PERFORMANCE OPTIMIZATION OF SATURATED PID CONTROLLER FOR ROBOT MANIPULATORS

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Abstract. In this paper a new approach to performance tuning of saturated PID controller for robot manipulators is presented. The proposed approach is based on construction of a parameter dependent Lyapunov function. With the appropriate choice of the free parameter, which is not included in stability conditions, a estimation of integral performance index is obtained. The performance index depends on controller parameters and few parameters which characterize the robot dynamics. The optimal values of the controller gains are obtained by minimization of the performance index. An example is given to demonstrate the obtained results.

Key Words. Robot control, performance optimization, global asymptotic stabilization.

1. INTRODUCTION

The most industrial robots are controlled by linear PID controllers which do not require any component of robot dynamics into its control law. A simple linear and decoupled PID feedback controller with appropriate control gains achieves the desired position without any steady-state error. This is the main reason why PID controllers are still used in industrial robots. However, a linear PID controller in closed loop with a robot manipulator guarantees only local asymptotic stability [1], [2]. This is the reason to believe that linear PID control is inadequate to cope with highly nonlinear systems like robot manipulators, since the design of the linear PID control law is based solely on local arguments.

The first nonlinear PID controller which ensure global asymptotic stability is proposed in [3]. In this work, which was inspired by results of Tomei [4], it is proven that global convergence is still preserved if regressor matrix is replaced by constant matrix. Since the regressor matrix is constant, the control law can be interpreted as a nonlinear PID controller which achieve GAS by normalization nonlinearities in the integrator term of the control law. The second approach to achieve GAS is the scheme of Arimoto [5] that uses a saturation function in the integrator to render the system globally asymptotically stable, just as the normalization did in [3]. A unified approach to both above controllers, which belong to the class of PD plus a nonlinear integral action (PD+NI) controllers, is given in [6].

Although the stability properties of PD+NI controllers for robot manipulators are well understood, there are no many results regarding to optimality and performance tuning rules, except of H_∞ optimality [7].

In this paper a new approach to performance tuning of PD+NI controller for robot manipulators is presented. The proposed approach is based on construction of a parameter dependent Lyapunov function. With the appropriate choice of the free parameter, which is not included in stability conditions, a estimation of integral performance index is obtained. The performance index depends on controller parameters and few parameters which characterize the robot dynamics. The optimal values of the controller gains are obtained by minimization of the performance index.

This paper is organized as follows. The system description is presented in Section 2. The stability criterion based on the Lyapunov's approach is derived in

Section 3. The performance index evaluation is presented in Section 4. In Section 5, an example is given to demonstrate the results. Finally, the concluding remarks are emphasized in Section 6.

2. SYSTEM DESCRIPTION

We consider a nonlinear mechanical system with n-degree of freedom in closed loop with a nonlinear PID controller.

2.1. Dynamics of Rigid Robot

The model of n-link rigid-body robotic manipulator with all revolute joints, in the absence of friction and disturbances, is represented by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u, \qquad (1)$$

where q is the $n \times 1$ vector of robot joint coordinates, $\dot{q}$ is the $n \times 1$ vector of joint velocities, u is the $n \times 1$ vector of applied joint torques, $M(q)$ is $n \times n$ inertia matrix, $C(q, \dot{q})\dot{q}$ is the $n \times 1$ vector of centrifugal and Coriolis torques, and $g(q)$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $U(q)$

$$g(q) = \frac{\partial U(q)}{\partial q}. \qquad (2)$$

The following well known properties of the robot dynamics, [8]- [10], are important for stability analysis.

Property 1. The inertia matrix $M(q)$ is a positive definite symmetric matrix which satisfies

$$\lambda_m\{M\}\|\dot{q}\|^2 \leq \dot{q}^T M(q)\dot{q} \leq \lambda_M\{M\}\|\dot{q}\|^2, \qquad (3)$$

where $\lambda_m\{M\}$ and $\lambda_M\{M\}$ denotes strictly positive minimum and maximum eigenvalues of $M(q)$, respectively.

