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A REDUNDANCY-BASED DESIGN BY EVENT-ORIENTED ANALYSIS

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Summary

The paper investigates a lifetime service of mechanical objects put through failures in order to illustrate the convenience of uncertainty modelling and redundancy based design by event-oriented system analysis (EOSA) in engineering. The numerical example presented in the paper provides results and comparisons with the traditional approach to the redundancy assessment based on reserve strength. It is demonstrated how EOSA identifies the system configuration and evaluates the service performance of potentially redundant objects in case of component failures and load redistribution. In the conclusion, EOSA appears to be an appropriate method for treating systems acting under uncertain conditions, and useful in the improvement of redundant system design in engineering.

Key words: *Mechanics, structures, information, probability, redundancy, safety*

PROJEKTIRANJE NA OSNOVI ZALIHOSTI PRIMJENOM DOGAĐAJIMA USMJERENE ANALIZE

Sažetak

U članku se ispituje cjeloživotno djelovanje mehaničkih objekata pri oštećivanju u cilju prikazivanja korisnosti ocjene zalihosti čvrstoće u modeliranju i projektiranju neizvjesnosti inženjerskih objekata primjenom sustavne analize usmjerene dagađajima. Numerički primjer u članku prikazuje rezultate i usporedbe sa tradicionalnim postupcima procjene zalihosti čvrstoće na osnovi pričuvne čvrstoće. Pokazano je kako se analizom usmjerene dagađajima mogu prepoznavati konfiguracije sustava i ocjenjivati svojstva objekata s mogućom zalihosti u slučaju oštećenja komponenti i prerasopdjele opterećenja. U zaključku, analiza usmjerena dagađajima izgleda kao odgovarajući postupak za ocjenu sustava koji djeluju u neizvjesnim okolnostima, te da se može primjeniti za poboljšanja inženjerskih projekata sa zalihostima.

Ključne riječi: *Mehanika, strukture, informacije, vjerojatnost, zalihost, sigurnost*

1. INTRODUCTION

The paper identifies the configuration of the simplest, potentially redundant object in terms of the event oriented system analysis (EOSA) [1], regardless of physical or any other properties. An example, typical in mechanical engineering, is subjected to numerical analysis in order to investigate the redundancy defined by the conditional entropy of operational modes [2]. The procedure in the paper employs the method for the analysis of multi-level systems of events [3] reduced to only two operational levels, considering discrete topology changes caused by step-wise system deterioration based on the theory presented in [4]. Commonly adopted time-invariant reliability methods can be applied to bring into relation the geometrical and physical properties of the considered object and the probabilities of occurrence of significant events in the lifetime [5], [6], [7], [8], [9], [10], [11] until a component failure occurs, but accounting for load redistribution after any component failure [12]. The entropy based redundancy measures are compared to the traditional probabilistic measure for redundancy viewed as the conditional probability of system survival given if any failure occurs [13], [14], [15], considering relative [16] and average measures for entropy [17]. The mechanical object investigated in the paper, Fig. 1, is selected using scantlings and materials typical in ship construction [18]. The conclusion offers the recognition of feasibility and usefulness of EOSA and a potential for improvement in the design of redundant technical object in lifetime service by an appropriate numerical support to EOSA.

By operational modes and effects analysis, all N, or at least all observable and important events ${}^{L}_{j}E^{s}_{i}$ in a lifetime service of a system, can be supposedly determined. The probabilities $p({}^{L}_{j}E^{s}_{i})$, $i = 1,2,...,N$, can be calculated by quantitative methods, where N is the total number of events constituting a system of events ${}^{L}_{j}S^{s}$. The lifetime functioning of an object in engineering can be represented by events grouped on functional levels denoted by "*L*", functional states and functional modes "j". The functional status "*s*" according to the common engineering reasoning may have one of the following meanings: *i*-intact, *c*-collapse, *t*-transitive, emerging, *n*-non-transitive, without emergent potentials, *o*-operational, *f*-failure, and combinations. The system modes are collected in the system of subsystems of events, which are denoted as the primary service profile of intact, transitive and collapse modes, represented as ${}^{L}\mathcal{S}^{itc}$ [4].

Any object of only one element provides one functional level with intact and collapse modes, denoted as simple alternatives and presented as a simple two-element system of events as shown:

$$^{1}\mathcal{S} = \left({}^{1}\mathcal{S}^{i} + {}^{1}\mathcal{S}^{c} \right) = \left({}^{1}E_{1}^{i} \quad {}^{1}E_{1}^{c} \right).$$

Such a system does not provide any transition and emergence of new functional levels as a precondition to system redundancy is not possible, Fig. 1b or 1c.

The simplest redundant object belonging to the class of 'fail-safe' systems with two functional states on the second level [4], Fig. 1, can be represented by a single functional state of one intact, two transitive and one collapse modes as a primary system of four events on the primary functional level as shown:

$$^{1}\mathcal{S} = \left({}^{1}\mathcal{S}^{i} + {}^{1}\mathcal{S}^{t} + {}^{1}\mathcal{S}^{c} \right) = \left({}^{1}E_{1}^{i} \quad {}^{1}E_{1}^{t} \quad {}^{1}E_{2}^{t} \quad {}^{1}E_{1}^{c} \right).$$

2. AN EXAMPLE OF A REDUNDANT STRUCTURAL OBJECT

The aim of this example is to illustrate the uncertainty assessment of redundant systems by using EOSA as theoretically explained in [4]. The notation and the equation numbers (n[4]) in this example follow reference [4], where 'n' is the equation number defined in [4].

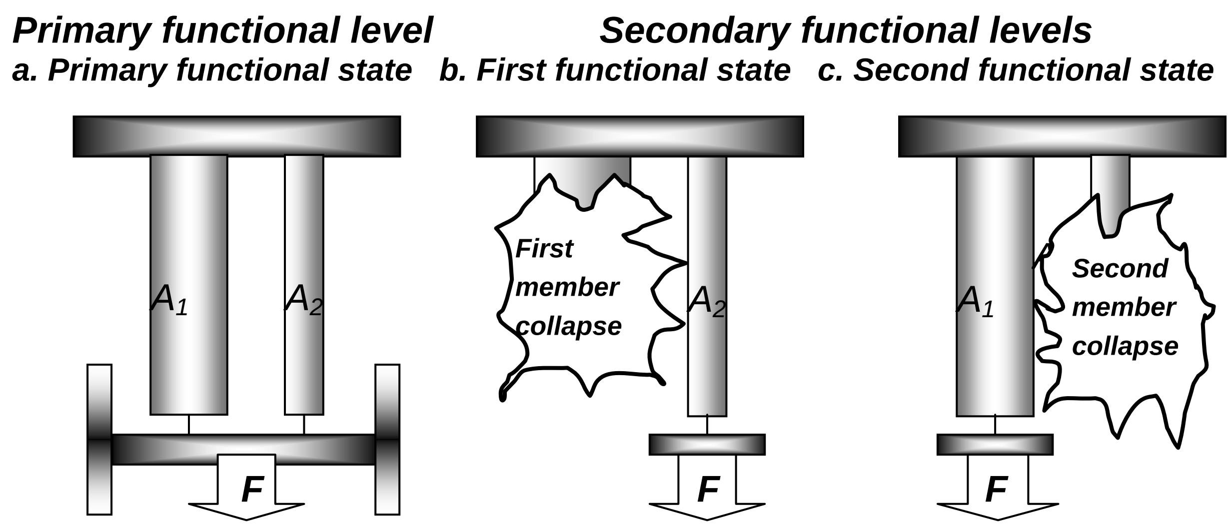

Figure 1. Simplest "fail-safe" or damage tolerant redundant object example

Let us apply EOSA to a simple redundant structural object exemplified as a two-member mechanical system of two parallel members (bars or ropes), Fig. 1, with deterministic sectional areas a_1 and a_2. A random tensile force F with mean value μ_F=1 MN and standard deviation of σ_F=0.3 MN vertically loads the object uniformly acting on both members.

