# PLC SIGNAL PROPAGATION MODELING 

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#### Abstract

Here we present a propagation model for Power-Line Communication (PLC) networks, based on a frequency domain analysis. First we set a simple two-port model of a network and calculate complex attenuation factor from the $z$-parameters, which are expressed in terms of impedances easily calculable/measurable from the network's ports. We provide all the elements for a simple and efficient propagation model, including propagation by crosstalk between different circuits in e.g. three phase networks, as well as influence of the loads connected to the ports of different circuits on such networks. We conclude that the model shown is suitable for PLC channels simulations, giving very accurate results and enabling modeling of wide band channel characteristics.


## Key Words

Power-Line Communications, Propagation, Model, Frequency Domain.

## 1. Introduction

In this article we present an accurate and straightforward model for propagation analysis in complex Power-Line Communication (PLC) networks. The model is based on the simulations of input impedances of network's ports. We also develop a measurement method grounded on the same principle. Models which first evaluate an impulse response of the network, switch to frequency domain by means of inverse fast Fourier transform, and then introduce correction for material loss in cables, are today widely spread and rather well developed [1, 2, 3]. Since distribution networks have very complex topologies and many ports, a multipath propagation phenomena occur. However, tracing of all possible impulse paths in multibranched networks, i.e. pure calculation of delays and amplitudes of all possible modes of impulse propagation, turns out to be very complex even for relatively simple networks. While developing our method, we were led by several firm principles:

- The propagation model should be as simple as possible, and it may require as input data only the network topology and a few cable properties for each cable type used.
- The model must be experimentally verified against another independent direct channel attenuation
measurement, performed on a model network designed for this purpose.
- The model must be based on easily calculable quantities, which must be related only to network terminals, and not to inner (hidden) quantities within the network.
- The model must define all the elements needed to make a software tool for propagation analysis. Besides pure complex transfer function, this tool should also be able to perform calculations and analyses of other quantities important for wide-band communication channel characterization.
So, our purpose is to develop a very simple and practical model with which one can act on real life networks, predicting channel attenuations and other important features. Thus this model is complementary to time domain based models, treating the same physical processes in an alternative way.


## 2. Basic Propagation Analysis

Fig. 1. shows the basic two-port network model with $z$ parameters. From the elementary analysis of this circuit, the input/output voltage ratio, which we will call attenuation, $\Gamma$, is:

$$
\begin{equation*}
\Gamma=\frac{V_{1}}{V_{2}}=\frac{\left(z_{22}+Z_{L}\right) \cdot z_{11}-z_{12}^{2}}{z_{12} Z_{L}} \tag{1}
\end{equation*}
$$



Fig. 1. Two-port model of PLC network.
Other attenuation factors could also be easily specified, e.g. $E / V_{2}, I_{1} / I_{2},\left(V_{1} I_{1} / V_{2} I_{2}\right)$, but throughout this article we assume attenuation is defined as port voltages ratio, (1). Next, we introduce the following quantities:

- $Z_{1 O}$ - impedance seen at port 1 , when port 2 is open;
- $Z_{1 S}$ - impedance seen at port 1 , when port 2 is short circuited;
- $Z_{20}$ - impedance seen at port 2 , when port 1 is open;
- $Z_{2 S}$ - impedance seen at port 2, when port 1 is short circuited.
Since $z$-parameters can be expressed in terms of three of the four of those impedances, (1) can be transformed to:

$$
\begin{equation*}
\Gamma=\frac{Z_{L} Z_{1 O}+Z_{1 S} Z_{2 O}}{Z_{L} \sqrt{Z_{2 O}\left(Z_{1 O}-Z_{1 S}\right)}} \tag{2}
\end{equation*}
$$

or to:

$$
\begin{equation*}
\Gamma=\frac{Z_{1 O}\left(Z_{L}+Z_{2 S}\right)}{Z_{L} \sqrt{Z_{1 O}\left(Z_{2 O}-Z_{2 S}\right)}} . \tag{3}
\end{equation*}
$$

