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Estimate of consumed energy at backward extrusion process by means of modelling approach

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Abstract

In order to predict the consumed energy in backward extrusion process (on Al 99.5F7 specimens), the analytic (10 models), numerical, stochastic and experimental modelling of deformation work on the basis of multi-factorial experimental designs (by means of the rotatable design of experiments) was done. Thus, results of backward extrusion force versus punch motion with five different coefficients of friction and five different wall thicknesses were obtained. The most important factor contributing to the accuracy of modelling is the plastic curve of material, for that reason the experimental investigations (compression testing on specimens Ø20 mm × 20 mm) were performed and results in form of Hollomon–Ludwik's power law were obtained. Investigations in this paper were supported with: data processing system, measuring sensors and Lab View software. Experimental research of deformation work was done for both the checking and the verification of obtained results.

Keywords: Modelling; Backward extrusion; Experiment

1. Introduction

Process of backward extrusion of axial-symmetric profiles, in comparison with deep drawing, has techno-economic advantages in spite of considerable investments in tools if deep drawing would be conducted in more steps [1]. For backward extrusion, it can be said that due to its material savings, different distributions of stresses in relation to similar processes, and increasingly reduced machining it has become one of the most promising manufacturing processes.

Problem is to determine the model type (analytic, stochastic or numerical) that describes the consumed energy (in the form of backward extrusion force versus punch motion) in backward extrusion process in the most accurate way in comparison with experiment. Because of that, 10 most common analytic models were examined. It is difficult, by means of analytic models, to determine the force of extrusion process exactly. Therefore, stochastic models especially provide wider possibilities in the solving of extrusion force [2].

It is useful to perform the stochastic modelling of the backward extrusion process before expensive manufacturing process. In this way, savings in process and tool improvements can be made at the start stage of process, before its establishing. When the parameters of process became better-understood, backward extrusion force by means of stochastic modelling can be determined. Thus, it is possible to find out the optimal force for this process.

Needs for the faster and cheaper solving of process are higher, therefore numerical modelling of process was performed. Experimental research for the checking and the verification of obtained results was done. All modelling and experiments were performed according to the rotatable design of experiments. Contribution and new in the paper are established stochastic model and the most accurate analytic model. According to authors' knowledge, no one did the comparison.

2. Experimental design

Analytic, stochastic, numerical modelling and experimental investigations of backward extrusion were performed according to the rotatable design of experiments. This is an active experimental design which is the special form of central composition plan applying in the modelling and adaptive control in the processes with more variables. This plan, besides an applicative features, has the property of optimality, thus it is suitable for optimization of processes. The design contains a basic part 2^k (k—number of varying variables in the process), a symmetric set points n_α around the centre of

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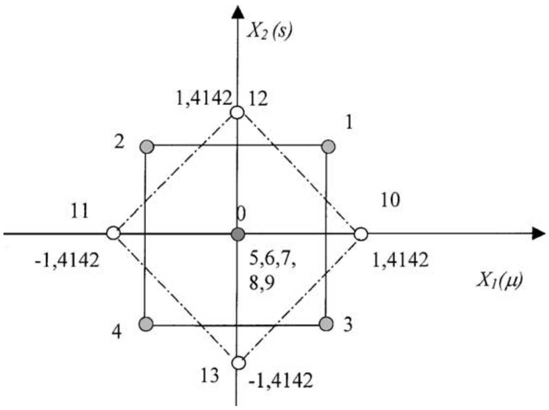

Fig. 1. Schematic outline of rotatable plan for two variables $X_1(\mu)$ and $X_2(s)$.

design, and point of repetition n_0 in the centre of design. The composition of rotatable design is represented in Fig. 1.

2.1. *Experimental investigations*

The basic parts of system for the experimental investigations are: measuring sensors, measuring converter, PCI (data processing system), and press HUO 250–400. Computer for experimental investigations was supported with Lab View software for Windows 95.