Property 2. The matrix $S(q, \dot{q}) = \dot{M}(q) - 2C(q, \dot{q})$ is skew-symmetric, i.e.,

$$z^T S(q, \dot{q})z = 0, \quad \forall z \in \mathbb{R}^n. \qquad (4)$$

This implies

$$\dot{M}(q) = C(q, \dot{q}) + C(q, \dot{q})^T. \qquad (5)$$

Property 3. The Coriolis and centrifugal term $C(q, \dot{q})\dot{q}$ satisfies

$$\|C(q, \dot{q})\dot{q}\| \leq k_c\|\dot{q}\|^2, \qquad (6)$$

for some bounded constant $k_c > 0$.

Property 4. There exists some positive constant k_g such that gravity vector satisfies

$$\|g(x) - g(y)\| \leq k_g\|x - y\|, \quad \forall\, x, y \in \mathbb{R}^n. \qquad (7)$$

Property 5. There exist positive diagonal matrix K_P such that the following two inequalities with specified constant $k_1 > 0$ are satisfied simultaneously

$$s(\tilde{q})^T K_P \tilde{q} + s(\tilde{q})^T (g(q) - g(q_d)) \geq k_1 s(\tilde{q})^T \tilde{q}, \quad (8)$$

$$\frac{1}{2}\tilde{q}^T K_P \tilde{q} + U(q) - U(q_d) - \tilde{q}^T g(q_d) \geq \frac{1}{2}k_1\|\tilde{q}\|^2, \quad (9)$$

where $k_1 = \lambda_m\{K_P\} - k_g \geq 0$, and $s(\tilde{q})$ is a continuous differentiable increasing vector function.

2.2. Nonlinear PID Controller

The nonlinear PID control law is given by

$$u = -\Psi_P(\tilde{q})\tilde{q} - K_D \dot{q} - K_I \nu, \qquad (10)$$
$$\dot{\nu} = s(\tilde{q}), \qquad (11)$$

where $\tilde{q} = q - q_d$ is the joint position error, K_D and K_I are constant positive-definite diagonal matrix, $s(\tilde{q})$ is continuous differentiable increasing vector function $s(\tilde{q}) = [s(\tilde{q}_1)\, s(\tilde{q}_2)\, \ldots\, s(\tilde{q}_n)]^T$ such that

$$s(\tilde{q}_i)\tilde{q}_i \geq 0, \qquad |s(\tilde{q}_i)| \leq s_M, \qquad 0 \leq \frac{ds(\tilde{q}_i)}{d\tilde{q}_i} \leq 1,$$

for all $\tilde{q}_i \in \mathbb{R}$. The function $\Psi_P(\tilde{q})$ is $(n \times n)$ positive definite diagonal matrix functions which can be written in the following form

$$\Psi_P(\tilde{q}) = K_P + \bar{K}_P \bar{\Psi}_P(\tilde{q}), \qquad (12)$$

where K_P, and $\bar{K}_P$ are constant positive-definite diagonal matrix and $\bar{\Psi}_P(\tilde{q})$ is $(n \times n)$ positive definite diagonal matrix function $\bar{\Psi}_P(\tilde{q}) = \text{diag}\{\bar{\psi}_P(\tilde{q}_1), ..., \bar{\psi}_P(\tilde{q}_n)\}$, which satisfies additional conditions

$$0 \leq \bar{\Psi}_P(\tilde{q}) \leq I, \quad \bar{\Psi}_P(0) = I, \quad \lim_{\tilde{q}\to\pm\infty} \bar{\Psi}_P(\tilde{q}) = 0,$$

where I is the identity matrix and 0 is the null matrix.

The function $s(\tilde{q})$ ensure global asymptotic stability and the function $\Psi_P(\tilde{q})$ provide performance improvement. The following properties of functions $s(\tilde{q})$ and $\Psi_P(\tilde{q})$ are important for stability analysis.

