

## LETTER TO THE EDITOR

# The importance of the nucleon-nucleon correlations for the $\eta\alpha$ S-wave scattering length, and the $\pi^0$ - $\eta$ mixing angle in the low energy $\eta\alpha$ scattering length model

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**Abstract.** Using the new set of  $dd \rightarrow \eta\alpha$  near threshold experimental data, the estimate of the importance of the nucleon-nucleon correlations for the  $\eta\alpha$  S-wave scattering length in the multiple scattering theory is obtained using the low energy scattering length model. The contribution turns out to be much bigger than previously believed. The  $\pi^0$ - $\eta$  mixing angle is extracted using the experimental data on the  $dd \rightarrow \eta\alpha$  and  $dd \rightarrow \pi^0\alpha$  processes. The model is dominated by the subthreshold extrapolation recipe for the  $\eta\alpha$  scattering amplitudes. When the recipe is chosen the model is completely insensitive to the  $\eta\alpha$  parameters for the subthreshold value of the  $\eta$  cm momentum of  $p_\eta^2 = -(0.46)^2 \text{ fm}^{-2}$ . Provided that the subthreshold extrapolation recipe is correct, a good estimate of the  $\pi^0$ - $\eta$  mixing angle is obtained if the experimental cross sections for the  $dd \rightarrow \pi^0\alpha$  reaction at the corresponding deuteron input energy are taken from the literature.

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In the calculation of the  $\eta\alpha$  S-wave scattering length, nucleon-nucleon correlation corrections to the impulse approximation in the multiple scattering theory have been previously investigated within different formalisms [1, 2, 3]. The agreement about its absolute value has not been reached. We have used the low energy  $\eta\alpha$  scattering length model to estimate that value on the basis of the experimental data for the  $dd \rightarrow \eta\alpha$  near threshold measurements [4, 5] and have shown that the obtained value is significantly bigger than in [3].

In spite of the fact that the  $dd \rightarrow \pi^0\alpha$  process is forbidden by the isospin conservation, nonzero values for the total cross section for that process have been reported [6]. Unfortunately, these measurements have been performed at the subthreshold energy for the isospin allowed  $\eta$  production. The near-threshold measurements for the  $dd \rightarrow \eta\alpha$  can not, therefore, be directly used to extract the  $\pi^0$ - $\eta$  isospin mixing parameter. The dominant  $\eta$ -production S-wave scattering function has to be extrapolated below  $\eta$  production threshold to determine a charge symmetry breaking  $\pi^0$ - $\eta$  mixing angle. It is shown that the extracted value of this angle almost completely depends on the subthreshold extrapolation recipe for the  $\eta\alpha$  scattering function, and it is extremely insensitive to the details of the  $\eta\alpha$  interaction.

When the experimental data quite near the  $\eta$  production threshold with the deuteron energy varying only several MeV are used, the scattering amplitude must be associated with the  $\eta\alpha$  final state interaction. Therefore, the scattering amplitude  $f_\eta$  of the process  $dd \rightarrow \eta\alpha$  can be written as [7]:

$$f_\eta = \frac{f_B}{p_\eta a_{\eta\alpha} \cot \delta - i p a_{\eta\alpha}} \quad (1)$$

$a_{\eta\alpha}, \delta$  ... the scattering length and the S-wave phase shift in the exit channel

$p_\eta$  ... the cm momentum in the exit channel

If the usual approximation for the weak transition to a channel with a strong final-state interaction is used [7] the function  $f_B$  is a slowly varying function near  $\eta$  production threshold. If near threshold expansion of the S-wave scattering phase shift  $\delta$  is applied [8]:

$$p_\eta \cot \delta = \frac{1}{a_{\eta\alpha}} \quad (2)$$

the formula Equation(1) is transformed to the following form:

$$f_\eta \approx \frac{f_B}{1 - i p_\eta a_{\eta\alpha}} \quad (3)$$

Finally, the square of the absolute value of the  $\eta\alpha$  scattering amplitude is expressed in

terms of the differential cross section as:

$$|f_\eta|^2 = \frac{p_d}{p_\eta} \frac{d\sigma}{d\Omega} \quad (4)$$

We suggest to parametrize the nucleon-nucleon correlations using the multiple-scattering theory, as it has been done in reference [3].