The nominal yield stresses of the elements in amount of R_1=235 N/mm^2 and R_2=355 N/mm^2 are taken for mild and high tensile, hot rolled shipbuilding steel elements.

The mean values of static yield stresses are biased with respect to nominal values and assessed as μ_{R1}=1.16·235=272 N/mm^2 and μ_{R2}=1.16·355=376 N/mm^2, and the appropriate standard deviations of yield stresses are assessed as σ_{R1}=0.07·272=19 N/mm^2 and σ_{R2}=0.06·376=22 N/mm^2 [18].

Let the cross sectional areas a_1 and a_2 be deterministic free design variables of a redundancy (33[4]) maximization problem for the presumed object weight proportional to the sum of the sectional areas a_1+a_2, satisfying the minimally acceptable reliability requirements (24[4]), stated in a form of the following mathematical program:

$$Max\, RED({}^{2}\mathcal{S}^{i})$$

Subjected to:

$$p({}^{2}_{j}\mathcal{S}^{i}) = p({}^{2}_{j}E^{i}_{1}) \geq p_{acc}({}_{j}^{2}\mathcal{S}^{i}),\ j = 1,2.$$

A preliminary optimisation study indicates a family of solutions with maximal redundancy $RED({}^{2}\mathcal{S}^{i})$ (33[4]) satisfying the minimal reliability requirement (24[4]) $p_{acc}({}^{2}_{j}\mathcal{S}^{i}) = \Phi(-{}^{2}_{j}\beta = 0.5) = 0.69146$, where Φ is the standard normal density function and β is the safety index.

Results of EOSA applied to the optimal solution for cross sectional areas a_1=4333.4 mm^2 and a_2=3068.5 mm^2, which maximize the redundancy for the least weight object satisfying the minimal reliability criteria, are presented in the sequel [4].

3. THE PRIMARY PROBABILITY ANALYSIS

If the force F is resolved uniformly and the components $F/2$ are acting on the intact members, see Fig. 1a, the mean values of primary member stresses are assessed as:

$\mu_{S_1} = (\mu_F / 2)/a_1 = 115.4$ N/mm^2 and $\mu_{S_2} = (\mu_F / 2)/a_2 = 162.9$ N/mm^2.

The standard deviations of stresses are assessed as:

$o_{S_1} = (\sigma_F / 2)/a_1 = 34.6$ N/mm^2 and $o_{S_2} = (\sigma_F / 2)/a_2 = 48.8$ N/mm^2.

The nominal primary safety factors are $f_1 = R_1/S_1 = 2.036$ and $f_2 = R_2/S_2 = 2.178$.

The central safety factors are $c_1 = \mu_{R1}/\mu_{S1} = 2.357$ and $c_2 = \mu_{R2}/\mu_{S2} = 2.307$.

Primary reliabilities and collapse probabilities for components A_1 and A_2 of the intact object, Fig. 2a, are assessed by the distribution-free, level II or second-moment reliability analysis using safety index β [5], [6], [7], [8], [9], [10], [11], giving results as follows:

$$p({}^1_1A^i_1) = \Phi\left(-{}^1_1\beta = \frac{\mu_{R1} - \mu_{S_1}}{\sqrt{\sigma^2_{R_1} + \sigma^2_{S_1}}} \right) = \Phi\left(-{}^1_1\beta = 3.966\right) = 0.999964,\quad p({}^1_1A^c_1) = 1 - p({}^1_1A^i_1) = 0.000036$$

$$p({}^1_2A^i_1) = \Phi\left(-{}^1_2\beta = \frac{\mu_{R_2} - \mu_{S_2}}{\sqrt{\sigma^2_{R_2} + \sigma^2_{S_2}}} \right) = \Phi\left(-{}^1_2\beta = 3.974\right) = 0.999965,\quad p({}^1_2A^c_1) = 1 - p({}^1_2A^i_1) = 0.000035\ .$$

The probabilities of all possible individual random events comprising a single functional state on the first functional level can be defined as follows:

$p({}^1E^i_1) = p({}^1_1A^i_1)p({}^1_2A^i_1) = 0.9999292$ Fully operational (status: intact - i)

$p({}^1E^t_1) = p({}^1_1A^c_1)p({}^1_2A^i_1) = 0.000036$ The first member failure (status: transitive - t)

$p({}^1E^t_2) = p({}^1_1A^i_1)p({}^1_2A^c_1) = 0.000035$ The second member failure (status: transitive - t)

$p({}^1E^c_1) = p({}^1_1A^c_1)p({}^1_2A^c_1) = 1.250 \cdot 10^{-9}$ Fully non-operational (status: collapse - c).

The intact object, Fig. 1a, can be represented as a "fail-safe" system, by a single functional state as a primary system of events on the first level, as:

$${}^1\mathcal{S} = \left({}^1\mathcal{S}^i + {}^1\mathcal{S}^t + {}^1\mathcal{S}^c \right) = \begin{pmatrix} {}^1E^i_1 & {}^1E^t_1 & {}^1E^t_2 & {}^1E^c_1 \\ 0.9999292 & 0.000036 & 0.000035 & 1.25 \cdot 10^{-9} \end{pmatrix}.$$

The probabilities of the primary intact level (1[4]), of transition (2[4]), of collapse (3[4]), and of non-transition (4[4]) are as follows:

$p({}^1\mathcal{S}^i) = p({}^1E^i_1) = 0.9999292$, $\quad p({}^1\mathcal{S}^t) = \sum_{i=1}^{2} p({}^1E^t_i) = 0.000071$,

$p({}^1\mathcal{S}^c) = p({}^1E^c_1) = 1.25 \cdot 10^{-9}$ and $\quad p({}^1\mathcal{S}^n) = p({}^1E^i_1) + p({}^1E^c_1) = 0.9999292$.

The probability of primary level (5[4]) $p({}^1\mathcal{S}) = 1$ indicates a complete system.

The primary level can be viewed as a compound of the operational mode ${}^1\mathcal{S}^o = \left({}^1E^o_1 + {}^1E^t_1 + {}^1E^t_2\right)$ and the collapse mode ${}^1S^c = \left({}^1E^1_1\right)$. The reliability with respect to operations at the primary level (6[4]) is $p({}^1\mathcal{S}^o) = p({}^1E^i_1) + p({}^1E^t_1) + p({}^1E^t_2) = 1 - 1.25 \cdot 10^{-9}$.