Property 1. The function $\Psi_P(\tilde{q})$ is lower bounded and satisfies the following inequalities

$$z^T \Psi_P(\tilde{q})z \geq \lambda_m\{K_P\}\|z\|^2, \quad \forall z \in \mathbb{R}^n. \quad (13)$$

Property 2. The following integrals are positive-definite functions for all $z \in \mathbb{R}$

$$0 \leq \int_0^z s(\xi)d\xi \leq \left\{ \begin{array}{ll} \frac{1}{2}|z|^2, & \text{if } |z| < s_M \\ s_M|z|, & \text{if } |z| \geq s_M \end{array} \right., \quad (14)$$

$$0 \leq \int_0^z \bar{\psi}_P(\xi)\xi d\xi \leq \frac{1}{2}z^2. \qquad (15)$$

3. STABILITY ANALYSIS

Although the stability of the saturated PID controller for robot manipulator is proven in [6] and [8], a slightly different approach is needful for construction of parameter dependent Lyapunov function which will be used for performance evaluation. Namely, the main request is elimination of the free parameter contained in the Lyapunov function from final stability conditions. On this way, the stability criterion doesn't depend on the mentioned parameter providing elegant determination of a integral performance index in the following section.

The stability analysis can be divided in four parts. First, error equations for closed loop system (1), (10), (11) is determined. Second, the Lyapunov function candidate is proposed. Then, a global stability criterion on system parameters is established. Finally, LaSalle invariance principle is invoked to guarantee the asymptotic stability.

The stationary state of the system (1), (10), (11) is $\tilde{q} = 0$, $\dot{q} = 0$, $\nu = \nu^*$, and ν^* satisfies $g(q_d) = -K_I \nu^*$. If a new variable $z = \nu - \nu^*$ is introduced, then system (1), (10), (11) becomes

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - g(q_d) = u, \quad (16)$$

$$u = -\Psi_P(\tilde{q})\tilde{q} - K_D\dot{q} - K_I z, \quad (17)$$

$$\dot{z} = s(\tilde{q}). \quad (18)$$

3.1. Construction of Lyapunov function

First, an output variable $y = \dot{q} + \alpha s(\tilde{q})$ with parameter $\alpha > 0$ is introduced, and inner product between (16) and y is made, resulting in a nonlinear differential form which can be separated in the following way

$$\frac{dV(\tilde{q},\dot{q},z;\alpha)}{dt} = -W(\tilde{q},\dot{q};\alpha), \quad (19)$$

where $V(\tilde{q},\dot{q},z;\alpha)$ is the Lyapunov function candidate parameterized by a positive parameter α.

For easier determination of conditions for positive-definiteness of function V and W, the following decompositions are made: $V(\tilde{q},\dot{q},z;\alpha) = V_1(\tilde{q},\dot{q};\alpha) + V_2(\tilde{q},z;\alpha)$ and $W(\tilde{q},\dot{q};\alpha) = W_1(\tilde{q},\dot{q};\alpha) + W_2(\tilde{q};\alpha)$, where

$$V_1 = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha s(\tilde{q})^T M(q)\dot{q} + $$
$$+ \alpha \sum_{i=1}^n K_{Di} \int_0^{\tilde{q}_i} s_i(\xi)d\xi, \quad (20)$$

$$V_2 = \frac{1}{2}\tilde{q}^T K_P \tilde{q} + U(q) - U(q_d) - \tilde{q}^T g(q_d) + $$
$$+ \sum_{i=1}^n \bar{K}_{Pi} \int_0^{\tilde{q}_i} \bar{\psi}_P(\xi)\xi d\xi + $$
$$+ \tilde{q}^T K_I z + \frac{1}{2}\alpha z^T K_I z, \quad (21)$$

