

Instanton Dominance of Topological Charge Fluctuations in QCD?

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Abstract

We consider the local chirality of near-zero eigenvectors from Wilson-Dirac and clover improved Wilson-Dirac lattice operators as proposed recently by Horváth et al. We studied finer lattices and repaired for the loss of orthogonality due to the non-normality of the Wilson-Dirac matrix. As a result we do see a clear double peak structure on lattices with resolutions higher than 0.1 fm. We found that the lattice artifacts can be considerably reduced by exploiting the biorthogonal system of left and right eigenvectors. We conclude that the dominance of instantons on topological charge fluctuations is not ruled out by local chirality measurements.

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1 Introduction

It is generally accepted that instanton field configurations play an important part in hadron phenomenology [1]. The space-time structure of instantons has been the target of a variety of lattice investigations in the past [2]. Recently Horváth et al. [3] have added a very interesting contribution to this discussion about the rôle of instantons. They studied the local chiral orientation of near-zero eigenfunctions on an ensemble of (rather coarse grained) quenched QCD vacuum configurations, but saw no evidence for instanton driven topological charge fluctuations. On the other hand, by applying the techniques of Horváth et al. to overlap [4] instead of standard Wilson fermions, DeGrand and Hasenfratz arrived at the opposite conclusion [5].

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Wilson-type actions have the merit of being ultralocal which is desirable for the study of local properties but at the cost of chiral symmetry breaking. On the other hand the overlap formulation has a remnant of chiral symmetry [6] but might wipe out local structures, as it implies couplings beyond nearest neighbours [7, 8].

In view of the implications of the above findings for our understanding of the QCD vacuum structure we shall focus in this letter on possible lattice artifacts due to the use of Wilson-type operators. At finite lattice spacing a these operators are non-normal. In Section 2 we shall remind the reader of some peculiarities encountered in the eigenmode expansion due to this non-normality [9, 10]. The latter induces a biorthogonal system of distinct right and left eigenvectors. This suggests to modify the local chiral orientation parameter X proposed in Ref. [3]. The resulting lattice improved form, X_{imp} , is based on projecting the chiral reduction of the right eigenvectors onto the *related left* eigenvectors. In Section 3 we shall demonstrate this improvement for the case of QED2. In Section 4 we shall apply X_{imp} to the case of QCD.

2 Local Chirality in Non-normal Scenario

Two commonly used discretizations of the Dirac operator are the Wilson-Dirac matrix, D_W , and the clover improved Wilson-Dirac matrix, D_{CW} . Both matrices are non-normal

$$[D, D^\dagger] \neq 0 . \quad (1)$$

This fact implies that both matrices are not diagonalizable by unitary transformations. Yet their diagonalization can be achieved by similarity transformations with non-unitary matrices

$$\Lambda = S^{-1} D S , \quad (2)$$

with Λ being the diagonal matrix. The columns of S are then the *right* eigenvectors, $|R_i\rangle$, and the rows of S^{-1} the *left* eigenvectors, $\langle L_i|$, of D (see e.g. [11]),

$$\begin{aligned} D|R_i\rangle &= \lambda_i |R_i\rangle, \\ \langle L_i|D &= \lambda_i \langle L_i| . \end{aligned} \quad (3)$$

It follows directly from $S^{-1} S = I$ that the left and right eigenvectors are *biorthogonal*

$$\langle L_i|R_j\rangle = \delta_{ij} . \quad (4)$$

In contrast, the right (or left) eigenvectors by themselves *do not* form an orthogonal system.

This biorthogonality has to be taken into account when expressing an abstract operator A in terms of a matrix representation by use of eigenfunctions

$$A = \sum_{i,j} |R_i\rangle \langle L_i| A |R_j\rangle \langle L_j| . \quad (5)$$

In the following we will consider the operator $(1 \pm \gamma_5)$, which can be expanded accordingly

$$1 \pm \gamma_5 = \sum_{i,j} |R_i\rangle \langle L_i| 1 \pm \gamma_5 |R_j\rangle \langle L_j| . \quad (6)$$

(Actually, we will need only the diagonal elements $\langle L_i| 1 \pm \gamma_5 |R_i\rangle$). Only in the continuum limit, where D becomes normal, the operators in Eqs. (5) and (6) can be expanded in terms of right eigenvectors only.