The primary level can also be viewed as a compound of the intact mode ${}^1\mathcal{S}^i = \left({}^1E^i_1\right)$ and the failure mode ${}^1\mathcal{S}^f = \left({}^1E^t_1 + {}^1E^t_2 + {}^1E^c_1\right)$. The appropriate failure probability (7[4]) amounts to $p({}^1\mathcal{S}^f) = p({}^1\mathcal{S}^t) + p({}^1\mathcal{S}^c) = p({}^1E^t_1) + p({}^1E^t_2) + p({}^1E^c_1) = 0.000071$.

4. THE SECONDARY PROBABILITY ANALYSIS

The redundant object in Fig. 1 is expected to remain operational, although at a lower carrying capacity and reduced safety level, even if a member collapses [4], providing two functional states on the second level, pertinent to "fail-safe" systems, Figs. 1b and 1c. The mean values of governing secondary member tensile stresses under full loading F are obtained as $\mu_{S1}=\mu_F/a_1$=231.0 N/mm^2 and $\mu_{S2}=\mu_F/a_2$=325.9 N/mm^2. The appropriate standard deviations are $\sigma_{S1}=\sigma_F/a_1$=69.2 N/mm^2 and $\sigma_{S2}=\sigma_F/a_2$=97.8 N/mm^2. The nominal secondary safety factors are f_1=1.017 and f_2=1.089. The central safety factors are c_1=1.177 and c_2=1.153.

A repeated reliability calculus for redistributed member load of full amount of F=1 MN, provides the probabilities of the intact and collapse modes of the primary and secondary functional states at the second functional level, Figs. 1b and 1c, as follows:

$$p({}^2_1E^i_1) = \Phi\left(-{}^2_1\beta = \frac{\mu_{R_2}-\mu_{S_2}}{\sqrt{\sigma^2_{R_2}+\sigma^2_{S_2}}}\right) = \Phi\left(-{}^2_1\beta = 0.500\right) = 0.69146\ ,\quad p({}^2_1E^c_2) = 1 - p({}^2_1E^i_1) = 0.30854$$

$$p({}^2_2E^i_1) = \Phi\left(-{}^2_2\beta = \frac{\mu_{R_1}-\mu_{S_1}}{\sqrt{\sigma^2_{R_1}+\sigma^2_{S_1}}}\right) = \Phi\left(-{}^2_2\beta = 0.571\right) = 0.71604\ ,\quad p({}^2_2E^c_2) = 1 - p({}^2_2E^i_1) = 0.28396\,.$$

The two functional states at the second operational level in Fig. 1a and 1b, can be presented by two appropriate complete systems of two events, as follows:

$${}^2_1\mathcal{S} = \begin{pmatrix} {}^2_1E^i_1 & {}^2_1E^c_1 \\ 0.69146 & 0.30854 \end{pmatrix},\qquad {}^2_2\mathcal{S} = \begin{pmatrix} {}^2_2E^i_1 & {}^2_2E^c_1 \\ 0.71604 & 0.28396 \end{pmatrix}.$$

To each secondary functional state, reliabilities of remaining intact component (14[4]) and probabilities of collapse (55), respectively, can be assigned as: $p({}^2_1\mathcal{S}^i) = p({}^2_1E^i_1) = 0.69146$, $p({}^2_1S^c) = p({}^2_1E^c_1) = 0.30854$, $p({}^2_2\mathcal{S}^i) = p({}^2_2E^i_1) = 0.71604$ and $p({}^2_2S^c) = p({}^2_2E^c_1) = 0.28396$.

Secondary functional states are complete systems, since $p({}_1{}^2\mathcal{S}) = 1$ and $p({}_2{}^2\mathcal{S}) = 1$.

The transitions from one level to the next level are symbolically presented by two transitive conditional subsystems of events, as shown:

$${}^2_1\mathcal{S}\cap{}^1E^t_1 = \begin{pmatrix} {}^2_1E^i_1\cap{}^1E^t_1 & {}^2_1E^c_1\cap{}^1E^t_1 \\ 0.0000249 & 0.0000111 \end{pmatrix},\qquad {}^2_2\mathcal{S}\cap{}^1E^t_2 = \begin{pmatrix} {}^2_2E^i_1\cap{}^1E^t_2 & {}^2_2E^c_1\cap{}^1E^t_2 \\ 0.0000249 & 0.0000099 \end{pmatrix}.$$

The probabilities of establishing the secondary functional states (16[4]) amount to $p({}^2_1S\cap{}^1E^t_1) = p({}^1E^t_1) = 0.0000360$ and $p({}^2_2S\cap{}^1E^t_2) = p({}^1E^t_2) = 0.0000348$.

The secondary functional level is presented as a system comprised of the first level non-transitive events as well as of second level events conditioned on primary transitive events:

$${}^2S = \begin{pmatrix} {}^1E^i_1 & {}^2_1E^i_1\cap{}^1E^t_1 & {}^2_1E^c_1\cap{}^1E^t_1 & {}^2_2E^i_1\cap{}^1E^t_2 & {}^2_2E^c_1\cap{}^1E^t_2 & {}^1E^c_1 \\ 0.9999292 & 0.0000249 & 0.0000111 & 0.0000249 & 0.0000099 & 1.25\cdot10^{-9} \end{pmatrix}.$$

The secondary service profile equals the primary level system of events:

$${}^2_{\ }\mathcal{S}^{itc\,\prime} = \begin{pmatrix} {}^1\mathcal{S}^i_1 & {}^1E^t_1 & {}^1E^t_2 & {}^1\mathcal{S}^c_1 \\ 0.9999292 & 0.0000360 & 0.0000348 & 1.25\cdot10^{-9} \end{pmatrix} \equiv {}^1\mathcal{S}\,.$$

The conditional probabilities of secondary intact and collapse modes (18, 19, 20[4]) are

$$p({}_1{}^2\mathcal{S}^i\cap{}^1E^t_1) = p({}^2_1E^i_1)p({}^1E^t_1) = 0.0000249,\ p({}_1{}^2\mathcal{S}^c\cap{}^1E^t_1) = p({}^2_1E^c_i)p({}^1E^t_1) = 0.0000111$$

$$p({}_2{}^2\mathcal{S}^i\cap{}^1E^t_2) = p({}^2_2E^i_1)p({}^1E^t_2) = 0.0000249\,,\ p({}_2{}^2\mathcal{S}^c\cap{}^1E^t_2) = p({}^2_2E^c_i)p({}^1E^t_2) = 0.0000099.$$

The secondary intact and collapse modes are composed as ${}^2_{\ }\mathcal{S}^i = \begin{pmatrix} {}^2_1E^i_1\cap{}^1E^t_1 & {}^2_2E^i_1\cap{}^1E^t_2 \\ 0.0000249 & 0.0000249 \end{pmatrix}$.