Now we have two equivalent formulas expressed in terms of port impedances rather than $z$-parameters. All of those impedances can easily be calculated or measured in PLC networks. Both propagation model is based on (2), or (3). For the two port network, at least three of four impedances defined above, seen from the two ports under defined conditions, are required. Of course, $z$-parameters can also be calculated by performing three measurements/simulations only from one port, requiring that the other can be loaded by three different precise terminations. We choose the approach with formulas (2) or (3) because it gives the simplest expressions, thus reducing error accumulation when measurements based on the same principle as the simulation method are involved. From purely computational (modeling) point of view, all the approaches mentioned above are equivalent.

Let us emphasize that along with the simulation method treated here we can also define a measurement method involving measurements of port impedances, and then calculation of the attenuation factor from them by (2) or (3). So, when ever we address impedance measurements, we also address impedance calculations, and vice versa.

We shall now explain extension of the method to multiport networks. Imagine an $N$-port network and its $z$ parameter matrix. All the diagonal $z$-parameters can obviously be identified with the impedances measured from appropriate ports, while all other ports are open. Transimpedances should be expressed in terms of impedances measurable from the ports, avoiding simultaneous measurements of voltages and currents on different ports, which might seem appropriate from the $z$ parameters definition, i.e. $z_{i j}=V_{i} / I_{j}$. Since

$$
\begin{equation*}
z_{i j}=\left.\frac{V_{i}}{I_{j}}\right|_{I_{k}=0 ; k \neq j}=\frac{V_{i}}{V_{j} / Z_{j O}}=\frac{z_{j j}}{\Gamma_{j, i}^{O}} \tag{4}
\end{equation*}
$$

we can extract all the $z$-parameters by single-sided impedances measurements/simulations. Here, indexes $i, j$
stand for respective ports, $Z_{j O}$ stands for the impedance seen at the port $j$ while all other ports are open, and $\Gamma_{j, i}^{O}$ stands for the complex voltage attenuation factor when the signal propagates from port $j$ to port $i$, and when all other ports are open. This factor can obviously be determined by two-port type expressions, like (2) or (3), so we can calculate all the $z$-parameters from single-sided impedance measurements. Employing two-port scheme, Fig. 1, $z_{i j}$ can be expressed as:

$$
\begin{align*}
& z_{i j}=z_{j i}=\frac{z_{j j}}{\Gamma_{j, i}^{O}}=\frac{z_{i i}}{\Gamma_{i, j}^{O}}=  \tag{5}\\
& =\sqrt{Z_{i O}\left(Z_{j O}-Z_{j S}\right)}=\sqrt{Z_{j O}\left(Z_{i O}-Z_{i S}\right)}
\end{align*}
$$

Here the indexes $O$ and $S$ mean "open" and "short". For each $z_{i i}$ one must measure one complex impedance, and for each $z_{i j}$ three, so the total number of measured port impedances needed for full description of $N$-port network is $N(3 N-1) / 2$.

Suppose that the port 1 is fed by a source with Thevenen's voltage $E_{1}$ and Thevenen's impedance $Z_{L 1}$. According to standard $z$-parameters definition, voltage on port 1 equals $E_{1}-I_{1} Z_{L 1}$. If the load impedance on $i$-th port is $Z_{L i}$, the matrix equation for currents and voltages is:

$$
\left[\begin{array}{c}
E_{1}  \tag{6}\\
0 \\
\vdots \\
0
\end{array}\right]=\left[\begin{array}{cccc}
z_{11}+Z_{L 1} & z_{12} & \cdots & z_{1 N} \\
z_{21} & z_{22}+Z_{L 2} & \cdots & z_{2 N} \\
\vdots & \vdots & \ddots & \vdots \\
z_{N 1} & z_{N 2} & \cdots & z_{N N}+Z_{L N}
\end{array}\right] \cdot\left[\begin{array}{c}
I_{1} \\
I_{2} \\
\vdots \\
I_{N}
\end{array}\right] .
$$