In the multiple-scattering expansion the  $\eta\alpha$  S-wave scattering length depends on the impulse approximation term (dependent on the  $\eta$ -nucleon S-wave scattering length) and the nucleon-nucleon correlations contribution  $\beta$  in the following way:

$$\frac{1}{a_{\eta\alpha}} = \frac{1}{4R a_{\eta N}} - \beta. \quad (5)$$

In reference [3] the  $\beta$  is assumed to be real and  $R = m_{red}(\eta\alpha)/m_{red}(\eta N) \approx 1.38$ . The subscript *red* means that the reduced masses are used. The assumption of reality of  $\beta$  bears no physical meaning, and is used to make the model more transparent. Introducing the imaginary part would just bring in an additional free parameter having no obvious physical interpretation.

However, the inputs to this equation have not been well defined until recently: both, the numerical value of the real part of the  $a_{\eta N}$  (S-wave scattering length), and the nucleon-nucleon correlation factor have been model dependent. The origin of the  $\eta N$  scattering length problem has been extensively discussed in ref. [9]. The real part of the  $\eta N$  S-wave scattering length was reported to have very different values: 0.27 [10]  $\leq \text{Real}(a_{\eta N}) \leq 0.98$  fm [11]. In all cases the imaginary part is quite well fixed by the optical theorem ( $\text{Im}(a_{\eta N}) \approx 0.26$  fm). Recently, a controversy is resolved, and a general agreement on the size of the real part has been reached [5, 12, 13, 14]. It is agreed that it is definitely bigger than 0.5 fm, and close to 0.72 fm. In this article we have used four values:  $\text{Real}(a_{\eta N}) = 0.35$  fm [15]; 0.48 fm [16]; 0.55 fm [17] and 0.72 fm [13] as an illustration of the problem.

The nucleon-nucleon correlation factor  $\beta$  is, on the other hand, theoretically estimated in the simple multiple-scattering expansion for the S-wave scattering length  $a_{\eta\alpha}$ . In reference [3] it has been approximated with  $\beta = 0.75 \langle \frac{1}{x} \rangle$ ,  $x$  being the separation between two nucleons in the  $\alpha$  target, and  $R$  is the afore defined ratio of the reduced masses. The factor  $3/4$  originates from the self-correlations, and the  $\langle \frac{1}{x} \rangle$  factor is for simplicity estimated in the rigid model of the  $\alpha$  particle to the value of  $0.375 \text{ fm}^{-1}$  [3]. The question arises whether such an approximation for the nucleon-nucleon correlations is compatible with the value obtained from the experiment, and that is what we have done using the low energy  $\eta\alpha$  scattering length model.

The Argand diagram for the  $\eta\alpha$  S-wave scattering length is shown in Figure 1 for several suggested values of the  $\eta N$  S-wave scattering length, and as a function of  $\beta$ . The open squares connected with the full line show the value of the  $\eta\alpha$  scattering length for the nucleon-nucleon correlation factor  $\beta = 0.28$ , corresponding to the estimate of

reference [3] for different values of  $a_{\eta N}$ . Full dots connected with dotted lines show the value of the  $a_{\eta\alpha}$  for different  $a_{\eta N}$  values and for different values of  $\beta$  in steps of  $0.05 \text{ fm}^{-1}$ , and  $0 < \beta < 0.6 \text{ fm}^{-1}$ . The meaning of the open triangle will be discussed later.

We have decided to test which value of  $\beta$  correspond to the experimental data in the following way:

We take squares of the experimental values of the scattering amplitude for the  $dd \rightarrow \eta\alpha$  process from the literature [4, 5]. We take the low energy scattering length model given by Equation (4), and normalize the value of the function  $|f_\eta|^2$  at the point  $p_\eta=0.15 \text{ fm}^{-1}$  to the value of  $|f_\eta|^2=27.0 \text{ nb/sr}$  [5] for different  $\beta$  values. The results for  $\beta = 0.28; 0.42$  and  $0.56 \text{ fm}^{-1}$  and for the  $\eta N$  S-wave scattering length of ref. [13] are shown in Figure 2, for  $p_\eta > 0$ . The agreement of the model with experiment for the  $\eta N$  S-wave scattering length of ref. [3] and  $\beta=0.28$  is not shown here, but the reader is referred to the original reference - see. [3], Fig.1. The other part of the figure ( $p_\eta < 0$ ) will be explained later. We conclude that the best agreement with experiment is obtained for the value  $\beta = 0.56 \text{ fm}^{-1}$ . Therefore, the nucleon-nucleon correlation corrections which reproduce the experimental numbers for  $dd \rightarrow \eta\alpha$  are much bigger then theoretically estimated in reference [3], and correspond to the  $\eta\alpha$  S-wave scattering length value of  $a_{\eta\alpha} = (-2.88 + i 0.71) \text{ fm}$ . That significantly differs from the value  $a_{\eta\alpha} = (0.396 + i 1.43)$  which would come out as a result of a simple impulse approximation and from the value  $a_{\eta\alpha} = (0.06 + i 6.02) \text{ fm}$  given in reference [3]. On the other hand, the obtained result quite well corresponds to the value  $a_{\eta\alpha} = (-2.2 + i 1.1) \text{ fm}$  of Willis et al. [5] which is extracted by direct fitting the same experimental data set but not in the multiple-scattering formalism basically defined by Equation (5). That value of  $\eta\alpha$  S-wave scattering length is represented by the inverse triangle in Figure 1.