The secondary collapse profile can be presented as ${}^{2}\mathcal{S}^{c\prime} = \left({}^{2}_{1}\mathcal{S}^{c} \cap {}^{1}E^{t}_{1}, {}^{2}_{2}\mathcal{S}^{c} \cap {}^{1}E^{t}_{2} \right) = {}^{2}\mathcal{S}^{c}$.

The secondary reliability (20[4]) : $p({}^{2}\mathcal{S}^{i}) = p({}^{2}_{1}E^{i}_{1})p({}^{1}E^{t}_{1}) + p({}^{2}_{2}E^{i}_{1})p({}^{1}E^{t}_{2}) = 0.0000498$.

The secondary failure probability (21[4]) : $p({}^{2}\mathcal{S}^{c}) = p({}^{2}_{1}E^{c}_{1})p({}^{1}E^{t}_{1}) + p({}^{2}_{2}E^{c}_{1})p({}^{1}E^{t}_{2}) = 0.000021$.

The overall reliability is $p({}^{2}\mathcal{S}^{o}) = p({}^{1}\mathcal{S}^{i} + {}^{2}\mathcal{S}^{i}) = p({}^{1}E^{i}_{1}) + p({}^{2}_{1}E^{i}_{1})p({}^{1}E^{t}_{1}) + p({}^{2}_{2}E^{i}_{1})p({}^{1}E^{t}_{2}) = 0.999979$.

The failure probability (53[4]) : $p({}^{2}\mathcal{S}^{f}) = p({}^{1}\mathcal{S}^{c} + {}^{2}\mathcal{S}^{c}) = p({}^{1}E^{c}_{1}) + p({}^{2}_{1}E^{c}_{1})p({}^{1}E^{t}_{1}) + p({}^{2}_{2}E^{c}_{1})p({}^{1}E^{t}_{2}) = 0.000021$.

5. THE PRIMARY LEVEL UNCERTAINTIES

The unconditional entropy of the primary level (8[4]) of four events amounts to:

$$H({}^{1}\mathcal{S}) = H({}_{1}{}^{1}\mathcal{S}) = -\sum_{i=1}^{4} p({}^{1}E_{i}) \log p({}^{1}E_{i}) = 0.0011482 \text{ bits } (2,\ 0.000574,\ 1.000796) \cdot$$

Note that maximal, relative [16] and average [17] values are listed in parentheses.

The unconditional entropy of the primary service profile (9[4]) of three modes is:

$$H({}^{1}\mathcal{S}^{itc\prime}) = -p({}^{1}\mathcal{S}^{i}) \log p({}^{1}\mathcal{S}^{i}) - p({}^{1}\mathcal{S}^{t}) \log p({}^{1}\mathcal{S}^{t}) - p({}^{1}\mathcal{S}^{c}) \log p({}^{1}\mathcal{S}^{c}) =$$
$$= 0.0010775 \text{ bits } (1.585,\ 0.00068,\ 1.000747).$$

The conditional entropy of the primary level with respect to the intact mode (10[4]), also denoted as redundancy, vanishes: $H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{i}) = RED({}^{1}\mathcal{S}^{i}) = -\dfrac{p({}^{1}E^{i}_{1})}{p({}^{1}\mathcal{S}^{i})} \log \dfrac{p({}^{1}E^{i}_{1})}{p({}^{1}\mathcal{S}^{i})} = 0 \text{ bits} \cdot$

The redundancy of the primary level with respect to the transitive mode (11[4]) is:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{t}) = RED({}^{1}\mathcal{S}^{t}) = -\sum_{i=1}^{2} \frac{p({}^{1}E^{t}_{i})}{p({}^{1}\mathcal{S}^{t})} \log \frac{p({}^{1}E^{t}_{i})}{p({}^{1}\mathcal{S}^{t})} = 0.999774 \text{ bits } (1,\ 0.999774,\ 1.999687)$$

The robustness of the first functional level with respect to the collapse mode (12[4]) vanishes: $H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{c}) = ROB({}^{1}\mathcal{S}^{c}) = -\dfrac{p({}^{1}E^{c}_{1})}{p({}^{1}\mathcal{S}^{c})} \log \dfrac{p({}^{1}E^{c}_{1})}{p({}^{1}\mathcal{S}^{c})} = 0 \text{ bits} \cdot$

The primary level uncertainty can also be viewed as the conditional entropy of the first functional level with respect to the service profile of intact, transitive and collapse modes. In this way it is shown how the uncertainty of the service profile reduces uncertainty (13[4]):

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{itc\prime}) = p({}^{1}\mathcal{S}^{t})RED({}^{1}\mathcal{S}^{t}) = H({}^{1}S) - H({}^{1}S^{itc\prime}) = 0.0000707 \text{ bits } (2, 0.0000353, 1.000049)$$

The redundancy of the primary level with respect to the operational mode amounts to:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{o}) = RED({}^{1}\mathcal{S}^{o}) = -\frac{p({}^{1}E^{i}_{1})}{p({}^{1}\mathcal{S}^{o})} \log \frac{p({}^{1}E^{i}_{1})}{p({}^{1}\mathcal{S}^{o})} - \sum_{i=1}^{2} \frac{p({}^{1}E^{t}_{i})}{p({}^{1}S^{o})} \log \frac{p({}^{1}E^{t}_{i})}{p({}^{1}S^{o})} =$$
$$= 0.0011482 \text{ bits } (1.585, 0.000724, 1.000796).$$

The entropy of the primary service profile with respect to operational and collapse modes, vanishes due to much higher operational probability than the probability of collapse:

$$H({}^{1}\mathcal{S}^{oc\prime}) = -p({}^{1}\mathcal{S}^{o}) \log p({}^{1}\mathcal{S}^{o}) - p({}^{1}\mathcal{S}^{c}) \log p({}^{1}\mathcal{S}^{c}) \approx 0 \text{ bits}.$$

The operational uncertainty is expressed by the conditional entropy of the first level with respect to the service profile of operational and collapse modes as:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{oc\prime}) = p({}^{1}\mathcal{S}^{o})RED({}^{1}\mathcal{S}^{o}) = H({}^{1}S) - H({}^{1}S^{oc\prime}) \approx H({}^{1}S) = 0.0011482 \text{ bits } (2, 0.000574, 1.000796).$$

The conditional entropy of the primary level with respect to the failure mode is equal to:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{f}) = RED({}^{1}\mathcal{S}^{f}) = -\sum_{i=1}^{2} \frac{p({}^{1}E^{t}_{i})}{p({}^{1}\mathcal{S}^{f})} \log \frac{p({}^{1}E^{t}_{i})}{p({}^{1}\mathcal{S}^{f})} - \frac{p({}^{1}E^{c}_{1})}{p({}^{1}\mathcal{S}^{f})} \log \frac{p({}^{1}E^{c}_{1})}{p({}^{1}\mathcal{S}^{f})} =$$
$$= 1.000061 \text{ bits } (1.585,\ 0.63096,\ 2.000085).$$