From the Cramer's rule, the expression for the attenuation factor when signal propagates from port 1 to port $i$ is:

$$
\begin{equation*}
\Gamma=\Gamma_{1, i}=\frac{V_{1}}{V_{i}}=\frac{\sum_{k=1}^{N} z_{1 k} \Delta_{k}}{\sum_{k=1}^{N} z_{i k} Z_{L i}}=-\frac{\sum_{k=1}^{N} z_{1 k} \Delta_{k}}{\Delta_{i} Z_{L i}} . \tag{7}
\end{equation*}
$$

Here $\Delta_{x}$ denotes the determinant of the $z$-matrix from (6) in which the $x$-th column is replaced by a column vector with all elements equal to zero, except the first one, equaling 1 .

One easily shows that the result (7), in both amplitude and phase parts, is sensitive to signs of the transimpedances from (6), which are calculated by (5), so the question of the square roots signs occurs. To ensure the correctness of the resulting transimpedance, one can take the following approach. Determine first the argument of the impedance $Z_{i O}$ from, for example, the left expression in (5), which can take values between $-\pi / 2$ and $+\pi / 2$. Then calculate the argument of the difference $Z_{j O}-Z_{j S}$, which can take any value between 0 and $2 \pi$.

Sum up the two results and divide by 2, so to get the argument of the resulting transimpedance. The module of it is calculated in a straightforward manner, so that we get the transimpedance in polar coordinates, which can easily be converted to Cartesian shape.

Equation (7) enables analysis of any port's load influence on attenuation factor. In general, the multi-port approach may not be very practical because in real networks load impedances are in most cases not known, or at least not sufficiently precisely known. So, with (7) we can in fact simulate load influences in the theoretic context, which we also find usable. However, for the development of the simulation software tool, or the measurement method, we shall apply the two-port model. Load variations are modeled by stochastic termination of all the network nodes according to appropriate distributions which simulate realistic situations.

## 3. Cable Parameters

We shall now briefly describe the building blocks for the development of an efficient and accurate software tool for PLC channel propagation modeling, starting with cable parameters. We present here a method for extraction of relevant parameters from simple measurements of input impedances on a piece of cable, while the other side of it is short circuited, or open. The cable used here for demonstrations was three-wire $0.75 \mathrm{~mm}^{2}$, PVC isolated. It was the cable with highest specific attenuation available on market, which is an advantage for demonstration of material attenuation modeling. Other types more commonly encountered in electrical installations have somewhat lower attenuation. From the basic transmission line theory we can derive the ratio between imaginary and real part of the impedance for an open line of the length $l$ :

$$
\begin{equation*}
X_{O} / R_{O}=-\sin (2 \beta l) / \sinh (2 \alpha l) \tag{8}
\end{equation*}
$$

For shorted line we obtain the same expression, only with opposite sign. Measured ratios for a 15 -meter peace of cable are given on Fig. 2. Information on wave number, $\beta$, is in zero crossings, whilst $\alpha$ can be calculated from the decay of the curves' envelopes. The wave number equals $2 \pi / \lambda=2 \pi f / v$, where $\lambda$ is the wavelength in the propagation medium, and $v$ is wave propagation velocity. The suitable model for $\alpha$ is [1]:

$$
\begin{equation*}
\alpha=\alpha_{0}+\alpha_{1} f^{K} \tag{9}
\end{equation*}
$$