The  $\pi^0$ - $\eta$  mixing angle is defined in the following way:

$\pi^0$  and  $\eta$  have identical quantum number with the exception of isospin, and the physical particles are formed as a mixture of pure isospin states. Then the mixing angle  $\theta$  is given as:

$$\begin{aligned} |\pi^0\rangle &= \cos\theta|\tilde{\pi}^0\rangle + \sin\theta|\tilde{\eta}\rangle \\ |\eta\rangle &= -\sin\theta|\tilde{\pi}^0\rangle + \cos\theta|\tilde{\eta}\rangle \end{aligned} \quad (6)$$

where  $|\tilde{\pi}^0\rangle$  and  $|\tilde{\eta}\rangle$  are isospin eigenstates. If we follow the formalism of reference [3], the  $\pi^0$ - $\eta$  mixing angle is extracted from the ratio of the subthreshold extrapolation of the  $dd \rightarrow \eta\alpha$  cross section to the cm  $\eta$  momentum value of  $p_\eta^2 = -(0.46)^2 \text{ fm}^{-2}$  and the measured value of the  $dd \rightarrow \pi^0\alpha$  scattering function at the corresponding point [6]. The subthreshold extrapolation of the  $\eta d$  amplitude is not known, and in the afore cited model it is defined by the assumption that the  $\eta$  momentum becomes complex, and the

absolute value keeps the negative sign. Then, the mixing angle  $\theta$  is extracted as:

$$\begin{aligned}\cos\theta &= \frac{1}{\sqrt{1+\lambda}} \\ \lambda &= \frac{p_d}{p_\pi F_N} \frac{d\sigma}{d\Omega}(dd \rightarrow \pi^0\alpha) \\ F_N &= N F F^* \\ F &= \frac{a_{\eta\alpha}}{1 - i a_{\eta\alpha} p_\eta}\end{aligned}\tag{7}$$

where  $p_d$  and  $p_\pi$  are deuteron and pion cm momentum values at the subthreshold  $\eta$  production momentum of  $p_\eta^2 = -(0.46)^2 \text{ fm}^{-2}$  and  $\lambda$  is the  $\pi^0$ - $\eta$  mixing parameter. The measured  $dd \rightarrow \eta\alpha$  cross sections are "hidden" in the  $a_{\eta\alpha}$  scattering length parameter. The  $N$  is a normalization constant which ensures that the low energy scattering amplitude expansion is reproducing the measured value of the square of the absolute value of the scattering amplitude of 27.0 nb/sr for the  $\eta$  momentum of  $0.15 \text{ fm}^{-1}$  for the  $dd \rightarrow \pi^0\alpha$  process [5] (effectively simulating the  $f_B$  function given in Equation (1)).

According to the ref. [6] the value of the  $dd \rightarrow \pi^0\alpha$  differential cross section at the deuteron kinetic energy of  $T_d=1.100 \text{ GeV}$ , more specifically 20 MeV below  $\eta$  production threshold is

$$\frac{d\sigma}{d\Omega} = (1.00 \pm 0.25) \frac{\text{pb}}{\text{sr}}$$

at the cm angle of  $73^\circ$ . However, this result has to be taken "with the grain of salt" because of the presence of the two photon (or  $e^+e^-$ ) events observed in the experiment. Combined with the cuts imposed by the acceptance and the analysis, this continuum might simulate a " $\pi^0$ " event of approximately the right mass [3]. Nevertheless, it is interesting to wonder, if we take the result at face value, how accurately can we estimate  $\pi^0$ - $\eta$  charge symmetry breaking parameter. As the result for the  $dd \rightarrow \pi^0\alpha$  process is to be confirmed, we try to estimate how much the  $\pi^0$ - $\eta$  mixing parameter depends on the uncertainty, and not only on the statistical one given in the article, but as well on the systematic one (still quite opened).