The entropy of the primary service profile comprised of the intact and failure modes is:

$$H({}^{1}\mathcal{S}^{if}{}') = -p({}^{1}\mathcal{S}^{i})\log p({}^{1}\mathcal{S}^{i}) - p({}^{1}\mathcal{S}^{f})\log p({}^{1}\mathcal{S}^{f}) = 0.0010775 \text{ bits } (1, 0.0010775, 1.000747) .$$

The conditional entropy of the first functional level, with respect to the service profile of the intact and failure modes, expresses the uncertainty in the intact condition as follows:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{if}{}') = p({}^{1}\mathcal{S}^{f})RED({}^{1}\mathcal{S}^{f}) = H({}^{1}\mathcal{S}) - H({}^{1}\mathcal{S}^{if}{}') = 0.0000707 \text{ bits } (2, 0.0000353, 1.000049).$$

The conditional entropy of primary level with respect to non-transitive mode vanishes:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{n}) = -\frac{p({}^{1}E_{1}^{i})}{p({}^{1}\mathcal{S}^{n})}\log\frac{p({}^{1}E_{1}^{i})}{p({}^{1}\mathcal{S}^{n})} - \sum_{i=1}^{2}\frac{p({}^{1}E_{i}^{c})}{p({}^{1}\mathcal{S}^{n})}\log\frac{p({}^{1}E_{i}^{c})}{p({}^{1}\mathcal{S}^{n})} = 0 \text{ bits} .$$

The entropy of the primary service profile of non-transitive and transitive modes is equal to $H({}^{1}\mathcal{S}^{nt}{}') = -p({}^{1}\mathcal{S}^{n})\cdot\log p({}^{1}\mathcal{S}^{n}) - p({}^{1}\mathcal{S}^{t})\cdot\log p({}^{1}\mathcal{S}^{t}) = 0.0010775 \text{ bits } (1, 0.0010775, 1.000747)$

The conditional entropy of the first functional level with respect to the service profile of non-transitive and transitive modes is as follows:

$$H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{nt}{}') = p({}^{1}\mathcal{S}^{t})\cdot H({}^{1}\mathcal{S}/{}^{1}\mathcal{S}^{t}) = H({}^{1}\mathcal{S}) - H({}^{1}\mathcal{S}^{nt}{}') = 0.0000707 \text{ bits } (2, 0.0000353, 1.000049$$

The probabilistic redundancy measures [13] [14] [15] (23[4]) are calculated on the basis of the residual strength (2[4]), amounting to $p({}^{1}\mathcal{S}^{t}) = 0.000078$, as follows:

$$R_{I} = \frac{p({}^{1}\mathcal{S}^{t})}{p({}^{1}\mathcal{S}^{f})} = 0.999823 , \quad R_{F} = \frac{p({}^{1}\mathcal{S}^{t})}{p({}^{1}\mathcal{S}^{c})} = 56552 \text{ and } R_{O} = \frac{p({}^{1}\mathcal{S}^{t})}{p({}^{1}\mathcal{S}^{o})} = 0.00007074 .$$

6. THE SECONDARY UNCERTAINTY ANALYSIS

The entropies of the two independent secondary functional states, each comprising two events [4] regardless of the transitional character of the considered system (25), amount to:

$$H({}_{1}^{2}\mathcal{S}) = -p({}_{1}^{2}E_{1}^{i})\cdot\log p({}_{1}^{2}E_{1}^{i}) - p({}_{1}^{2}E_{1}^{c})\cdot\log p({}_{1}^{2}E_{1}^{c}) = 0.891471 \text{ bits } (1, 0.891471, 1.8551)$$

$$H({}_{2}^{2}\mathcal{S}) = -p({}_{2}^{2}E_{1}^{i})\cdot\log p({}_{2}^{2}E_{1}^{i}) - p({}_{2}^{2}E_{1}^{c})\cdot\log p({}_{2}^{2}E_{1}^{c}) = 0.860792 \text{ bits } (1, 0.860792, 1.8160) .$$

The secondary system of events is a complete system of six events since $p({}^{2}\mathcal{S}) = 1$.

The unconditional entropy of the secondary level (27[4]) is calculated as follows:

$$H({}^{2}\mathcal{S}) = \sum_{all\, E\in{}^{2}\mathcal{S}} p(E)\log p(E) = 0.0012102 \text{ bits } (2.5849, 0.0004682, 1.000839) .$$

The conditional entropy of the secondary level with respect to the primary level (28[4]) is equal to:

$$H({}^{2}\mathcal{S}/{}^{1}\mathcal{S}) = \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t})\cdot H({}_{j}^{2}\mathcal{S}) = 0.000062 \text{ bits } (2, 0.000031, 1.000043) .$$

The unconditional entropy of the secondary service profile (29[4]) of four modes is equal to $H({}^{2}\mathcal{S}^{itc}{}') = H({}^{1}\mathcal{S}) = -p({}^{1}\mathcal{S}^{i})\log p({}^{1}\mathcal{S}^{i}) - \sum_{i=1}^{2} p({}^{1}E_{i}^{t})\log p({}^{1}E_{i}^{t}) - p({}^{1}\mathcal{S}^{c})\log p({}^{1}\mathcal{S}^{c}) =$

$$= 0.0011482 \text{ bits } (2, 0.0005741, 1.000796).$$

The uncertainty of the secondary functional level can be expressed in this example by the secondary conditional entropy with respect to the primary level (30[4]), as shown:

$$H\left({}^{2}\mathcal{S}/{}^{2}\mathcal{S}^{itc}{}'\right) = H\left({}^{2}\mathcal{S}/{}^{1}\mathcal{S}\right) = \sum_{j=1}^{2} p({}^{1}E_{j}^{t})H({}_{j}^{2}\mathcal{S}) = 0.0000620 \text{ bits } (2, 0.0000310, 1.000043) .$$

The increments in unconditional uncertainties of functional levels and service profiles due to transition from the primary to the secondary level are calculated as shown:

$$H({}^{2}\mathcal{S}) - H({}^{1}\mathcal{S}) = \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t}) \cdot H({}_{j}^{2}\mathcal{S}) = 0.0000620 \text{ bits}$$

$$H({}^{2}\mathcal{S}^{itc}{}') - H({}^{1}\mathcal{S}^{itc}{}') = p({}^{1}\mathcal{S}^{t}) \cdot RED({}^{1}\mathcal{S}) = 0.0000707 \text{ bits,}$$ respectively.

The conditional entropies of the individual secondary functional states (31, 32[4]), for $j=1,2$, vanish, since they are series systems with only one intact and collapse mode:

$$H({}_{j}^{2}\mathcal{S} / {}_{j}^{2}\mathcal{S}^{i}) = RED({}_{j}^{2}\mathcal{S}) = 0 \text{ bits} \quad \text{and} \quad H({}_{j}^{2}\mathcal{S} / {}_{j}^{2}\mathcal{S}^{c}) = ROB({}_{j}^{2}\mathcal{S}) = 0 \text{ bits} .$$

The secondary level does not provide any system redundancy, only robustness, since each functional state renders only one intact mode and possibly a number of collapse modes.