The least squares method employed to (8) and (9) produces very complicated system of simultaneous equations with four unknowns ( $\alpha_{0}, \alpha_{1}, K$ and $\beta$ ). With this system we could make use of all 259 measured for each curve from Fig. 2. Instead we use only the data on zero crossings for calculation of $\beta$, and data on extreme values for $\alpha_{0}, \alpha_{1}$, and $K$. Looking at Fig. 2, the average frequency interval between zero points is 2.87 MHz , and the standard deviation of it is 0.078 MHz . We derive the
least squares condition by enforcing the function $y=$ $\sin (C f)$ to run as close as possible to zero crossings, yielding:
$\sum f_{i} \sin \left(2 C f_{i}\right)=2 \sum y_{i} f_{i}=0$.
All summations hereafter run from $i=1$ to the number of data points used, $N$. Here $C$ is a parameter. The left-hand side of (10) must equal zero because all $y_{i}$ equal zero (zero crossings). Equation (10) has many solutions and is very sensitive to $C$, so the result should be controlled by comparing the solution with average interval between zero crossings, $\Delta f$, which was here 2.87 MHz . Relation between $C$ and $\Delta f$ is simple: $C=\pi / \Delta f$. For our 15-meter test peace of cable we obtained $C=1.141 \mathrm{rad} / \mathrm{MHz}$, implying $v=4 \pi l / C=1.652 \mathrm{~m} / \mathrm{s}$, i.e. 0.55 times light speed in vacuum. Parameter $C$ is constant within the frequency band of our interest ( 5 to 30 MHz ), and so is the wave velocity, $v$.

Parameters $\alpha_{0}, \alpha_{1}$, and $K$ can be extracted from the points where the envelopes touch the curves from Fig. 2, i.e. where $\sin (C f)=1$. Those data points are special because their ordinate values do not depend on $C$. For them we calculate:
$\alpha_{i}=\frac{1}{2 l} \mathrm{~A} \sinh \frac{1}{|X / R|_{\sin \left(C f_{i}\right)=1}}$.
The $X / R$ term applies to both open and short impedance imaginary and real parts. From our two curves we can obtain $N=18$ values for $\alpha_{i}$. Now, using (9), one can easily show that $K$ is the solution of the following implicit equation:


Fig. 2. Measured ratios of imaginary and real part of the impedances of 15 -meter cable (one pair in a 3-wire cable), when the other end is open, or shorted.


Fig. 3. Three-wire cable as a four-port network.

$$
\left[\begin{array}{c}
\frac{N \sum \alpha_{i} f_{i}^{K}-\sum f_{i}^{K} \sum \alpha_{i}}{N \sum\left(f_{i}^{K}\right)^{2}-\left(\sum f_{i}^{K}\right)^{2}}-  \tag{12}\\
-\frac{N \sum \alpha_{i} f_{i}^{K} \ln f_{i}-\sum f_{i}^{K} \ln f_{i} \sum \alpha_{i}}{N \sum\left(f_{i}^{K}\right)^{2} \ln f_{i}-\sum f_{i}^{K} \sum f_{i}^{K} \ln f_{i}}
\end{array}\right]=0 .
$$

This can be solved by a simple numerical procedure. Knowing $K$, we obtain $\alpha_{1}$ as any of the two left-hand side expressions from (12). Finally, $\alpha_{0}$ can be calculated as:

$$
\begin{equation*}
\alpha_{0}=(1 / N) \cdot\left(\sum \alpha_{i}-\alpha_{1} \sum f_{i}^{K}\right) \tag{13}
\end{equation*}
$$

For our cable we obtained: $\quad K=0.709$;

$$
\begin{aligned}
& \alpha_{1}=0.00095 \mathrm{~m}^{-1} \mathrm{MHz}^{-1} \\
& \alpha_{0}=0.00307 \mathrm{~m}^{-1} .
\end{aligned}
$$

These figures are in agreement with values found in literature, see e.g. [1], and they also enabled very accurate simulations.

The wave impedance, $Z_{0}$, was found from the data as the geometrical mean between short- and open line impedances:

$$
\begin{equation*}
Z_{0}=\sqrt{Z_{S} Z_{O}} \tag{14}
\end{equation*}
$$

This was calculated for all measured points at 259 different frequencies, and then all results were averaged. The statistical processing showed very small frequency dependence of the resulting best linear fit throughout the frequency band from 5 to 30 MHz . Real part of $Z_{0}$ was never different from the absolute value of it more than $1.5 \%$, and in $96 \%$ of total frequency band the difference was less than $0.5 \%$. Resulting $Z_{0}$ was $90.442 \Omega$. Using the same procedure, we also extracted the wave impedance when the two of three cable's wires were shorted at both ends. In such a propagation mode, the wave impedance was $67 \Omega$.