and

$$W_1 = \dot{q}^T(K_D - \alpha s_{\tilde{q}}(\tilde{q})M(q))\dot{q} + $$
$$+ \alpha s(\tilde{q})^T(\dot{M}(q) - C(q,\dot{q}))\dot{q}, \quad (22)$$

$$W_2 = s(\tilde{q})^T(\alpha \Psi_P(\tilde{q}) - K_I)\tilde{q} + $$
$$+ \alpha s(\tilde{q})^T(g(q) - g(q_d)). \quad (23)$$

where $s_{\tilde{q}}(\tilde{q}) = \text{diag}\{s_{\tilde{q}_1}(\tilde{q}_1),...,s_{\tilde{q}_n}(\tilde{q}_n)\}$. We can see that function V_1 contains positive-definite parts in variables $\tilde{q}$ and $\dot{q}$ and cross-term in the same variables. Also, first and third part of function V_2 is positive-definite in variables $\tilde{q}$ and z respectively, and second part is cross-term in same variables.

In this way, the problem of determination of conditions for positive-definiteness of function V, which contains three variables, is transformed to two simpler problems of determination of conditions for positive-definiteness of functions $V_1(\tilde{q},\dot{q};\alpha)$ and $V_2(\tilde{q},z;\alpha)$, which contain only two variables.

3.2. Stability criterion determination

First, we consider function V_1 which can be rearranged to be of the form

$$V_1 = \frac{1}{2}(\dot{q} + \alpha s(\tilde{q}))^T M(q)(\dot{q} + \alpha s(\tilde{q})) - $$
$$- \frac{1}{2}\alpha^2 s(\tilde{q})^T M(q)s(\tilde{q}) + \alpha \sum_{i=1}^n K_{Di} \int_0^{\tilde{q}_i} s(\xi)d\xi,$$

and using property (3) we get

$$V_1 \geq \alpha \sum_{i=1}^n f(\tilde{q}_i) \geq 0, \quad (24)$$

where

$$f(\tilde{q}_i) = \lambda_m\{K_D\} \int_0^{\tilde{q}_i} s(\xi)d\xi - \frac{1}{2}\alpha\lambda_M\{M\}s(\tilde{q}_i)^2,$$

that is positive-definite function if $\tilde{q}_i f_{\tilde{q}_i}(\tilde{q}_i) \geq 0$,

$$\tilde{q}_i f_{i\tilde{q}_i}(\tilde{q}_i) = \lambda_m\{K_D\}\tilde{q}_i s(\tilde{q}_i) - $$
$$- \alpha\lambda_M\{M\}\tilde{q}_i s(\tilde{q}_i)s_{\tilde{q}_i}(\tilde{q}_i) = $$
$$= \tilde{q}_i s(\tilde{q}_i)(\lambda_m\{K_D\} - \alpha\lambda_M\{M\}s_{\tilde{q}_i}(\tilde{q}_i)) \geq $$
$$\geq \tilde{q}_i s(\tilde{q}_i)(\lambda_m\{K_D\} - \alpha\lambda_M\{M\}) \geq 0, \quad (25)$$

for $i = 1,...,n$, where we used property $s_{\tilde{q}_i}(\tilde{q}_i) \leq 1$. The above mentioned expression is the positive definite if the following condition is satisfied

$$\frac{\lambda_m\{K_D\}}{\lambda_M\{M\}} > \alpha. \quad (26)$$

Further, we consider function V_2 which can be rearranged to be of the form

$$V_2 \geq \frac{1}{2}\left(\sqrt{\alpha}z + \frac{1}{\sqrt{\alpha}}\tilde{q}\right)^T K_I\left(\sqrt{\alpha}z + \frac{1}{\sqrt{\alpha}}\tilde{q}\right) + $$
$$+ \frac{1}{2}k_1\|\tilde{q}\|^2 - \frac{1}{2\alpha}\tilde{q}^T K_I \tilde{q} \geq $$
$$\geq \frac{1}{2}\left(k_1 - \frac{1}{\alpha}\lambda_M\{K_I\}\right)\|\tilde{q}\|^2, \quad (27)$$