The conditional entropy of the second level with respect to the intact mode (33[4]) is expressed as: $H({}^{2}\mathcal{S} / {}^{2}\mathcal{S}^{i}) = RED({}^{2}\mathcal{S}^{i}) - \sum_{j=1}^{2}\sum_{i=1}^{1} \frac{p({}^{1}E_{j}^{t})p({}_{j}^{2}E_{i}^{i})}{p({}^{2}\mathcal{S}^{i})} \log \frac{p({}^{1}E_{j}^{t})p({}_{j}^{2}E_{i}^{i})}{p({}^{2}\mathcal{S}^{i})} = 1 \text{ bit } (1,1,2)$.

The conditional entropy of the second level with respect to collapse state (34[4]) is expressed as $H({}^{2}\mathcal{S} / {}^{2}\mathcal{S}^{c}) = ROB({}^{2}\mathcal{S}^{c}) = -\sum_{j=1}^{2}\sum_{i=1}^{1} \frac{p({}^{1}E_{j}^{t})p({}_{j}^{2}E_{i}^{c})}{p({}^{2}\mathcal{S}^{c})} \log \frac{p({}^{1}E_{j}^{t})p({}_{j}^{2}E_{i}^{c})}{p({}^{2}\mathcal{S}^{c})} =$

$= 0.997477$ bits (1, 0.997477, 1.996506).

Equations (34, 35 and 36[4]) result with the same values in this example.

The unconditional entropy of the secondary service profile of primary non-transitive and emerged intact and collapse modes (37[4]) is calculated as follows:

$$H({}^{2}\mathcal{S}^{ic}{}') = -p({}^{1}\mathcal{S}^{i})\log p({}^{1}\mathcal{S}^{i}) - p({}^{2}\mathcal{S}^{i})\log p({}^{2}\mathcal{S}^{i}) - p({}^{2}\mathcal{S}^{c})\log p({}^{2}\mathcal{S}^{c}) - p({}^{1}\mathcal{S}^{c})\log p({}^{1}\mathcal{S}^{c}) =$$

$= 0.001139$ bits (2,0.0005695,1.000790).

The conditional entropy of the secondary collapse mode with respect to collapse profile (38[4]) relates the state and mode robustness, but in this example it vanishes:

$$H({}^{2}\mathcal{S}^{c} / {}^{2}\mathcal{S}^{c}{}') = \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t}) p({}_{j}^{2}\mathcal{S}^{c}) ROB({}_{j}^{2}\mathcal{S}^{c}) = p({}^{2}\mathcal{S}^{c})\left[ROB({}^{2}\mathcal{S}^{c}) - ROB({}^{2}\mathcal{S}^{c}{}')\right] = 0 \text{ bits} .$$

The unconditional entropy of the secondary service profile of intact and collapse modes is calculated according (39[4]) as follows:

$$H({}^{2}\mathcal{S}^{ic}{}'') = -p({}^{2}\mathcal{S}^{i}) \cdot \log p({}^{2}\mathcal{S}^{i}) - p({}^{2}\mathcal{S}^{c}) \cdot \log p({}^{2}\mathcal{S}^{c}) - \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t}) p({}_{j}^{2}\mathcal{S}^{c}) \log p({}^{1}E_{j}^{t}) p({}_{j}^{2}\mathcal{S}^{c}) =$$

$= 0.001160$ bits $(2.3219, 0.000499, 1.000805)$.

The conditional entropy of the secondary level with respect to the service profile of primary and secondary intact and collapse modes (40[4]), relates the primary and secondary redundancy and robustness, and can be calculated as follows:

$$H({}^{2}\mathcal{S} / {}^{2}\mathcal{S}^{ic}{}') = +p({}^{2}\mathcal{S}^{i}) RED({}^{2}\mathcal{S}^{i}) + p({}^{2}\mathcal{S}^{c}) ROB({}^{2}\mathcal{S}^{c}) =$$

$= H({}^{2}\mathcal{S}) - H({}^{2}\mathcal{S}^{ic}{}') = 0.000071$ bits (2.5849, 0.0000274, 1.000049).

The conditional entropy of the secondary level with respect to the service profile of all primary and secondary intact and collapse modes (41[4]) is calculated as follows:

$$H({}^{2}\mathcal{S} / {}^{2}\mathcal{S}^{ic}{}'') = p({}^{1}\mathcal{S}^{c}) ROB({}^{1}\mathcal{S}^{c}) + p({}^{2}\mathcal{S}^{i}) RED({}^{2}\mathcal{S}^{i}) + \sum_{j=1}^{{}^{1}N^{t}} p({}^{1}E_{j}^{t}) p({}_{j}^{2}\mathcal{S}^{c}) ROB({}_{j}^{2}\mathcal{S}^{c}) =$$

$= p({}^{2}\mathcal{S}^{i}) RED({}^{2}\mathcal{S}^{i}) = H({}^{2}\mathcal{S}) - H({}^{2}\mathcal{S}^{ic}{}'') = 0.000051$ bits (2.5849, 0.0.000019 34, 1.000035).

The secondary conditional entropies with respect to the intact and collapse modes are:

$H({}^{2}\mathcal{S} / {}^{2}\mathcal{S}^{o}) = RED({}^{2}\mathcal{S}^{o}) = 0.000833$ bits (1.584, 0.000525, 1.000578)

$H({}^{2}\mathcal{S} / {}^{2}\mathcal{S}^{f}) = RED({}^{2}\mathcal{S}^{f}) = 0.998341$ bits (1,584, 0.629883, 1.99770), respectively.

7. PARAMETRIC STUDIES AND REDUNDANCY-BASED DESIGN

Parametric studies in the sequel demonstrate the usage of EOSA in the design of fail-safe redundant objects in engineering [4]. First, the primary level probabilities (1, 2, 3[4]), the primary redundancy expressed by the conditional entropy of transitive events (11[4]) and the redundancy indices (23[4]) for a redundant object of two members without redistribution of loads, Fig. 1a, are subjected to a parametric study. The range of the first member reliability $p({}_1^1A_1^i)$ from 0.2 to 1.0 related to the second member reliability as $p({}_2^1A_1^i) = 1.2 - p({}_1^1A_1^i)$ is selected for the study, Fig. 2. The study indicates that the increase of the residual strength probability expressed by the increase of redundancy indices R_I and R_O (23[4]) implies simultaneous diminution of intact (1[4]) and collapse (2[4]) probabilities. The non-uniform distribution of the reserve strength between the members indicates that the object may remain operational mostly due to the failure of the member with lower operational probability. It is due to the fact that it provides very low reserve strength in case of the failure of the member with higher operational probability, i.e. with higher reserve strength. This means that there is practically only one secondary functional state, because another one is almost improbable. Moreover, for the maximal values of redundancy indices the object is practically not redundant at all, since only one member provides the maximal reserve strength for limiting reliability values of 0.2 and 1.0, Fig. 2.

