GLOBAL ASYMPTOTIC STABILIZATION OF ROBOT MANIPULATORS WITH MIXED REVOLUTE AND PRISMATIC JOINTS [1]

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Abstract: In this paper a class of globally stable controllers for robotic manipulators with mixed revolute and prismatic joints is proposed. The global asymptotic stabilization is achieved by adding a nonlinear proportional and derivative term to the linear PID controller. By using Lyapunov's direct method, explicit conditions on controller parameters which ensure global asymptotic stability are obtained. Further, the Lyapunov function is employed for the evaluation of a performance index and determination of optimal values of controller parameters. Finally, an example is given to demonstrate the obtained results.

Keywords: robot control, global stability, PID control, nonlinear control, Lyapunov methods, robotic manipulators.

1. INTRODUCTION

It is well known that a PD plus gravity compensation controller can globally asymptotically stabilize a rigid-joints manipulator (Takegaki and Arimoto, 1981). This approach has drawbacks that gravitational torque vector which depends on some parameters, usually uncertain, is assumed to be known accurately. To overcome parametric uncertainties on the gravitational torque vector, an adaptive version of PD controller has been introduced in (Tomei, 1991), guaranteeing global asymptotic stability. The main weakness of this approach is that the structure of the gravitational torque vector has to be known.

On the other hand, most industrial robots are controlled by linear PID controllers which do not require any component of robot dynamics into its control law. A simple linear and decoupled PID feedback controller with appropriate control gains achieves the desired position without any steady-state error. This is the main reason why PID controllers are still used in industrial robots. However, a linear PID controller in closed loop with a robot manipulator guarantees only local asymptotic stability (Arimoto and Miyazaki, 1986; Kelly, 1995). By looking at the proof it can be seen that the quadratic terms in joint velocities contained in the Coriolis matrix hampers the global asymptotic stability. This is the reason to believe that linear PID control is inadequate to cope with highly nonlinear systems like robot manipulators, since the design of the linear PID control law is based solely on local arguments.

The first nonlinear PID controller which ensures global asymptotic stability (GAS) is proposed in

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(Kelly, 1993). In this work, which was inspired by the results of (Tomei, 1991), it is proven that global convergence is still preserved if the regressor matrix is replaced by the constant matrix. Since the regressor matrix is constant, the control law can be interpreted as a nonlinear PID controller which achieves GAS by normalization nonlinearities in the integrator term of the control law. The second approach to achieving GAS is the scheme of Arimoto (Arimoto, 1994) that uses a saturation function in the integrator to render the system globally asymptotically stable, just as the normalization did in (Kelly, 1993). A unified approach to both above controllers, which have a linear derivative term, linear or saturated proportional term, and nonlinearities in the integrator, is given in (Kelly, 1998).

An alternative approach to global asymptotic stabilization of robot manipulator is "delayed PID" (PI_dD) (Loria *et al.*, 2000). PI_dD can be understood as a simple PD controller to which an integral action is added after some transient of time. The idea of this approach consists of "patching" a global and a local controller. The first drives the solutions to an arbitrarily small domain, while the second, yields local asymptotic stability.

All mentioned approaches can be applied to robot manipulators with revolute joints only. In this paper an approach to GAS of robot manipulators with revolute and prismatic joints is presented. In this approach GAS is achieved by adding a nonlinear proportional and derivative term to the linear PID controller. Explicit conditions on controller parameters which guarantee GAS are given. Also, a performance index is evaluated on the base of the Lyapunov function.

This paper is organized as follows. The system description is presented in Section 2. The stability criterion based on the Lyapunov approach is derived in Section 3. The performance tuning is presented in Section 4. In Section 5, an example is given to demonstrate the results. Finally, the concluding remarks are emphasized in Section 6.

2. SYSTEM DESCRIPTION

We consider a robot manipulator with n-degree of freedom in a closed loop with a nonlinear PID controller.