If we take the value of  $\beta = 0.56 \text{ fm}^{-1}$  for  $a_{\eta N} = (0.72 + i 0.26) \text{ fm}$ , the values of the  $\pi^0$ - $\eta$  mixing parameters are uniquely extracted:

$$\theta = 0.986^\circ \quad \lambda = 0.017\tag{8}$$

However, it is very interesting to observe that the values of the square of the absolute value of the  $\eta\alpha$  amplitude are extremely insensitive to the nucleon-nucleon correlations parameter  $\beta$ , and henceforth to the overall  $\eta\alpha$  S-wave scattering length, see Figure 2. The value of  $|f_\eta|^2$  at  $p_\eta^2 = -(0.46)^2 \text{ fm}^{-2}$  is almost independent of  $\beta$ . The insensitivity to the  $\eta\alpha$  input originates from the fact that within the low energy model, only the

subthreshold extrapolation of the  $\eta N$  S-wave scattering amplitude determines the shape of the  $|f_\eta|^2$  curve. As it can be seen in Figure 2 the tail is mostly insensitive to the details of the  $\eta\alpha$  S-wave scattering length, and that is exactly the domain where the  $\pi^0$ - $\eta$  mixing angle determination is performed. Introducing the  $\eta\alpha$  effective range might change the afore conclusion slightly, but as it is a completely unknown parameter it will not improve the predictive power of the model.

The insensitivity of the model to the  $a_{\eta\alpha}$  is used to make a correlation of the  $\pi^0$ - $\eta$  mixing parameters and the measured  $dd \rightarrow \pi^0\alpha$  cross section at that energy. In Table 1 we show that correspondence assuming that the systematic error for the  $dd \rightarrow \pi^0\alpha$  measurement can range from 1 pb/sr to maximally 5 pb/sr. Otherwise, the signal would be clearly detected.

**Table 1.** The experimental value of the  $dd \rightarrow \pi^0\alpha$  cross section as a function of different  $\pi^0$ - $\eta$  mixing parameters at  $p_\eta = -(0.46)^2 fm^{-1}$ .

$d\sigma/d\Omega$ (pb/sr)	$\lambda$	$\theta^0$
1	0.017	0.980
2	0.024	1.394
3	0.030	1.707
4	0.034	1.971
5	0.038	2.204

We can offer two general conclusions:

- The nucleon-nucleon correlation contributions to the impulse approximation for the calculation of the  $\eta\alpha$  scattering length in the multiple-scattering theory are much higher than previously anticipated [3] and the  $\eta\alpha$  S-wave scattering length of  $a_{\eta\alpha} = (0.06 + i 6.02)$  fm obtained in that article should not be taken as realistic.
- The precision of experimental resolution of the deuteron beam kinetic energy is seen as a possible problem (the lab kinetic energy for dd initial state goes in steps of 1 Mev at the GeV level). However, we have taken the published numbers at a face value and we are not discussing how reliable they are. We analyze the impact of the published experimental data upon theoretical models. We would not dare to go beyond that, and estimate the reliability of the experimental procedure itself.
- Either increasing the confidence of the existing measurement of the  $dd \rightarrow \pi^0\alpha$  cross section at the present energy or further approaching the  $\eta$  production threshold can improve the confidence of the  $\pi^0$ - $\eta$  mixing angle extraction. However, allowing even for the extremely high systematic error of the  $dd \rightarrow \pi^0\alpha$  process (factor 5) the  $\pi^0$ - $\eta$  mixing angle can not be higher than 2.204<sup>0</sup>.