where we used properties (9) and (15). The above mentioned expression is the positive definite if the following condition is satisfied

$$\alpha > \frac{\lambda_M\{K_I\}}{k_1}. \quad (28)$$

If we compare (28) and (26) then we obtain

$$k_1\lambda_m\{K_D\} > \lambda_M\{K_I\}\lambda_M\{M\}. \quad (29)$$

Note that in the above-stated condition the unspecified positive constant α is eliminated.

Next step is condition which ensure that time derivative of Lyapunov function is negative definite function, i.e., $W \geq 0$. First, we consider function W_1. Applying properties (3), (5), (6) we get

$$W_1 \geq \lambda_m\{K_D\}\|\dot{q}\|^2 - \alpha\lambda_M\{M\}\|\dot{q}\|^2 - \\ - \alpha k_c s_M\|\dot{q}\|^2 \geq 0, \quad (30)$$

that is positive-definite if the following condition is satisfied

$$\frac{\lambda_m\{K_D\}}{\lambda_M\{M\} + k_c s_M} > \alpha. \quad (31)$$

Further, we consider function W_2. Using properties (8) and (13) we get

$$W_2 \geq (\alpha k_1 - \lambda_M\{K_I\})\tilde{q}^T s(\tilde{q}), \quad (32)$$

that is positive-definite if we have

$$\alpha > \frac{\lambda_M\{K_I\}}{k_1}. \quad (33)$$

Comparing (31) with (33) the following condition is obtained

$$k_1\lambda_m\{K_D\} > \lambda_M\{K_I\}(\lambda_M\{M\} + k_c s_M). \quad (34)$$

Also, in the above-stated condition the unspecified positive constant α is eliminated. Notice that the condition (29) is trivially implied by the condition (34). So, the condition (34) is the final stability condition which guaranty global stability. Finally, invoking the LaSalle's invariance principle we conclude asymptotic stability.

4. PERFORMANCE OPTIMIZATION

The Lyapunov function V and its time derivative $\dot{V} = -W$ contain free parameter $\alpha > 0$ which is not included in stability condition. This fact can be employed for the evaluation of the following performance index

$$I = I_1 + \tau^2 I_2, \quad (35)$$

where the constant τ^2 is the weighting factor, and

$$I_1 = \int_0^\infty \tilde{q}^T s(\tilde{q})dt, \qquad I_2 = \int_0^\infty \|\dot{q}\|^2 dt. \quad (36)$$

Also, in this section, because of compactness, following shortened notation is introduced: $k_{jm} = \lambda_m\{K_j\}$, $k_{jM} = \lambda_M\{K_j\}$, $\bar{m} = \lambda_M\{M\} + k_c s_M$, $\mu_j = \lambda_M\{K_j\}/\lambda_m\{K_j\}$, where $j = P, I, D$.

The performance index (35) can be evaluated using Lyapunov function (20), (21) and its time derivative. From the equation (19) we can get

$$V(t) - V(0) \leq - \int_0^t W(\tilde{q}(s), \dot{q}(s))ds, \quad (37)$$

and, for $t \to \infty$,

$$V(0) \geq \int_0^\infty W(\tilde{q}(s), \dot{q}(s))ds, \quad (38)$$

because $V(\infty) = 0$. Putting (30) and (32) in (38) we get

$$V(0) \geq (k_{Dm} - \alpha\bar{m})I_2 + (\alpha k_1 - k_{IM})I_1. \quad (39)$$

The next step is the estimation of the upper bounds on $V(0)$. We have $\tilde{q}(0) = -q_d$, $\dot{q}(0) = 0$, $z(0) = -\nu^* = K_I^{-1}g(q_d)$, so that $V(0)$ satisfies the following expression

$$V(0) = \frac{1}{2}q_d^T K_P q_d + \frac{1}{2}\alpha g(q_d)^T K_I^{-1} g(q_d) - \\ - U(q_d) + \sum_{i=1}^n \bar{K}_{Pi} \int_0^{-q_{di}} \bar{\psi}_P(\xi)\xi d\xi + \\ + \alpha \sum_{i=1}^n K_{Di} \int_0^{-q_{di}} s_i(\xi)d\xi. \quad (40)$$