2.1 Dynamics of Rigid Robot

The model of n-link rigid-body robotic manipulator, in the absence of friction and disturbances, is represented by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u, \qquad (1)$$

where q is the $n \times 1$ vector of robot joint coordinates, $\dot{q}$ is the $n \times 1$ vector of joint velocities, u is the $n \times 1$ vector of applied joint torques and forces, $M(q)$ is $n \times n$ inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of centrifugal and Coriolis torques and $g(q)$ is the $n \times 1$ vector of gravitational torques and forces, obtained as the gradient of the robot potential energy $U(q)$

$$g(q) = \frac{\partial U(q)}{\partial q}. \qquad (2)$$

The following properties of the robot dynamics, are important for stability analysis (see e.g. (Ortega *et al.*, 1998) for properties 1, 4 and 5, and (Pervozvanski and Freidovich, 1999) for properties 2 and 3).

Property 1. The matrix $S(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e.,

$$z^T S(q,\dot{q}) z = 0, \quad \forall z \in \mathbb{R}^n. \qquad (3)$$

This implies

$$\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T. \qquad (4)$$

Property 2. The inertia matrix $M(q)$ is a positive definite symmetric matrix which satisfies

$$a_1\|z\|^2 \le z^T M(q) z \le \bar{a}_2(\|q\|)\|z\|^2, \qquad (5)$$

for all $z, q \in \mathbb{R}^n$, where

$$\bar{a}_2(\|q\|) = a_2 + c_2\|q\| + d_2\|q\|^2, \qquad (6)$$

and $a_1, a_2 > 0$, $c_2, d_2 \ge 0$.

Property 3. The Coriolis and centrifugal terms $C(q,\dot{q})\dot{q}$ satisfy

$$\|C(q,\dot{q})\dot{q}\| \le (c_1 + d_1\|q\|)\|\dot{q}\|^2, \qquad (7)$$

for all $q, \dot{q} \in R^n$, where $c_1, d_1 \ge 0$.

Property 4. There exists a positive constant k_g such that the gravity vector satisfies

$$\|g(x) - g(y)\| \le k_g\|x - y\|, \quad \forall\, x, y \in \mathbb{R}^n. \qquad (8)$$

Property 5. There exists a positive diagonal matrix $K_P(x)$ such that the following two inequalities with specified constant $k_1 > 0$ are satisfied simultaneously

$$\tilde{q}^T K_P(x)\tilde{q} + \tilde{q}^T(g(q) - g(q_d)) \ge k_1\|\tilde{q}\|^2, \qquad (9)$$

$$\frac{1}{2}\tilde{q}^T K_P(x)\tilde{q} + \bar{U}(\tilde{q}) \ge \frac{1}{2}k_1\|\tilde{q}\|^2, \qquad (10)$$

where

$$\bar{U}(\tilde{q}) = U(q) - U(q_d) - \tilde{q}^T g(q_d), \qquad (11)$$

$$k_1 = \lambda_m\{K_P\} - k_g \ge 0. \qquad (12)$$

Here and below we use the notation: $\|\cdot\|$ for the Euclidean norm of the vector "$\cdot$", $\lambda_M\{\cdot\}$ and $\lambda_m\{\cdot\}$ for the maximal and minimal eigenvalues of the symmetric matrix "$\cdot$", I for the identity matrix of the appropriate dimension.

The assumptions (5)-(10) are valid for all practically used robot manipulators. If the robot has no prismatic joints then $d_1, d_2, c_2 = 0$. The boundedness of gravity forces does not hold for manipulators having non-horizontal prismatic joints, but it is not required for stability analysis.

2.2 Nonlinear PID Controller

The nonlinear PID control law is given by

$$u = -\Psi_P(\tilde{q})\tilde{q} - \Psi_D(\tilde{q})\dot{q} - K_I\nu, \tag{13}$$
$$\dot{\nu} = \tilde{q}, \tag{14}$$

where $\tilde{q} = q - q_d$ is a joint position error, K_I is a constant positive definite diagonal matrix, $\Psi_j(\tilde{q})$, $j = P, D$ are $(n \times n)$ positive definite diagonal matrix functions which can be written in the following form

$$\Psi_j(\tilde{q}) = K_j + k_j^{(1)}\|\tilde{q}\|I + k_j^{(2)}\|\tilde{q}\|^2 I, \tag{15}$$

where K_j, $j = P, D$ are constant positive definite diagonal matrix, and $k_j^{(1)}$, $k_j^{(2)}$ are positive constants.