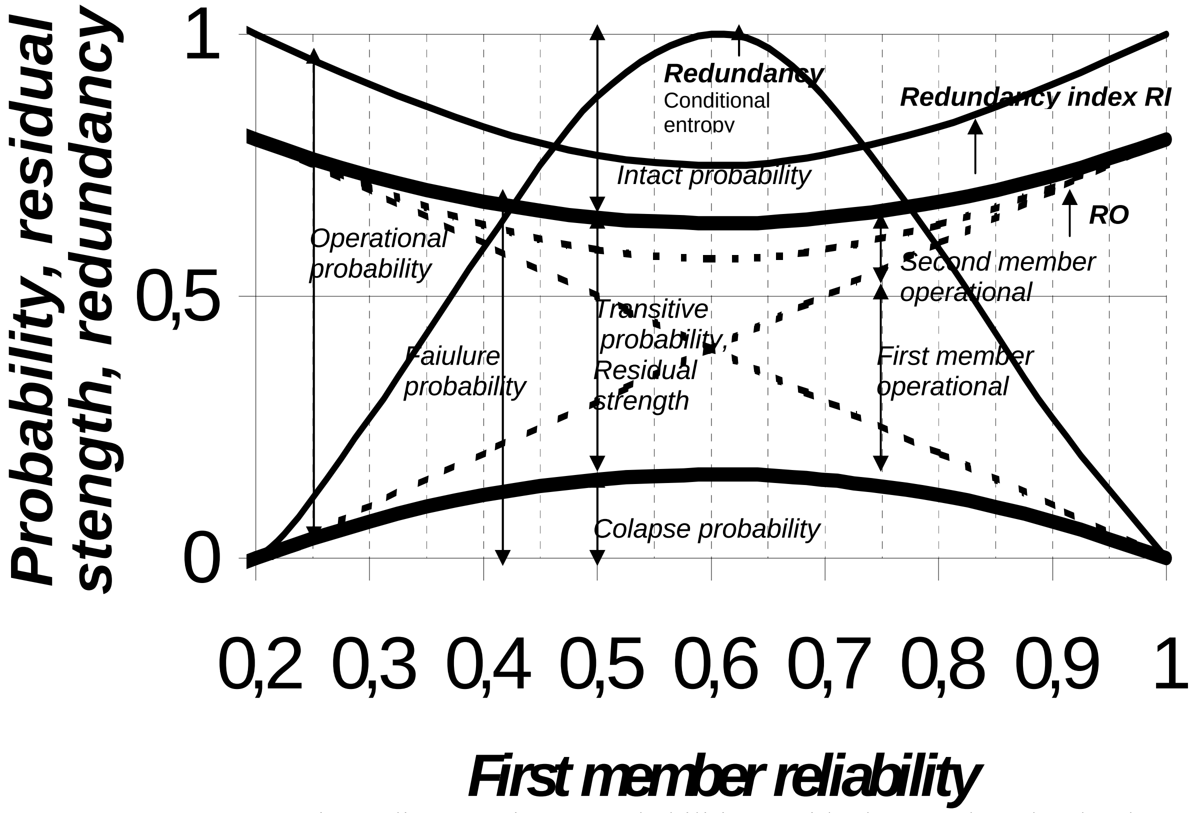

Figure 2. Parametric studies on primary probabilities, residual strength and redundancy

The minimal values of redundancy indices are attained for uniform distribution of reserve strength between the members and for the maximal attainable intact probability. The maximal primary redundancy expressed by the conditional entropy of transitive mode (11[4]) indicates the object with identical probabilities of transitive events, Fig. 2. However, the maximal primary redundancy (11[4]) encountered for $p({}_1^1A_1^i) = 0.6$ and $p({}_2^1A_1^i) = 0.6$, does not indicate the highest probability of the reserve strength, but the uniform distribution of member's reserve strength and the highest system's intact probability.

The following points are outlined from the first parametric study:

- The maximal secondary redundancy (33[4]) $RED({}^2\mathcal{S}^i) = \log {}^2N^i = \log 2 = 1\,\text{bit}$, is attained for a_1=4089 mm^2 and a_2=2915 mm^2, indicating the most uniformly distributed secondary probabilities of intact modes for each member (16[4]) $p({}_j^2\mathcal{S}^i \cap {}^1E_j^t),\ j = 1,2$.
- Consequently, the attained minimal object weight corresponding to the minimal overall cross-sectional area of a_1+a_2=7000 mm^2 is very close to the solution with maximal secondary redundancy, Fig. 3. Such a solution indicates that the uniform reliability distribution involves a rational distributon of materials for redundant objects.
- However, the redundancy index (23[4]) in this example attains its minimal value very close to the solution for maximal redundancy (33[4]), Fig. 3, indicating the minimal probability of the reserve strength and maximal primary intact probability.
- The increase of the probabilistic index (23[4]) indicates an undesired diminution of the probability of the primary intact mode (1[4]) $p({}^1\mathcal{S}^i)$. The desired common sense option would be rather the diminution of the primary collapse mode (3[4]) $p({}^1\mathcal{S}^c)$. In order to assure the maximal probability of the primary intact mode, the redundancy index (23[4]) has to be as low as possible under the condition that the secondary functional states satisfy the minimal safety requirements.

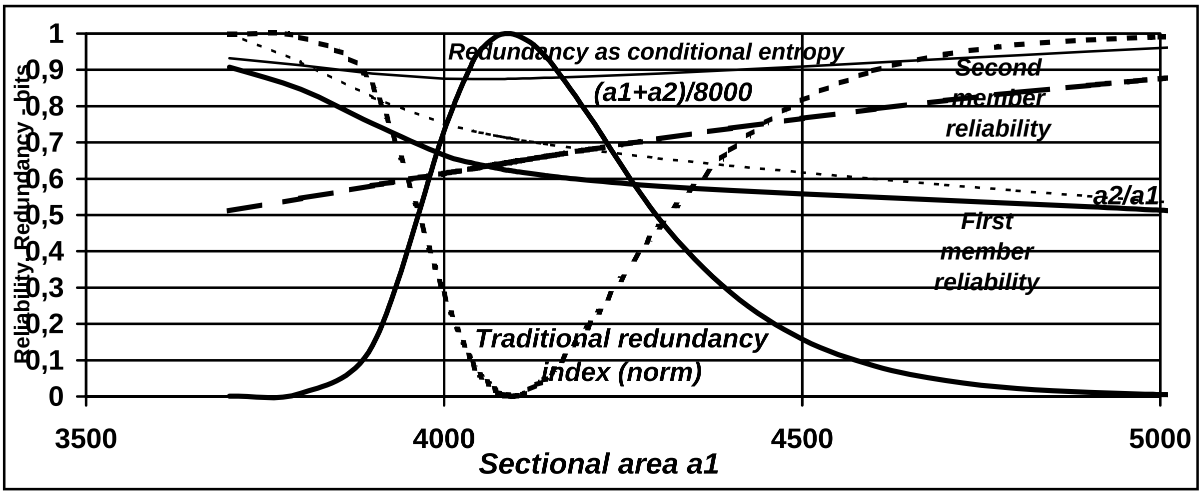

Figure 3. Secondary member reliabilities, redundancy and weight for reliability of *0.9999*

Next, a range of member cross-sectional areas a_1 and a_2 for a requested probability of primary and secondary intact modes denoted as overall reliability of $p({}^1\mathcal{S}^i + {}^2\mathcal{S}^i) = 0.9999$ is subjected to EOSA. The aim of this study is to evaluate the member secondary reliability (14[4]) $p({}_1^2\mathcal{S}^i)$ and $p({}_2^2\mathcal{S}^i)$, as well as the secondary redundancy (33[4]) $RED({}^2\mathcal{S}^i)$ and the redundancy index R_I normalized with respect to its minimal value [4], Fig. 3.