So, we can estimate the upper bounds

$$V(0) \leq \frac{1}{2}(k_{PM} + \bar{k}_{PM})\|q_d\|^2 + \frac{1}{2}\alpha k_{IM}^{-1}\|g(q_d)\|^2 \\ + \alpha k_{DM} \sum_{i=1}^n \int_0^{-q_{di}} s_i(\xi)d\xi. \quad (41)$$

Because of (7) and $\lambda_M\{K_I^{-1}\} = 1/\lambda_m\{K_I\}$ we have

$$V(0) \leq w_2 \left(k_{PM} + \bar{k}_{PM} + \alpha \frac{k_g^2}{k_{Im}} \right) + \\ + \alpha w_s k_{DM}, \quad (42)$$

where $w_2 = \frac{1}{2}\|q_d\|^2$ and

$$w_s = \begin{cases} \frac{1}{2}\|q_d\|^2, & \text{if } \|q_d\| < s_M \\ s_M\|q_d\|, & \text{if } \|q_d\| \geq s_M \end{cases}, \quad (43)$$

where w_s satisfies

$$w_s \geq \sum_{i=1}^n \int_0^{-q_{di}} s_i(\xi)d\xi. \quad (44)$$

Finally, comparing (39) and (42) we have

$$(k_{Dm} - \alpha\bar{m})I_2 + (\alpha k_1 - k_{IM})I_1 \leq w_s\alpha k_{DM} +$$

$$+ w_2\left(k_{PM} + \bar{k}_{PM} + \alpha\frac{k_g^2}{k_{Im}}\right). \qquad (45)$$

From the above mentioned expression we can get integral terms I_1 and I_2 in the following way. Because the choice of the free parameter α is not limited by stability conditions (34), we can put $\alpha = k_{Dm}/\bar{m}$ in expression (45) so that

$$I_1 \leq \frac{w_2}{S_M}\left((k_{PM} + \bar{k}_{PM})\bar{m} + k_{Dm}\frac{k_g^2}{k_{Im}}\right) +$$

$$+ \frac{w_s}{S_M}k_{Dm}k_{DM}, \qquad (46)$$

where

$$S_M = k_1 k_{Dm} - k_{IM}\bar{m} > 0. \qquad (47)$$

The positivity of S_M follows from stability conditions (34). Similarly, if we put $\alpha = k_{IM}/k_1$ in expression (45) we get

$$I_2 \leq \frac{w_2}{S_M}\left((k_{PM} + \bar{k}_{PM})k_1 + k_{IM}\frac{k_g^2}{k_{Im}}\right) +$$

$$+ \frac{w_s}{S_M}k_{IM}k_{DM}. \qquad (48)$$

Finally, if we put expressions (46) and (48) in (35) we get

$$I \leq \hat{I} = \frac{1}{S_M}(k_P^* + w_s\mu_D(k_{Dm}^2 + \tau^2 k_{Dm}k_{IM})) +$$

$$+ \frac{\bar{w}_2}{S_M}\left(\frac{k_{Dm}}{k_{IM}} + \tau^2\right), \qquad (49)$$

where $\hat{I}$ is the estimation of the upper bounds of the performance index (35), $\bar{w}_2 = w_2\mu_I k_g^2$, and

$$k_P^* = w_2(\bar{m} + \tau^2 k_1)(k_{PM} + \bar{k}_{PM}). \qquad (50)$$

We want to chose k_{Dm} and k_{IM} which will minimize the performance index (49)

$$\frac{\partial\hat{I}}{\partial k_{Dm}} = 0, \qquad \frac{\partial\hat{I}}{\partial k_{IM}} = 0. \qquad (51)$$