The following properties of functions $\Psi_j(\tilde{q})$, $j = P, D$ are important for stability analysis.

Property 1. Functions $\Psi_j(\tilde{q})$, $j = P, D$, are lower bounded and satisfy the following inequalities

$$\begin{aligned} z^T\Psi_j(\tilde{q})z &\geq (\lambda_m\{K_j\} + k_j^{(1)}\|\tilde{q}\| + k_j^{(2)}\|\tilde{q}\|^2)\|z\|^2 \geq \\ &\geq \lambda_m\{K_j\}\|z\|^2, \quad \forall z \in \mathbb{R}^n. \end{aligned} \tag{16}$$

Property 2. The following property of the Euclidean norm holds

$$\frac{d}{dt}\left(\frac{1}{k}\|\tilde{q}\|^k\right) = \|\tilde{q}\|^{k-2}\tilde{q}^T\dot{q}, \quad k \geq 2. \tag{17}$$

3. STABILITY ANALYSIS

The stability analysis is based on Lyapunov's direct method, and can be divided in four parts. First, error equations for the closed-loop system (1), (13), (14) is determined. Second, the Lyapunov function (LF) candidate is proposed. Then, a global stability criterion on system parameters is established. Finally, the LaSalle invariance principle is invoked to guarantee the asymptotic stability.

3.1 Error Equations

The stationary state of the system (1), (13), (14) is $\tilde{q} = 0$, $\dot{q} = 0$, $\nu = \nu^*$, and ν^* satisfies $g(q_d) = -K_I\nu^*$.

If a new variable $z = \nu - \nu^*$ is introduced, then the system (1), (13), (14) becomes

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - g(q_d) = u, \tag{18}$$
$$u = -\Psi_P(\tilde{q})\tilde{q} - \Psi_D(\tilde{q})\dot{q} - K_I z, \tag{19}$$
$$\dot{z} = \tilde{q}. \tag{20}$$

3.2 Construction of the Lyapunov function

First, an output variable $y = \dot{q} + \alpha\tilde{q}$ with some $\alpha > 0$ is introduced, and the inner product between (18) and y is made, resulting in a nonlinear differential form which can be separated in the following way

$$\frac{dV(\tilde{q},\dot{q},z)}{dt} = -W(\tilde{q},\dot{q}), \tag{21}$$

where $V(\tilde{q},\dot{q},z)$ is the Lyapunov function candidate.

For easier determination of conditions for positive-definiteness of function V and W, the following decompositions are made: $V(\tilde{q},\dot{q},z) = V_1(\tilde{q},\dot{q}) + V_2(\tilde{q},z)$ and $W(\tilde{q},\dot{q}) = W_1(\tilde{q},\dot{q}) + W_2(\tilde{q})$, where

$$\begin{aligned} V_1(\tilde{q},\dot{q}) &= \frac{1}{2}\dot{q}^T M(q)\dot{q} + U(q) - U(q_d) - \tilde{q}^T g(q_d) + \\ &+ \alpha\tilde{q}^T M(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_P\tilde{q} + \\ &+ \frac{1}{3}k_P^{(1)}\|\tilde{q}\|^3 + \frac{1}{4}k_P^{(2)}\|\tilde{q}\|^4, \end{aligned} \tag{22}$$

$$\begin{aligned} V_2(\tilde{q},z) &= \frac{1}{2}\alpha z^T K_I z + \tilde{q}^T K_I z + \frac{1}{2}\alpha\tilde{q}^T K_D\tilde{q} + \\ &+ \frac{1}{3}\alpha k_D^{(1)}\|\tilde{q}\|^3 + \frac{1}{4}\alpha k_D^{(2)}\|\tilde{q}\|^4, \end{aligned} \tag{23}$$

and

$$\begin{aligned} W_1(\tilde{q},\dot{q}) &= -\alpha\dot{q}^T M(q)\dot{q} + \dot{q}^T\Psi_D(\tilde{q})\dot{q} + \\ &+ \alpha\tilde{q}^T(\dot{M}(q) - C(q,\dot{q}))\dot{q}, \end{aligned} \tag{24}$$

$$\begin{aligned} W_2(\tilde{q}) &= -\tilde{q}^T(K_I - \alpha\Psi_P(\tilde{q}))\tilde{q} + \\ &+ \alpha\tilde{q}^T(g(q) - g(q_d)). \end{aligned} \tag{25}$$

In this way, the problem of determination of conditions for positive-definiteness of function V, which contains three variables (Kelly, 1995), is transformed into two simpler problems of determination of conditions for positive-definiteness of functions $V_1(\tilde{q},\dot{q})$ and $V_2(\tilde{q},z)$, which contain only two variables. The second advantage of the above mentioned decomposition of functions V and W is the elimination of unspecified constant α from the final stability condition.