## 4. Crosstalk Propagation

An important building block for the simulation tool is a procedure for crosstalk propagation calculations. It enables modeling of situations when the signal is injected into one wire pair and forwarded to other network parts from the other pair, as well as modeling of the influence of a load impedance connected to the wire pair other than the one used for signal transmission. Both situations are commonly encountered e.g. in three-phase networks. On the other hand, with the new methods of PLC network conditioning [1], where the signal is injected into a neutral-protection wire pair, and when other wires are shorted together with the protection one by small capacity condensers, crosstalk propagation model may not be of much importance. Therefore, we shall briefly present here only a model for transmission through two coupled lines section with common wire in a
geometrically symmetrical arrangement. For shortness, we will only give the results, without thorough derivations. We start with Fig. 3, where a three-wire cable is presented as a four-port network consisting of two transmission lines $l$ meters long, between which the signal can propagate by crosstalk. The $z$-parameters of such a structure can be found in [4]:

$$
\begin{align*}
& z_{11}=z_{22}=z_{33}=z_{44}=\frac{Z_{0 e}+Z_{0 o}}{2} \operatorname{coth}[\alpha(f) l+j \beta l] ; \\
& z_{12}=z_{21}=z_{34}=z_{43}=\frac{Z_{0 e}-Z_{0 o}}{2} \operatorname{coth}[\alpha(f) l+j \beta l] ; \\
& z_{13}=z_{31}=z_{24}=z_{42}=\frac{Z_{0 e}-Z_{0 o}}{2 \sinh [\alpha(f) l+j \beta l]} ;  \tag{15}\\
& z_{14}=z_{41}=z_{23}=z_{32}=\frac{Z_{0 e}+Z_{0 o}}{2 \sinh [\alpha(f) l+j \beta l]} .
\end{align*}
$$

It is understood that the cable is geometrically symmetric, and that the common wire is not necessarily at ground potential. Here $Z_{0 e}$ is the even-mode wave impedance, and $Z_{0 o}$ is the odd-mode wave impedance, which is always smaller or equal to $Z_{0 e}$. Our aim is to express the impedances that can be seen from port 1 , when the port 3 is open or shorted, and to express the impedance seen from the port 3 , when the port 1 is open. Those three impedances are sufficient to calculate the $z$-parameters of the two-port network having the ports 1 and 3 from Fig. 3. We assume the ports 2 and 4 are open. Let $Z_{(1,4) O}$ denote the impedance seen from the port 1 , when the port 4 (and all the others) is open, and let $Z_{(1,4) S}$ denote the impedance seen from port 1 when port 4 is shorted, and all the other ports are open. Let $Z_{(1,3) S}$ be the impedance seen from port 1 , when port 3 is shorted, and let $Z_{0}$ denote the wave impedance for the transmission line between ports 1 and 4, which can be measured as the geometrical mean between the latter two impedances, when the ports 2 and 3 are open, as described in Chapter 3. We have proven that the equation holds:

$$
\begin{equation*}
Z_{(1,3) S}=\frac{4 Z_{0 e} Z_{0 o}}{\left(Z_{0 e}+Z_{0 o}\right)^{2}} Z_{(1,4) O}+\left(\frac{Z_{0 e}-Z_{0 o}}{Z_{0 e}+Z_{0 o}}\right)^{2} Z_{(1,4) S} \tag{16}
\end{equation*}
$$

The impedance $Z_{(1,3) S}$ is a linear combination of $Z_{(1,4) O}$ and $Z_{(1,4) S}$. With $a=4 Z_{0 e} Z_{0 o} /\left(Z_{0 e}+Z_{0 o}\right)^{2}$, (16) takes the shape:
$Z_{(1,3) S}=a \cdot Z_{(1,4) O}+(1-a) \cdot Z_{(1,4) S}$.
Note that the factor $a$ can be viewed as another cable parameter and thus added to the set of parameters discussed in Chapter 4. In fact, the propagation model would not be complete without this parameter. Knowledge of $Z_{0}$ and $a$ is equivalent to the knowledge of the pair of wave impedances $Z_{0 e}$ and $Z_{0 o}$. From our previous considerations one can easily derive the connection between those two sets of parameters:
$Z_{0 e}=Z_{0}(1+\sqrt{1-a}) ; \quad Z_{0 o}=Z_{0}(1-\sqrt{1-a})$.