The solution of equations (51) is the following set of polynomial equations regarding to variables k_{Dm} and k_{IM}

$$a_D k_{Dm}^2 - b_D k_{Dm} - c_D = 0, \qquad (52)$$
$$a_I k_{IM}^2 + b_I k_{IM} - c_I = 0, \qquad (53)$$

where

$$a_D = k_1 w_s\mu_D, \qquad b_D = 2\bar{m}w_s\mu_D k_{IM},$$
$$c_D = \bar{m}(w_s\mu_D\tau^2 k_{IM}^2 + \bar{w}_2) + k_1(k_P^* + \bar{w}_2\tau^2),$$

and

$$a_I = \bar{m}(k_P^* + \bar{w}_2\tau^2) + w_s\mu_D(\bar{m} + k_1\tau^2)k_{Dm}^2,$$
$$b_I = 2\bar{m}\bar{w}_2 k_{Dm}, \qquad c_I = k_1\bar{w}_2 k_{Dm}^2.$$

We can rewrite the equations (52) and (53) in the following way

$$k_{Dm} = \frac{1}{2a_D}\left(b_D + \sqrt{b_D^2 + 4a_D c_D}\right), \quad (54)$$

$$k_{IM} = \frac{1}{2a_I}\left(-b_I + \sqrt{b_I^2 + 4a_I c_I}\right). \quad (55)$$

We can find solution of the set of nonlinear equations (54) and (55) applying simple iterative procedure.

It is well known that it is impossible to select fixed gains for a linear PID controller that will prevent overshoots for all configurations of a given robot system. A way to reduce the overshoots is selection of high proportional gain and appropriate derivative and integral gain. A drawback of this approach is high control jump during the transient response, because of large error at the beginning of control action, $u(0) \approx -K_P\tilde{q}(0) = K_P q_d$. This problem can be avoided by introducing a nonlinear proportional gain

$$\psi_{Pi}(\tilde{q}_i) = K_{Pi} + \bar{K}_{Pi}\exp(-\frac{\tilde{q}_i^2}{2\sigma_P}). \qquad (56)$$

In this way, we ensure high proportional gain $\Psi_P(\tilde{q}) \approx K_P + \bar{K}_P$, when the system state is near the stationary state, $\tilde{q} \approx 0$, preventing a large overshoot in the transient response. On the other side, for large error, $\tilde{q} \approx -q_d$, we have small gain $\Psi_P(\tilde{q}) \approx K_P$, what prevent high control jump during the transient response. The parameter σ_P defines a bandwidth around stationary state $\tilde{q}_i = 0$ with high proportional gains influence.

So, the maximal value of proportional gain K_P is determined by the maximal allowed control variable $u_{\max}$,

$$k_{PM} \leq \left|\frac{u_{\max}}{q_{d,\max}}\right|, \qquad (57)$$

where $q_{d,\max}$ is the maximal value of q_d.

5. SIMULATION EXAMPLE

The manipulator used for simulation is a two revolute jointed robot (planar elbow manipulator) considered in [11]. The numerical values of robot parameters have been taken from [2]. The entries of the inertia matrix $M(q)$ are given by

$$M_{11}(q) = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)) +$$
$$+ I_1 + I_2,$$
$$M_{12}(q) = M_{21}(q) = m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2,$$
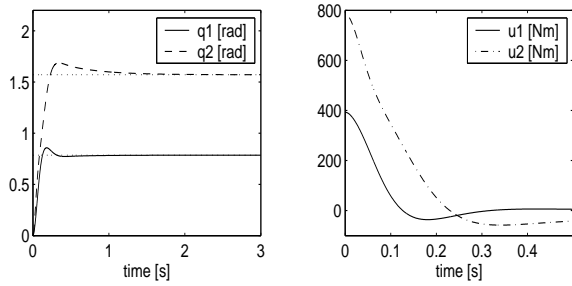$$M_{22}(q) = m_2 l_{c2}^2 + I_2.$$

Figure 1: The transient response for positions and control torque for $K_P = \mathrm{diag}\{500\}$, $\bar{K}_P = \mathrm{diag}\{0\}$, and for the optimal values of K_D and K_I.