3.3 Stability criterion determination

In this section, because of compactness, the following shortened notation is introduced: $k_{jm} = \lambda_m\{K_j\}$, $k_{jM} = \lambda_M\{K_j\}$, $j = P, I, D$.

3.3.1. Conditions of positive-definiteness of function V. First, we consider function V_1 which can be rearranged to be of the following form

$$V_1 = \frac{1}{2}\left(\dot{q} + \alpha\tilde{q}\right)^T M(q)\left(\dot{q} + \alpha\tilde{q}\right) - \frac{1}{2}\alpha^2\tilde{q}^T M(q)\tilde{q} + \\ + \frac{1}{2}\tilde{q}^T\Psi_P(\tilde{q})\tilde{q} + U(q) - U(q_d) - \tilde{q}^T g(q_d), \quad (26)$$

and using properties (10) and (5) we get

$$V_1 \geq \frac{1}{2}(k_1 + k_P^{(1)}\|\tilde{q}\| + k_P^{(2)}\|\tilde{q}\|^2)\|\tilde{q}\|^2 - \\ - \frac{1}{2}\alpha^2(a_2 + c_2\|q\| + d_2\|q\|^2)\|\tilde{q}\|^2 \geq 0. \quad (27)$$

Using triangle inequality $\|q\| \leq \|\tilde{q}\| + \|q_d\|$, and rearranging the previous expression we get

$$V_1 \geq \frac{1}{2}(k_1 - \alpha^2\bar{m})\|\tilde{q}\|^2 + \frac{1}{2}(k_P^{(1)} - \alpha^2\bar{m}_1)\|\tilde{q}\|^3 + \\ + \frac{1}{2}(k_P^{(2)} - \alpha^2 d_2\|\tilde{q}\|^4, \quad (28)$$

where

$$\bar{m} = a_2 + c_2\|q_d\| + d_2\|q_d\|^2, \quad (29)$$

$$\bar{m}_1 = c_2 + 2d_2\|q_d\|. \quad (30)$$

The function V_1 is positive-definite if the following conditions are satisfied

$$k_1 > \alpha^2\bar{m}, \quad k_P^{(1)} > \alpha^2\bar{m}_1, \quad k_P^{(2)} > \alpha^2 d_2. \quad (31)$$

Further, we consider function V_2 which can be rearranged to be of the form

$$V_2 = \frac{1}{2}\left(\sqrt{\alpha}z + \frac{1}{\sqrt{\alpha}}\tilde{q}\right)^T K_I\left(\sqrt{\alpha}z + \frac{1}{\sqrt{\alpha}}\tilde{q}\right) + \\ + \frac{1}{2}\tilde{q}^T\left(\alpha K_D - \frac{1}{\alpha}K_I\right)\tilde{q} + \\ + \frac{1}{3}\alpha k_D^{(1)}\|\tilde{q}\|^3 + \frac{1}{4}\alpha k_D^{(2)}\|\tilde{q}\|^4. \quad (32)$$

If we apply properties (16) then

$$V_2 \geq \frac{1}{2}\left(\alpha k_{Dm} - \frac{1}{\alpha}k_{IM}\right)\|\tilde{q}\|^2, \quad (33)$$

that is positive-definite if the following condition is satisfied

$$\alpha^2 > \frac{k_{IM}}{k_{Dm}}. \quad (34)$$

Comparing (34) with (31), the following conditions for positive definiteness are obtained

$$k_1 k_{Dm} > k_{IM}\bar{m}, \quad (35)$$

$$k_P^{(1)} k_{Dm} > k_{IM}\bar{m}_1, \quad (36)$$

$$k_P^{(2)} k_{Dm} > k_{IM} d_2. \quad (37)$$

Note that in the above-stated conditions the unspecified positive constant α is eliminated.