This is a relation with general validity. We found the set $\left(Z_{0}, a\right)$ much more practical than $\left(Z_{0 e}, Z_{0 o}\right)$ because both $Z_{0}$ and $a$ are in fact very easy to measure on a single peace of cable. Since the impedance seen from port 1 when the port 3 (and all the others) is open equals $z_{11}$, as well as to $z_{33}$, from (15), which is also equal to $Z_{(1,4) O}$, we have managed to express all the impedances needed to fully describe the crosstalk behavior in a structure from Fig. 3. with the impedances $Z_{(1,4) O}$ and $Z_{(1,4) S}$ only. The latter two are very easy to simulate, because they are simply: $Z_{(1,4) O}$ $=Z_{0} \operatorname{coth}(\gamma l)$ and $Z_{(1,4) S}=Z_{0} \tanh (\gamma l)$. This is essential if one wants to make a practical simulation tool.

## 5. Propagation Analysis Tool

Having all the elements explained, we may describe how the analysis tool works. It is grounded on impedance calculations. The complex $\Gamma$ factor, which is reciprocal to the complex transfer function, is calculated according to (2) or (3). Assume one wants to calculate $\Gamma$ for the propagation between ports A and B in a network. The program must calculate at least three impedances, for example:

- impedance seen from $A$ when $B$ is open;
- impedance seen from $A$ when $B$ is short;
- impedance seen from $B$ when $A$ is open.

In a distribution network of any complexity those calculations are easy to perform because we have explicit formulas for impedance transformations available. On Fig. 4. one can see a typical junction where four lines meet. Say we want to calculate the impedance seen from the port A , when ports $\mathrm{B}, \mathrm{C}$, and D are loaded by impedances $Z_{B}, Z_{C}$ and $Z_{D}$. Those loads can be anything, for example another line sections or parallel combinations of line sections. Let all four sections have the length $l$ and the complex propagation constant $\gamma$. The load $Z_{B}$ is transformed by the line of the length $l$, so that at the junction point it looks like:
$Z_{B}^{*}=Z_{0} \frac{Z_{B}+Z_{0} \tanh (\gamma l)}{Z_{0}+Z_{B} \tanh (\gamma l)}$.
The other two loads transform in the same manner. In the junction point, the line leading towards port A is loaded by the parallel combination of those transformed loads, so that:

$$
\begin{equation*}
Z_{J}=Z_{B}^{*}\left\|Z_{C}^{*}\right\| Z_{D}^{*} \tag{20}
\end{equation*}
$$

Finally we get the impedance seen from the port A :
$Z_{A}=Z_{0} \frac{Z_{J}+Z_{0} \tanh (\gamma l)}{Z_{0}+Z_{J} \tanh (\gamma l)}$.
This algorithm can obviously be performed easily over arbitrary complex distribution network containing such line branches. Another situation can occur if we have to model the crosstalk propagation. Then the program must calculate the $z$-parameters of such a section $\left(z_{11}, z_{22}\right.$ and
$z_{12}$ ). Impedance transformation by a two-port loaded with certain $Z_{L}$ is simply:

$$
\begin{equation*}
Z_{L}^{*}=z_{11}-\frac{z_{12}^{2}}{z_{22}+Z_{L}} \tag{22}
\end{equation*}
$$

Note that the program has to calculate nothing else, but impedance transformations, using explicit formulas of the types (19) or (21). It is self-understandable that the program must be given by the following inputs:

- cable parameters for each network branch;
- length of each branch;
- network topology;
- additional parameters for crosstalk sections;
- loads connected to each port.