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### Figure captions

*Figure 1.*

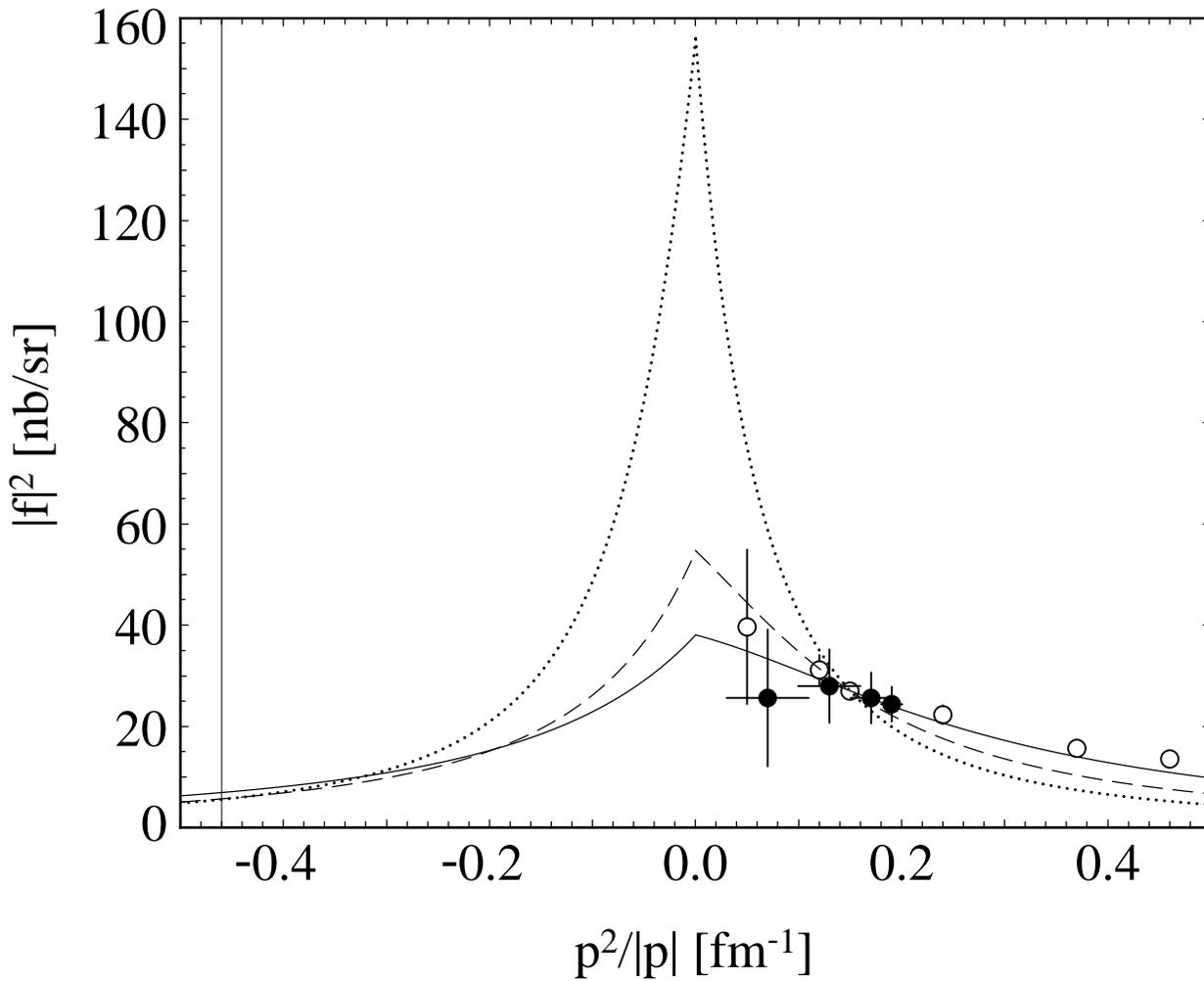
Argand diagram for the  $\eta\alpha$  S-wave scattering length. The open squares connected with the full line show the value of the  $\eta\alpha$  scattering length for the nucleon-nucleon correlation factor  $\beta = 0.28$ , corresponding to the estimate of reference [3] for different values of  $a_{\eta N}$ . Full dots connected with dotted lines show the value for the  $a_{\eta\alpha}$  for different  $a_{\eta N}$  values and for different values of  $\beta$  in steps of  $0.05 \text{ fm}^{-1}$ , and  $0.0 < \beta < 0.6 \text{ fm}^{-1}$ . The open triangle represents the  $\eta\alpha$  S-wave scattering length value without constraints imposed by Equation(5) obtained in reference [3].

*Figure 2.*

The square of the absolute value of the  $\eta\alpha$  scattering function as a function of  $\eta$  cm momentum. Full circles are from reference [4] and open circles are from reference [5]. Full, dashed and dotted lines correspond to the nucleon-nucleon correlation factor values  $\beta = 0.56; 0.42$  and  $0.28 \text{ fm}^{-1}$  respectively for the  $\eta N$  S-wave scattering value of ref. [13].