3.3.2. Conditions of positive-definiteness of function W. The next step is the condition which ensures that the time derivative of LF is a negative definite function, i.e., $W \geq 0$. First, we consider function W_1. Applying properties (4), (5), (7) and (16) we get

$$W_1 \geq (\lambda_m\{K_D\} + k_D^{(1)}\|\tilde{q}\| + k_D^{(2)}\|\tilde{q}\|^2)\|\dot{q}\|^2 - \\ - \alpha(a_2 + c_2\|q\| + d_2\|q\|^2)\|\dot{q}\|^2 - \\ - \alpha(c_1 + d_1\|q\|)\|\tilde{q}\|\|\dot{q}\|^2 \geq 0. \quad (38)$$

Using triangle inequality $\|q\| \leq \|\tilde{q}\| + \|q_d\|$, we get

$$W_1 \geq [\lambda_m\{K_D\} - \alpha\bar{m}]\|\dot{q}\|^2 + \\ + [k_D^{(1)} - \alpha(\bar{m}_1 + \bar{k}_c)]\|\tilde{q}\|\|\dot{q}\|^2 + \\ + [k_D^{(2)} - \alpha(d_1 + d_2)]\|\tilde{q}\|^2\|\dot{q}\|^2, \quad (39)$$

where $\bar{k}_c = c_1 + d_1\|q_d\|$.

The function W_1 is positive-definite if the following conditions are satisfied

$$k_{Dm} > \alpha\bar{m}, \quad (40)$$

$$k_D^{(1)} > \alpha(\bar{m}_1 + \bar{k}_c), \quad (41)$$

$$k_D^{(2)} > \alpha(d_1 + d_2). \quad (42)$$

Further, we consider function W_2. Using property (9) we get

$$W_2 \geq (\alpha k_1 - k_{IM})\|\tilde{q}\|^2, \quad (43)$$

that is positive-definite if we have

$$\alpha > \frac{k_{IM}}{k_1}. \quad (44)$$

Comparing (44) with (40)-(42) the following conditions for positive definiteness are obtained

$$k_1 k_{Dm} > k_{IM}\bar{m}, \quad (45)$$

$$k_D^{(1)} k_1 > k_{IM}(\bar{m}_1 + \bar{k}_c), \quad (46)$$

$$k_D^{(2)} k_1 > k_{IM}(d_1 + d_2). \quad (47)$$

Also, in the above-stated conditions, the unspecified positive constant α is eliminated.

3.3.3. A choice of parameters which ensure GAS. *Proposition 1.* The following choice of parameters $k_j^{(i)}$, $j = P, D$, $i = 1, 2$,

$$k_P^{(1)} = \frac{\bar{m}_1}{\bar{m}} k_1, \;\; k_P^{(2)} = \frac{d_2}{\bar{m}} k_1, \tag{48}$$

$$k_D^{(1)} = \frac{\bar{m}_1 + \bar{k}_c}{\bar{m}} k_{Dm}, \; k_D^{(2)} = \frac{d_1 + d_2}{\bar{m}} k_{Dm}, \tag{49}$$

will satisfy stability conditions (36), (37), (46) and (47).

Proof. From (35) or (45) we have

$$\frac{k_1}{\bar{m}} > \frac{k_{IM}}{k_{Dm}}, \quad \frac{k_{Dm}}{\bar{m}} > \frac{k_{IM}}{k_1}. \tag{50}$$

Putting first inequality in (48) we get (36) and (37). Further, putting second inequality in (49) we get (46) and (47). □

4. PERFORMANCE EVALUATION

The Lyapunov function V and its time derivative $\dot{V} = -W$ contain free parameter α. This fact can be employed for the evaluation of the following performance index

$$I = I_1 + \tau^2 I_2, \tag{51}$$

where the constant τ^2 is the weighting factor, and

$$I_1 = \int_0^\infty \|\tilde{q}\|^2 dt, \quad I_2 = \int_0^\infty \|\dot{q}\|^2 dt. \tag{52}$$

Also, in this section, because of compactness, the following shortened notation is introduced

$$\mu_j = \frac{\lambda_M\{K_j\}}{\lambda_m\{K_j\}}, \quad w_p = \frac{1}{p}\|q_d\|^p, \tag{53}$$

where $j = P, I, D$, and $p = 2, 3, 4$.