Basic data for material (A1 99.5F7): yield strength $k_0 = 41\,\text{N/mm}^2$, ultimate tensile strength $R_\text{m} = 70\text{–}75\,\text{N/mm}^2$, state: soft annealed, diameter: $d_0 = 32\,\text{mm}$, chemical structure: 0.15% Si; 0.20% Fe; 0.03% Cu; 0.03% Mn; 0.03% Mg; 0.04% Zn; 0.02% others. Basic data for lubrication: paraffin (light) $\mu = 0.171$, glycerine $\mu = 0.15$, Zn stearat $\mu = 0.1$, MoS_2 (liqui molly) $\mu = 0.05$, MoS_2 and oil for the lubrication of stainless steel $\mu = 0.029$. Basic data for machine (Press HUO 250–400): manufacturer: Litostroj (Slovenija), maximum force: 2500 kN, maximum stroke: 400 mm. Experimental compression test on Al 99.5F7 (specimen Ø20 mm × 20 mm) was performed and result in the form of Hollomon–Ludwik's power law [3] is obtained as

$$k_\text{f} = 135.9\varphi^{0.266} \tag{1}$$

The matrix of experimental design with varying variables (μ and s) is formed on the basis of rotatable design and it is shown in Table 1. The diameter of workpiece used in these experiments has been 32 mm, but both coefficient of friction and wall thickness of workpiece has been varied according to experimental design. All data regarding the workpiece as well as the results of backward extrusion energy (obtained in experiments as deformation work) are shown in Table 2.

3. Analytic modelling

On the basis of different models (Dipper's model, Siebel's model, Storozev–Popov's model, Tirosh's model, Kudo's model, Tarnovski's model, Beisel's model, Romanowski model, Hribar's model and Anikin–Lukasin's model) by means of different criteria of yielding with rotatable design of experiments the analytic modelling was derived [4]. According rotatable design of experiments the points of design have following features presented in Tables 1 and 2.

Table 1
Matrix of experimental design

Number of experiment	Coded values			Physical values	
	X_0	X_1	X_2	μ	s (mm)
1	1	+1	+1	0.15	4.5
2	1	−1	+1	0.05	4.5
3	1	+1	−1	0.15	2.5
4	1	−1	−1	0.05	2.5
5	1	0	0	0.1	3.5
6	1	0	0	0.1	3.5
7	1	0	0	0.1	3.5
8	1	0	0	0.1	3.5
9	1	0	0	0.1	3.5
10	1	1.414	0	0.171	3.5
11	1	−1.414	0	0.029	3.5
12	1	0	1.414	0.1	4.9
13	1	0	−1.414	0.1	2.1

Regarding a grid element twisting (Fig. 1), according to the most referenced theory on backward extrusion (Dipper's theory [5] of double upsetting), a total strain φ_u is derived as addition of axial φ_{h_1} (in range 1 beyond punch) and radial φ_{r_2} (in range 2 between wall of die and punch) strain:

$$\varphi_u = |\varphi_{h_1}| + |\varphi_{r_2}| = \left|\ln\frac{h_1}{h_0} + \frac{d_1}{8s}\ln\frac{h_1}{h_0}\right| = \varphi_{h_1}\left(1 + \frac{d_1}{8s}\right). \tag{2}$$

Also the same theory gives the equation for calculating a pressure on the bottom of punch:

Table 2
Results of deformation work in experimental investigations

Number of experiment	Basic data				Deformation work, W_exp (Nm)
	μ	h_p (mm)	h_1 (mm)	d_1 (mm)	
1	0.15	5.59	16.057	23	815
2	0.05	5.59	16.057	23	723
3	0.15	1.596	8.04	27	318
4	0.05	1.596	8.04	27	250
5	0.1	3.42	12.28	25	551
6	0.1	3.42	12.28	25	558
7	0.1	3.42	12.28	25	554
8	0.1	3.42	12.28	25	556
9	0.1	3.42	12.28	25	554
10	0.171	3.42	12.28	25	570
11	0.029	3.42	12.28	25	442
12	0.1	6.51	17.46	22.2	775
13	0.1	1	6.2	27.8	203