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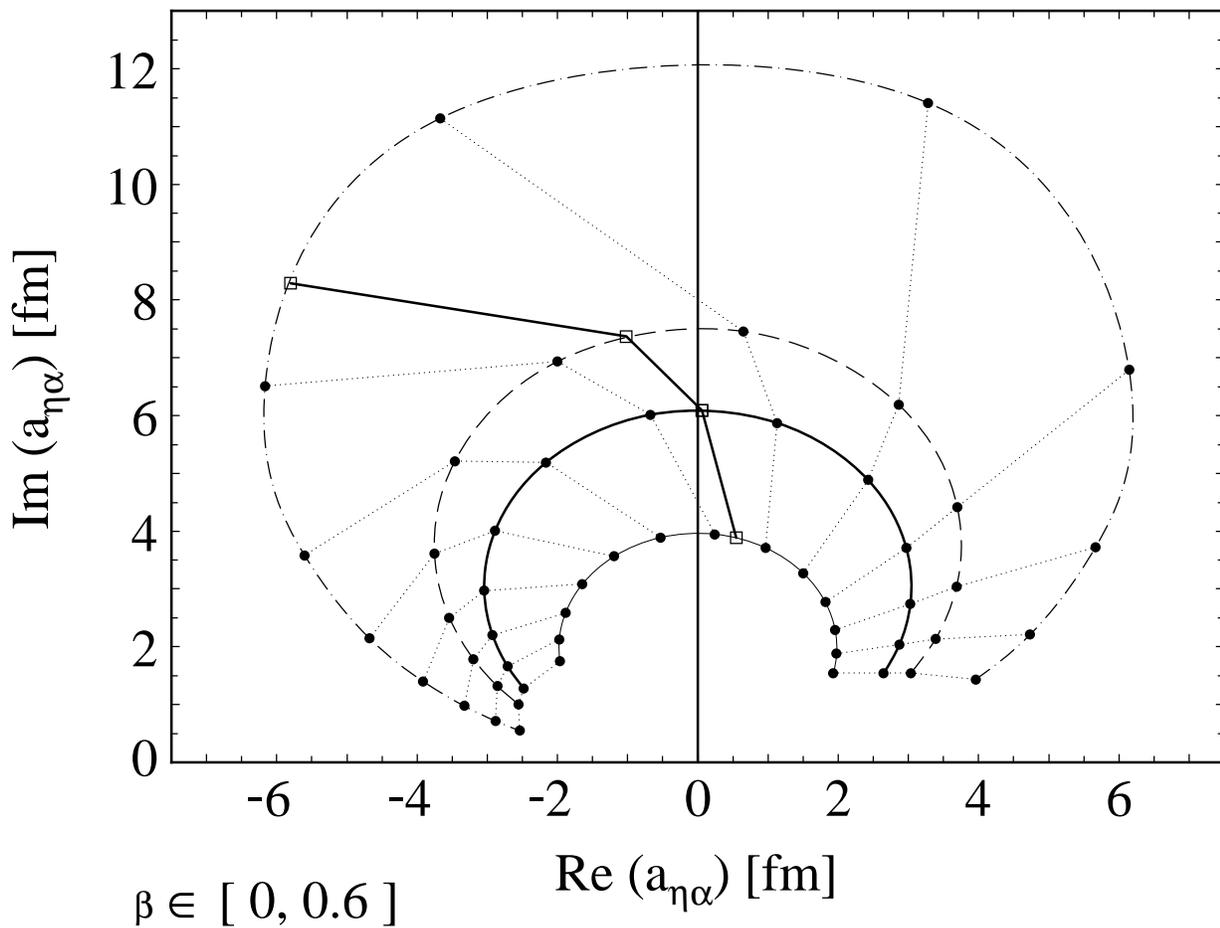
$$a = (0.72 + 0.26 i) \text{ fm}$$

Curves are normalized at  $p = 0.15 \text{ fm}^{-1}$

dotted line:  $\beta = 0.28 \text{ fm}^{-1}$

dashed line:  $\beta = 0.42 \text{ fm}^{-1}$

full line:  $\beta = 0.56 \text{ fm}^{-1}$



thin full line  $a_{\eta N} = (0.35 + 0.28 i) \text{ fm}$

thick full line  $a_{\eta N} = (0.48 + 0.28 i) \text{ fm}$

dashed line  $a_{\eta N} = (0.55 + 0.28 i) \text{ fm}$

dot-dashed line  $a_{\eta N} = (0.72 + 0.26 i) \text{ fm}$