The performance index (51) can be evaluated using the Lyapunov function (22), (23) and its time derivative. From the equation (21) we can get

$$V(t) - V(0) = -\int_0^t W(\tilde{q}(s), \dot{q}(s)) ds, \tag{54}$$

and, for $t \to \infty$,

$$V(0) = \int_0^\infty W(\tilde{q}(s), \dot{q}(s)) ds, \tag{55}$$

because $V(\infty) = 0$. Putting (38) and (43) in (55) we get

$$V(0) \ge (k_{Dm} - \alpha\bar{m}) I_2 + (\alpha k_1 - k_{IM}) I_1 + \\ + (k_D^{(1)} - \alpha(\bar{m}_1 + \bar{k}_c)) \int_0^\infty \|\tilde{q}\|\|\dot{q}\|^2 dt + \\ + (k_D^{(2)} - \alpha(d_1 + d_2)) \int_0^\infty \|\tilde{q}\|^2\|\dot{q}\|^2 dt. \tag{56}$$

The third and fourth term on the right side of the above-mentioned expression are positive because of (41) and (42), so that

$$V(0) \ge (k_{Dm} - \alpha\bar{m}) I_2 + (\alpha k_1 - k_{IM}) I_1. \tag{57}$$

The next step is the estimation of the upper bounds on V(0). We have $\tilde{q}(0) = -q_d$, $\dot{q}(0) = 0$, $z(0) = -\nu^* = K_I^{-1} g(q_d)$, so that $V(0)$ satisfies the following expression

$$V(0) = -U(q_d) + \frac{1}{2} q_d^T K_P q_d + \frac{1}{2}\alpha q_d^T K_D q_d + \\ + \frac{1}{2}\alpha g(q_d)^T K_I^{-1} g(q_d) + \frac{1}{3} k_P^{(1)} \|q_d\|^3 + \\ + \frac{1}{4} k_P^{(2)} \|q_d\|^4 + \frac{1}{3}\alpha k_D^{(1)} \|q_d\|^3 + \frac{1}{4}\alpha k_D^{(2)} \|q_d\|^4. \tag{58}$$

So, we can estimate the upper bounds

$$V(0) \le w_2(k_{PM} + \alpha k_{DM}) + \frac{1}{2}\alpha k_{IM}^{-1} \|g(q_d)\|^2 + \\ + w_3 k_P^{(1)} + w_4 k_P^{(2)} + \alpha w_3 k_D^{(1)} + \alpha w_4 k_D^{(2)} \tag{59}$$

Because of (8) and $\lambda_M\{K_I^{-1}\} = 1/\lambda_m\{K_I\}$ we have

$$V(0) \le w_2 \left[k_{PM} + \alpha \left(k_{DM} + \frac{k_g^2}{k_{Im}} \right) \right] + w_3 k_P^{(1)} \\ + w_4 k_P^{(2)} + \alpha w_3 k_D^{(1)} + \alpha w_4 k_D^{(2)}. \tag{60}$$

Finally, comparing (57) and (60) we have

$$(k_{Dm} - \alpha\bar{m}) I_2 + (\alpha k_1 - k_{IM}) I_1 \le \\ \le w_2 \left[k_{PM} + \alpha \left(k_{DM} + \frac{k_g^2}{k_{Im}} \right) \right] + \\ + w_3 k_P^{(1)} + w_4 k_P^{(2)} + \alpha w_3 k_D^{(1)} + \alpha w_4 k_D^{(2)}. \tag{61}$$

From the above-mentioned expression we can get integral terms I_1 and I_2 in the following way. If we put $\alpha = k_{Dm}/\bar{m}$ in expression (61) and apply (48) and (49) we get

$$I_1 \le \frac{w_2}{S_M} \left[\bar{m} k_{PM} + k_{Dm} \left(k_{DM} + \frac{k_g^2}{k_{Im}} \right) \right] + \\ + \frac{1}{S_M} (\bar{A} k_1 + \bar{C} k_{Dm}^2), \tag{62}$$

where $S_M = k_1 k_{Dm} - \bar{m} k_{IM}$, and

$$\bar{A} = w_3 \bar{m}_1 + w_4 d_2, \quad \bar{B} = w_3 \bar{k}_c + w_4 d_1, \\ \bar{C} = (\bar{A} + \bar{B})/\bar{m}. \tag{63}$$