μ is the coefficient of friction between tool, die and workpiece, d_1 the diameter of punch, s the wall thickness, h_1 the bottom thickness, h_p the punch stroke.

$$p_{\mathrm{be}} = k_{f_1}\left(1 + \frac{1}{3}\mu\frac{\mathrm{d}_1}{h_1}\right) + k_{f_2}\left[1 + \frac{h_1}{s}\left(\frac{\mu}{2} + 0.25\right)\right]. \quad (3)$$

According to (3) backward extrusion force have a form:

$$F_{\mathrm{be}} = \frac{\pi d_1^2}{4}\left[k_{f_1}\left(1 + \frac{1}{3}\mu\frac{d_1}{h_1}\right) + k_{f_2}\left[1 + \frac{h_1}{s}\left(\frac{\mu}{2} + 0.25\right)\right]\right], \quad (4)$$

where p_{be} is the pressure of backward extrusion, k_{f_1}, k_{f_2} the flow stress in ranges 1 and 2, respectively.

Fig. 2 shows both schematic representation of backward extrusion process and a grid pattern deforming as well as a grid element moving according to Dipper's theory. The expression (1) (also known as Ramberg–Osgood's expression) has to be included to the analysis as true stress versus the logarithmic plastic strain φ for calculating k_{f_1}, k_{f_2} in all analytic models.

All 10 referred models for calculating backward extrusion force (as on Dipper's theory in above example) can be found in reference [6], and on the basis of mentioned models and punch stroke h_{p} (Table 2), it can be formed Table 3 presenting results of backward extrusion deformation work (Nm) of all models in comparison with experimental results in the points of rotatable design.

According to Table 2 the best results in comparison with experiment, with Dipper's (at 4 points) and Anikin–Lukasin's (at 3 points) model were achieved. The main disadvantage of Tarnovski's and Hribar's models (the best results at 1 point) is that there is not influence of friction. The best model where there is influence of friction is Dipper's model. In generally analytic model determining a deformation work sufficient exactly in this extrusion process is Dipper's.

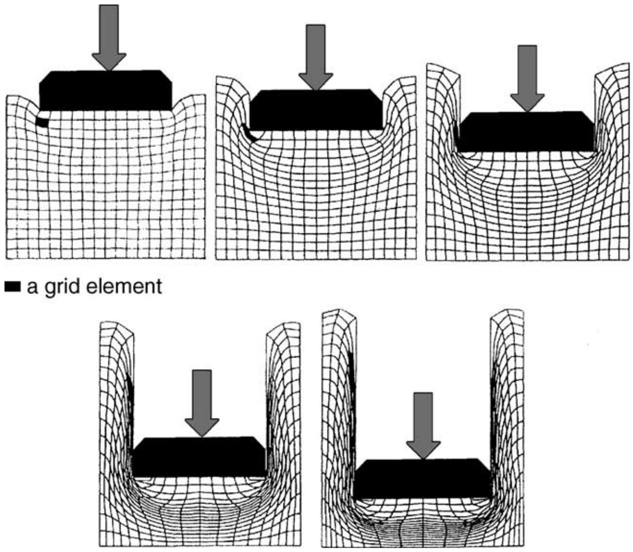

Fig. 2. A grid pattern deforming and a grid element moving.

Table 3
Results of analytic models and experiments

Design point	Dipper model	Siebel model	Storozev–Popov model	Tirosh model	Tarnovski model	Beisel model	Romanowski model	Hribar model	Kudo model	Anikin–Lukasin model	Experiment
1	824.94	930.31	960.22	301.94	652.94	867.36	607.33	716.78	380.42	715.20	815
2	757.14	930.31	960.22	301.94	652.94	867.36	567.58	498.93	380.42	633.56	723
3	329.69	612	574.28	244.09	256.73	450.32	251.53	314.25	264.64	416.22	318
4	286.18	612	574.28	244.09	256.73	450.32	498.24	219	264.64	258.65	250
5–9	572.49	860.54	832.88	172.68	480.56	709.83	434.91	457.16	396.42	501.5	554.6
10	615.98	860.54	832.88	172.68	480.56	709.83	456.56	595.72	396.42	588.63	570
11	528.99	860.54	832.88	172.68	480.56	709.83	413.26	352.12	396.42	453.84	442
12	856.61	916.66	977.72	355.60	699.80	900.28	630.07	631.67	No condition	718.83	775
13	200.70	457.67	430.46	39.85	165.14	322.18	162.62	178.54	192.30	214.20	203