The subsequent study investigates how the maximal attainable system's secondary redundancy (33[4]) $RED({}^2\mathcal{S}^i) = \log {}^2N^i = \log 2 = 1\,\text{bit}$ affects the object weight and the overall reliability of the system. Two related important safety aspects of a "fail-safe" object are the reliabilities of independent secondary functional states (14[4]) $p({}_1^2\mathcal{S}^i)$, $p({}_2^2\mathcal{S}^i)$ Fig. 4, and the overall system reliability $p({}^1\mathcal{S}^i + {}^2\mathcal{S}^i)$ [4] accounting for both, primary and secondary level effectiveness, Fig. 5. The maximal redundancy in this example implies identical compound probabilities of intact modes for each member (16[4]) $p({}_1^2\mathcal{S}^i \cap {}^1E_1^t) = p({}_2^2\mathcal{S}^i \cap {}^1E_2^t)$.

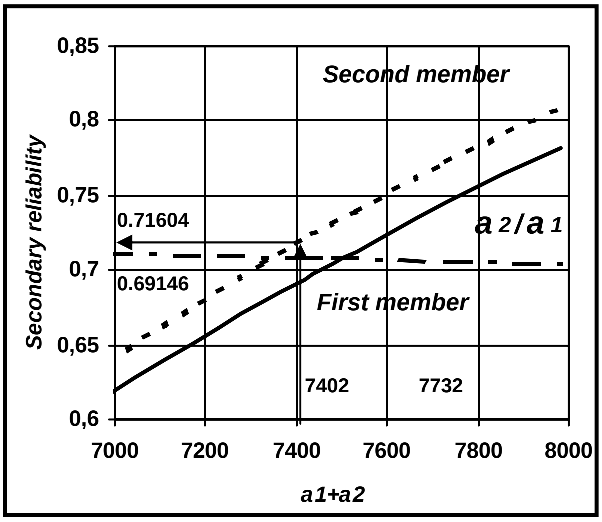

Figure 4. Reliabilities of secondary states for maximal secondary redundancy

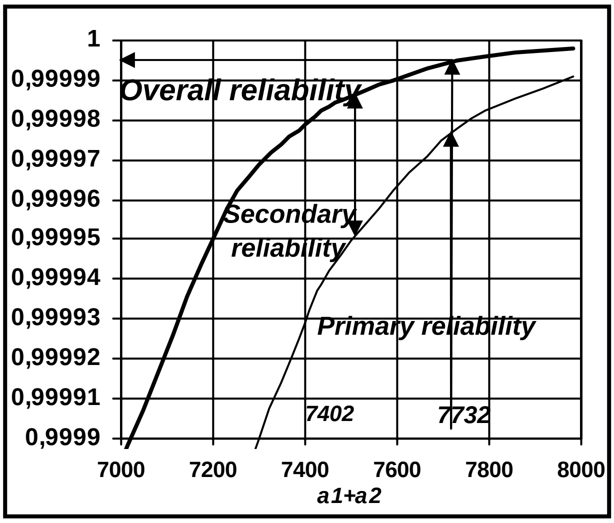

Figure 5. Overall system secondary reliability for maximal secondary redundancy

The following points are outlined from the second parametric study:

- The minimal reliability of all secondary functional states (24[4]) is expressed in terms of safety indices as $Min({}_{j}^{2}\beta,\ j = 1,2) \geq 0.5$. The task is accomplished by the optimisation study at the beginning of the example, for a_1=4333.4 mm^2, a_2=3068.5 mm^2, providing the first and second member reliabilities (44[4]) of $p({}_{1}^{2}\mathcal{S}^{i}) = \Phi(-{}_{1}^{2}\beta = 0.5) = 0.69146$ and $p({}_{2}^{2}\mathcal{S}^{i}) = \Phi(-{}_{2}^{2}\beta = 0.571) = 0.71604$, Fig. 4. The minimal object weight corresponds to the minimal overall cross-sectional area of a_1+a_2=7302 mm^2, yielding the overall reliability of $p({}^{1}\mathcal{S}^{i} + {}^{2}\mathcal{S}^{i}) = p({}^{1}\mathcal{S}^{i}) + p({}^{2}\mathcal{S}^{i}) = 0.9999292 + 0.0000498 = 0.9999790$, Fig. 5.

- The secondary conditional reliability (20[4]) $p({}^{2}\mathcal{S}^{i})$ adds little to the overall reliability $p({}^{1}\mathcal{S}^{i} + {}^{2}\mathcal{S}^{i})$, which is dominated by the primary reliability (1[4]) $p({}^{1}\mathcal{S}^{i})$, due to small transitive probability (2[4]) $p({}_{1}^{1}\mathcal{S}^{t})$, Fig. 5.

- The overall reliability of, let us say, $p({}^{1}\mathcal{S}^{i} + {}^{2}\mathcal{S}^{i}) = 0.999995$, Fig. 4, is accomplished for a_1=4532 mm^2, a_2=3200 mm^2, with the minimal weight corresponding to the minimal overall cross-sectional area of a_1+a_2=7732 mm^2, and with the secondary member reliabilities (14[4]) $p({}_{1}^{2}\mathcal{S}^{i}) = \Phi(-{}_{1}^{2}\beta = 0.659) = 0.745188$ and $p({}_{2}^{2}\mathcal{S}^{i}) = \Phi(-{}_{2}^{2}\beta = 0.745) = 0.772040$, Fig. 5.

- The study presented herein allows a design selection based on maximal redundancy, for different levels of primary, secondary and overall reliabilities, Figs. 4 and 5.

- The primary redundancy index (23[4]), Fig. 5, shows an inappropriate increase due to increase of the primary reserve strength, in spite of the diminution of the secondary system reliability after component failures and redistribution of loads.

8. CONCLUSION

This paper identifies first the simplest configuration of potentially redundant object of two members in terms of event-oriented analysis as a system of four events appropriate to a "fail-safe" or damage tolerant concept in engineering. However, neither the primary intact configuration of the object nor the primary system of events apparently appropriate to a potentially redundant object can affirm that the object performs its service, without checking the reliabilities of secondary operational modes after component failures and load redistribution. Therefore, a detailed numerical investigation of the example presented in the paper provides comparative and illustrative results in order to demonstrate the feasibility and usefulness of EOSA. In the conclusion, EOSA is an appropriate method to assess the system performance under uncertain conditions in full extent since it provides probabilities of successive operational levels and functional states after component failures. EOSA allows the assessment of redundancy expressed by the conditional entropy of operational modes, accounting simultaneously for all the events and the distribution of their probabilities. Such an approach may contribute to design improvement taking into consideration the redundancy level and may lead to more appropriate lifetime service of engineering objects under uncertain circumstances.

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