Similarly, if we put $\alpha = k_{IM}/k_1$ then

$$I_2 \leq \frac{w_2}{S_M}\left[k_{PM}k_1 + k_{IM}\left(k_{DM} + \frac{k_g^2}{k_{Im}}\right)\right] +$$
$$+ \frac{1}{S_M}\left(\frac{\bar{A}}{\bar{m}}k_1^2 + \bar{C}k_{Dm}k_{IM}\right). \qquad (64)$$

Finally, if we put expressions (62) and (64) in (51) including (53) we get

$$I \leq \hat{I} = \frac{1}{S_M}(k_P^* + A(k_{Dm}^2 + \tau^2 k_{Dm}k_{IM})) +$$
$$+ \frac{B}{S_M}\left(\frac{k_{Dm}}{k_{IM}} + \tau^2\right), \qquad (65)$$

where $\hat{I}$ is the estimation of the upper bounds of the performance index (51), and

$$k_P^* = (\bar{m} + \tau^2 k_1)\left(w_2 k_{PM} + \frac{\bar{A}}{\bar{m}}k_1\right),$$
$$A = w_2\mu_D + \bar{C}, \quad B = w_2\mu_I k_g^2. \qquad (66)$$

Expression (65) can be employed to find the optimal values of the controller gains

$$\frac{\partial \hat{I}}{\partial k_{Dm}} = 0, \qquad \frac{\partial \hat{I}}{\partial k_{IM}} = 0. \qquad (67)$$

The solution of above set of nonlinear algebraic equations can be found by applying simple iterative procedure.

5. SIMULATION EXAMPLE

We consider a 2-DOF manipulator with rotational and translational degrees of freedom,

$$(m_1 l_c^2 + m_2(l+q_2)^2)\ddot{q}_1 + 2m_2(l+q_2)\dot{q}_1\dot{q}_2 +$$
$$+(m_1 l_c + m_2(l+q_2))g\sin(q_1) = 0, \qquad (68)$$
$$m_2\ddot{q}_2 - m_2(l+q_2)\dot{q}_1^2 - mg\cos(q_1) = 0. \qquad (69)$$

as shown in Fig. 1. The values of parameters can be determined comparing (68) and (69) with (5), (7) and (8), $a_1 = \min\{m_1 l_c^2, m_2\}$, $c_1 = c_2 = 2m_2 l$, $d_2 = m_2$, $a_2 = \max\{m_1 l_c^2 + m_2 l^2, m_2\}$, $d_1 = 2m_2$, $k_g = 2g(m_1 l_c + m_2 l) + 2gm_2(2 + q_{d2})$. The numerical values of the model parameters are: $m_1 = 2$ kg, $m_2 = 0.5$ kg, $l_c = 0.7$ m, $l = 1$ m. In Fig. 1. we can see transient response of the link positions for $k_{Pm} = 100$, and for the optimal values $k_{Dm} = 16.6$ and $k_{IM} = 17.2$ which minimize the performance index (65).

6. CONCLUSION

In this paper a new class of globally stable controllers for robot manipulators with revolute and prismatic joints has been presented. The stability criterion in terms of Lyapunov's direct method is proposed to guarantee the global asymptotic stability. Also, a performance index is obtained providing determination of the optimal values of the controller gains.

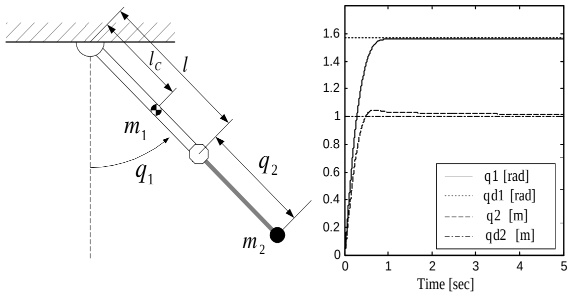

Fig. 1. The robot manipulator and the transient response of the position variables.

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