4. Stochastic modelling

Defining of stochastic model starts with identification of set of all process or system parameters (Fig. 3). Working out stochastic model is founded on the statistic processing of experimental data, when conditions are programmed according to the mathematical theory of experimental design (active experiment). That has been achieved by the change of input parameters determining the limit of varying in the conditions of real process. In this way, accurate mathematical model with minimal number of experimental data has been defined [7].

For modelling of backward extrusion force the second-order model has been introduced:

$$Y = b_0X_0 + b_1X_1 + b_2X_2 + b_{12}X_{12} + b_{11}X_1^2 + b_{22}X_2^2. \quad (5)$$

According to introduced model the table of two-factorial design with interaction is shown (Table 4).

Examining of dispersion homogeneity of extrusion force experimental results has been performed as (Cochran's criterion for level of reliability $P = 0.95$):

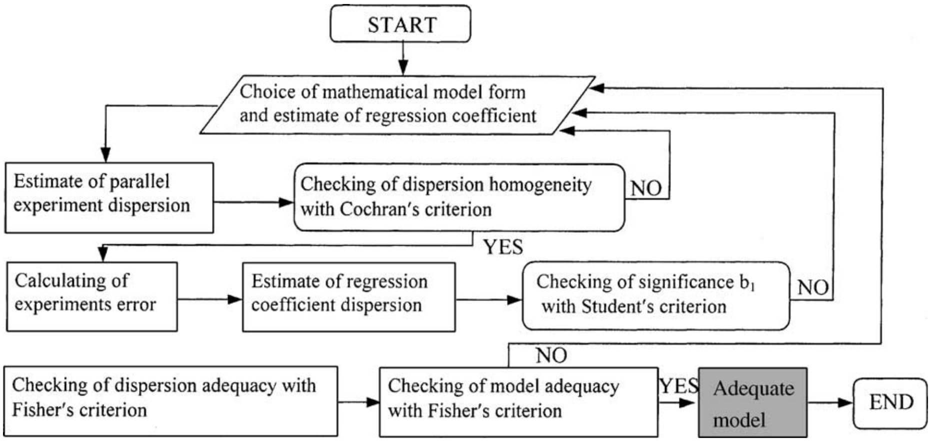

Fig. 3. Algorithm of results processing of active experiment.

$$K_{\mathrm{h}} = \frac{\max S_j^2}{\sum_{j=1}^{N} S_j^2} \leq K_{t(f_j,n_0)} \tag{6}$$

where K_t is the value according to Cochran's criterion for degrees of freedom f_j and N, f_j the degree of freedom ($f_j = n_j - 1$), n_j the repetition number on design, $\sum_{j=5}^{9} S_j^2 = S_0^2$ the variance of central points of rotatable design, $\max S_j^2$ the maximal variance at central design.

After checking of dispersion the mathematical model coefficients have been calculated:

$$b_0 = 554.6, \quad b_1 = 42.624, \quad b_2 = 222.351,$$
$$b_{12} = 6, \quad b_{11} = -17.05, \quad b_{22} = -25.55.$$

Now, coded mathematical model has a form:

$$Y = 554.6 + 42.624X_1 + 222.351X_2 + 6X_{12} - 17.05X_1^2 - 25.55X_2^2. \tag{7}$$

After checking of significance according to Student's criterion, mathematical model has a form:

$$Y = 554.6 + 42.624X_1 + 222.351X_2 + 6X_{12} - 17.05X_1^2 - 25.55X_2^2. \tag{8}$$

Checking of adequacy according to F-criterion has been examined. After model decoding,

$$X_1 = \left(\frac{\mu - 0.1}{0.05}\right) \quad \text{and} \quad X_2 = \left(\frac{s - 3.5}{1}\right), \tag{9}$$

the stochastic model of deformation work in physical form has been obtained:

$$W_{\mathrm{s}} = -6820\mu^2 - 25.55s^2 + 389.201s + 1796.48\mu + 120\mu s - 648.064. \tag{10}$$

A comparison of experimental and stochastic results is shown in Table 5.

