filter structures and "impedance tapering" design technique applied on theirs sub-circuits. The "tapered" component values significantly decrease the sensitivity to passive component tolerances and the improvement comes free of charge; component count and topology remain unchanged. Thus, thank to impedance tapering, the application of single-opamp "biquads" in building high-order narrow BP filter has become realistic, because of additional decrease of structure's magnitude sensitivity. Although structures with "tapered" single-opamp "biquads" have larger sensitivity than the structures having GP-2 sections, the sensitivity of the former approaches to the latter. Main advantage is that single-opamp sub-circuits provide low power consumption, because the number of opamps in whole structure has been significantly reduced. Obviously, the LF structure has minimum sensitivity.

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