The elements of the Coriolis matrix $C(q, \dot{q})$ are

$$C_{11}(q, \dot{q}) = -m_2 l_1 l_{c2} \sin(q_2)\dot{q}_2,$$
$$C_{12}(q, \dot{q}) = -m_2 l_1 l_{c2} \sin(q_2)(\dot{q}_1 + \dot{q}_2),$$
$$C_{21}(q, \dot{q}) = m_2 l_1 l_{c2} \sin(q_2)\dot{q}_1, \quad C_{22}(q, \dot{q}) = 0.$$

The elements of the gravitational torque vector $g(q)$ are given by

$$g_1(q) = (m_1 l_{c1} + m_2 l_1)g \cos(q_1) + g_2(q),$$
$$g_2(q) = m_2 l_{c2} g \cos(q_1 + q_2).$$

The numerical values of the constants k_g, k_c and $\lambda_M\{M\}$ are: $k_g = 75.46$ Nm, $k_c = 0.7$ kg m^2, $\lambda_M\{M\} = 1.33$ kg m^2. The nonlinear integral term is $s(\tilde{q}) = \frac{1}{2}\tanh(2\tilde{q})$ and $\tau = 0.01$.

In Fig. 1-2 we can see comparison between controller with $\bar{K}_P = 0$ and controller with $\bar{K}_P \neq 0$. To make the comparison fair, the value of $\lambda_M\{\Psi_P(\tilde{q})\}$ will be same in both cases. The values of controller parameters in the first case (Fig. 1) are: $K_P = \mathrm{diag}\{500\}$, $\bar{K}_P = \mathrm{diag}\{0\}$, $K_D = \mathrm{diag}\{37.4\}$, $K_I = \mathrm{diag}\{921.4\}$ and $\sigma_P = 0.5$. The values of controller parameters in the second case (Fig. 2) are: $K_P = \mathrm{diag}\{150\}$, $\bar{K}_P = \mathrm{diag}\{350\}$, $K_D = \mathrm{diag}\{44.8\}$, $K_I = \mathrm{diag}\{367.1\}$ and $\sigma_P = 0.5$.

We can see that for almost same quality of the transient response, controller in Fig. 2. has not a high jump of the control variable which can be seen for the controller in Fig. 1.

6. CONCLUDING REMARKS

In this paper a new approach to performance tuning of saturated PID controller for robot manipulator is proposed. The proposed tuning rules provide fast transient response without oscillations and large overshoots, overcoming undesirable effect of high control jumps which is characteristic for conventional linear PID controllers. The performance tuning rule involve only few parameters which characterize the robot dynamics.
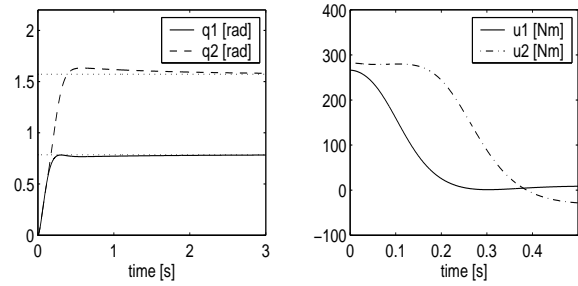
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Figure 2: The transient response for positions and control torque for $K_P = \mathrm{diag}\{150\}$, $\bar{K}_P = \mathrm{diag}\{350\}$, and for the optimal values of K_D and K_I.

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