The results obtained from stochastic analysis (by means of the Box–Wilson's multi-factorial experimental designs—rotatable design) and experimental investigations show that results of backward extrusion established in stochastic

Table 4
Matrix of coded variables and measuring results of deformation work

Experiment	Coded values						Physical values		Deformation work, Y_j (Nm)
	X_0	X_1	X_2	X_{12}	X_1^2	X_2^2	μ	s	
1	1	+1	+1	1	1	1	0.15	4.5	815
2	1	−1	+1	1	1	1	0.05	4.5	723
3	1	+1	−1	−1	1	1	0.15	2.5	318
4	1	−1	−1	−1	1	1	0.05	2.5	250
5	1	0	0	0	0	0	0.1	3.5	551
6	1	0	0	0	0	0	0.1	3.5	558
7	1	0	0	0	0	0	0.1	3.5	554
8	1	0	0	0	0	0	0.1	3.5	556
9	1	0	0	0	0	0	0.1	3.5	554
10	1	1.414	0	0	2	0	0.171	3.5	570
11	1	−1.414	0	0	2	0	0.029	3.5	442
12	1	0	1.414	0	0	2	0.1	4.9	775
13	1	0	−1.414	0	0	2	0.1	2.1	203

Table 5
Comparison the experimental and stochastic results

Rotatable design points	Stochastic obtained results (Nm)	Experimental obtained results (Nm)	Difference (Nm)
1	782.975	815	32.025
2	685.727	723	37.273
3	326.273	318	8.273
4	253.025	250	3.025
5–9	554.6	554.6	0
10	580.746	570	10.746
11	459.69	442	17.69
12	815.813	775	40.813
13	193.23	203	9.77

modelling are very close to experimental ones, especially according to results in 3rd, 4th, 5–10th and 13th point of experimental design. Also, experimental designs (rotatable design) can be used in stochastic modelling to determine state of stresses and forming force very successful.

5. Numerical modelling

In this analysis the model consisted of 600 axisymmetric quadrilateral elements with four nodes of reduced integration (TYPE = CAX4R). They are recommended as much more appropriate for such large plastic deformation that take place [8]. In backward extrusion during the material flow the element undergo a mesh distortion and that is the most important problem during the numerical analysis in a sever software. A problem is overcome with this type of elements and its interpolation functions. It was found that deformation is different with other type of element. Number of elements, their size and nodes that are needed for correct description of simulation are very important. The lines and curves that define the die and punch were interpreted as rigid unmovable body. They are fixed with a determined point on them. It means, there were used a geometrical way for description of tools. Punch and die are in contact with the workpiece which is a deformable body. At the contact between a workpiece and tool the nodes do not penetrate the tools. Because of symmetry of the process, one half of workpiece and tools can be taken into consideration [9]. Boundary conditions assure the full symmetry of process. The moving in X and Y directions is constrained by means of S instructions. There is automatic remeshing procedure in this software, but mesh was created by hand (elements and nodes are set by means of increments). When the mesh was established, one of the most important factors contributing to the accuracy of the solution is the plastic curve of material that describes the plastic characteristic of material. In order to assess its effect on the predictions, the investigations on material were carried out. Compression test was performed and result in the form of Eq. (1) is obtained. Further, the material is assumed to be rigid–plastic and it obeys the von Mises yield criterion. The elastic characteristics of material are governed by Poisson's ratio and modulus of elasticity.