We have tested the simulation model and the measurement method on a simple network made of the cable used for all investigations described in this article. The network had a shape of the letter H. All five branches were 15 meters long. The propagation between two points on diagonally opposite ends of the network was measured and simulated. We ran several tests, with various port terminations, and with the propagation by crosstalk, or influence of the load connected to a wire pair different than the one used for transmission. The results were very good. On Fig. 5. we present an example of a typical result, comparing the following:

- simulation using measured cable parameters;
- transfer function determined from measured port impedances, as described in this article;
- direct measurements, with signal source on one side, and the spectrum analyzer on the other, performed at 26 frequencies within the band from 5 to 30 MHz .


Fig. 4. A typical element of a complex network's structure for explanation of the algorithm used in the simulation software tool.


Fig. 5. On the experimental verification of the propagation model.
One can see excellent agreement, especially between direct attenuation measurement and simulation with the software tool. For impedance measurements we used the reflectometer built in the Anritsu Site Master S114B. We also checked for the agreement between the phase functions obtained by impedance measurements and by simulations. The results were also very good. Complex reflection coefficients with respect to instrument's impedance, $50 \Omega$, were converted to impedances with:

$$
\begin{align*}
& R=50 \Omega \cdot \frac{1-\rho^{2}}{1+\rho^{2}-2 \rho \cos \varphi}  \tag{23}\\
& X=50 \Omega \cdot \frac{2 \rho \sin \varphi}{1+\rho^{2}-2 \rho \cos \varphi}
\end{align*}
$$

We also used the spectrum analyzer of the same instrument for received signal measurements. The signal source in the range from 5 to 30 MHz was Wandel Goltermann PSM 139. We used a digital oscilloscope Metrix OX 2000 for input voltage control. We took utmost care of the connections between instruments and the network ports and made sure that all the measurements are carried out with the exactly the same mechanical arrangement of the equipment and model network. We also examined the effect of mismatch between the network impedance and the spectrum analyzer inner impedance and factored it out.

As regards simulation software tool, we made for demonstration a program that operated over a distribution network of an imagined office building with 150 termination ports. It is self-understood that the network of such complexity cannot be modeled with time-domain approach in an economic way, unless some channel impulse response measurements are taken. Our program needed fractions of a second to simulate the complex transfer function of a single channel. Since this function can depend heavily on the loads connected to network ports, especially if they are placed in the vicinity of the
transmitter or the receiver, we developed another interesting feature.
Our program was generating loads for every termination port stochastically, according to a given distribution. Such a procedure can be performed a number of times. We used to have 1,000 simulations for each channel, meaning that the set of terminating impedances were stochastically changed 1,000 times. In such numerical experiment it is possible to generate a waste of data on various physical quantities important for PLC channel properties, enabling statistical analysis of them. We found this approach very useful because PLC network are in real life conditions hardly predictable. With this feature we have been able to analyze:

- transfer function (amplitude, phase; real and imaginary part, various representations and visualizations);
- group delay versus frequency;
- delay spread according to various definitions;
- network impulse response by means of appropriately adjusted fast inverse Fourier transform;
- delay and delay spread for ultra wideband excitation of the channel;
- full impedance analysis (module, phase, real and imaginary part, Smith chart representation);
- thorough analysis of the channel's Shannon capacity (here the noise scenarios were needed, too).
A full statistical analysis for each quantity mentioned above in a desired frequency band was carried out. Due to a limited scope and volume of this paper, we shall not present the simulation results in any further detail here.


## 6. Conclusion

We have presented a propagation model in frequency domain suitable for PLC channel transfer function simulations. We also provide a measurement method based on the same principles. In both model and measurement method the transfer function is calculated from the impedances that can be calculated/measured on network ports. We have explained simple methods for cable parameters extraction, as well as necessary building blocks for the adequate software tool. We have experimentally verified the proposed model. The simulation tool can be used for extensive channel analyses.

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