On the basis of subroutine that determines the total load placed on the workpiece, the force punch–stroke curve has been constructed. By means of numerical integration at this curve, the deformation work (W_n) is calculated. It is presented in Fig. 4, for the central point of rotatable design.

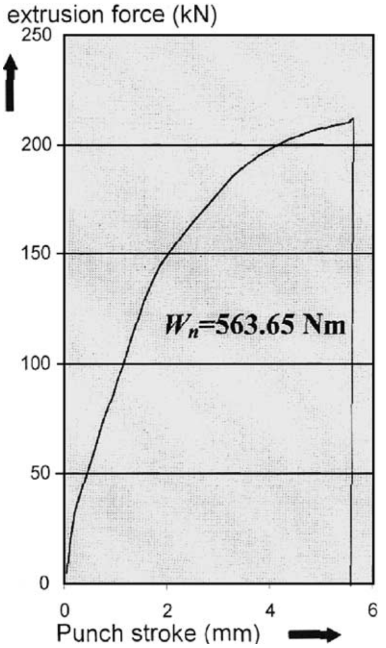

Fig. 4. Deformation work for the central point of rotatable design.

The curve was constructed as the composition of two diagrams. The first one, Fig. 5, represents extrusion stroke–time diagram and other one, Fig. 6, represents force–time diagram. In this way, it is possible to compose two diagrams into one. Results of other extrusion forces obtained by numerical simulation on all points of rotatable design are represented in Table 6.

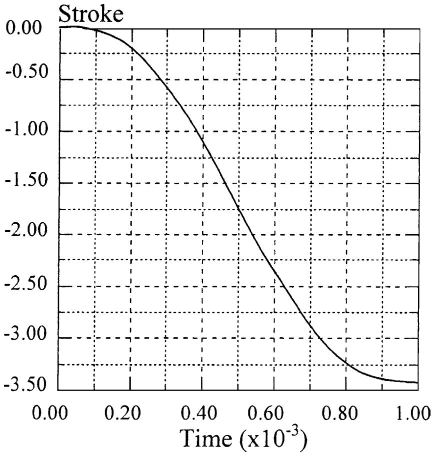

Fig. 5. Stroke–time diagram.

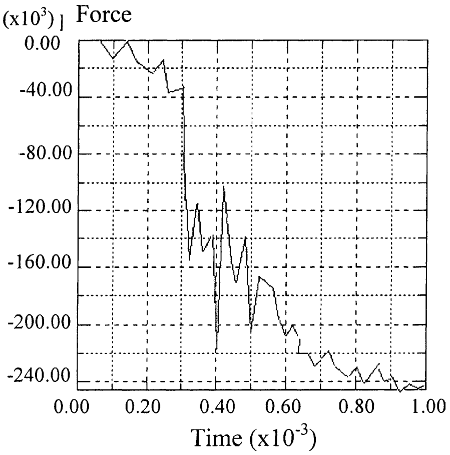

Fig. 6. Force–time diagram.

Table 6
Comparison the experimental and numerical results

Rotatable design points	Numerical obtained results (Nm)	Experimental obtained results (Nm)	Difference (Nm)
1	846.57	815	31.57
2	733.37	723	10.37
3	323.96	318	5.96
4	254.85	250	4.85
5–9	563.65	554.6	9.05
10	606.69	570	36.69
11	467.57	442	25.57
12	791.79	775	16.79
13	216.27	203	13.27

6. Conclusions

Generally, the most accurate analytic model is Dipper's model (at 1st and 13th point of design), then follows Anikin–Lukasin's model (at 11th point), at these models there is the influence of friction, and Hribar's at 3rd point, there is no the influence of friction. The best results in modelling were obtained by means of stochastic modelling, (at 4th, 5–10th, point of design) but disadvantage of this type of modelling is an expensive experiment. According to the experiment numerical modelling is very satisfactory (the most accurate results at 2nd and 12th point). Experimental research has been performed for the checking, the correction and the verification of obtained results. It can be concluded that coefficient of friction has a stronger effect on backward extrusion force. Thus, lubrication has a significant effect on reduction of stresses and forces in this process.

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