Fortunately, though, one encounters no duplication of the costs to compute the left eigenvectors of D_W and D_{CW} because these operators obey the γ_5 -hermiticity property

$$D^\dagger = \gamma_5 D \gamma_5 . \quad (7)$$

This symmetry allows for a straightforward construction of the left eigenvectors from the right ones:

$$\langle R_i| \gamma_5 D = \lambda_i^* \langle R_i| \gamma_5 . \quad (8)$$

Hence the left eigenvector to the eigenvalue λ_i^* is nothing but the hermitian conjugate of the right eigenvector to λ_i multiplied by γ_5 from the right:

$$\langle L_i| D = \langle R_i| \gamma_5 D = \lambda_i^* \langle L_i| . \quad (9)$$

Horváth et al. method. In order to quantify the rôle of instantons in the topological charge fluctuations in QCD Horváth et al. define the local “chiral orientation” parameter X of an eigenmode $|\psi\rangle$ at every lattice point (cf. Eq. (19) in [3]),

$$\tan\left(\frac{\pi}{4}(1+X)\right) = \frac{|\psi_-|}{|\psi_+|} . \quad (10)$$

The absolute values $|\psi_-|$ and $|\psi_+|$ stand for ¹

$$|\psi_\pm|^2 \equiv \langle \psi_\pm | \psi_\pm \rangle = \langle \psi | 1 \pm \gamma_5 | \psi \rangle , \quad (11)$$

where the scalar products are meant in colour and spin space, on an individual lattice site. The analysis is carried out on 1 % of the lattice sites with maximal values $\langle \psi | \psi \rangle$. As a result they produce histograms of X -distributions from a number of near zero-eigenmodes and gauge configurations. This is motivated by a continuum consideration according to

¹We use ψ_- and ψ_+ instead of their ψ_L and ψ_R to avoid confusion with the left and right eigenvectors.

which QCD instantons produce distributions in the quantity X that peak at $X = \pm 1$.

This procedure can be applied directly to normal Ginsparg-Wilson fermions where the choice of the bra-vector $\langle\psi|$ in Eq. (11) is not to be questioned since the left and right eigenvectors coincide. The situation is less clear once the normality is lost. In our notation Horváth et al. start from right eigenmodes and use²

$$\tan\left(\frac{\pi}{4}(1+X)\right) = \frac{|\langle R_i|1 - \gamma_5|R_i\rangle|^{1/2}}{|\langle R_i|1 + \gamma_5|R_i\rangle|^{1/2}}. \quad (12)$$

For non-normal operators, however, the question arises whether it would be more economical to adjust to the above described biorthogonal structure of operator representations. This suggests to replace in Eq. (11)

$$\langle\psi_\pm|\psi_\pm\rangle \rightarrow \langle L_i|1 \pm \gamma_5|R_i\rangle, \quad (13)$$

for Wilson-type fermions.

We emphasize that this modified prescription will coincide with the previous one in the continuum limit, $a \rightarrow 0$, where the Dirac operator is normal. Nevertheless one might hope that a lattice adapted definition of X will suppress lattice effects. With this motivation we propose here to make use of an “improved” quantity X_{imp} ,

$$\tan\left(\frac{\pi}{4}(1+X_{\text{imp}})\right) = \frac{|\langle L_i|1 - \gamma_5|R_i\rangle|^{1/2}}{|\langle L_i|1 + \gamma_5|R_i\rangle|^{1/2}}. \quad (14)$$

The selection of test points is consistently chosen with respect to $|\langle L_i|R_i\rangle|$ (instead of $\langle R_i|R_i\rangle$).

Throughout this letter, we shall refer to the two definitions as follows:

$$\langle R_i|1 \pm \gamma_5|R_i\rangle \cong \text{RR definition}, \quad (15)$$

$$\langle L_i|1 \pm \gamma_5|R_i\rangle \cong \text{LR definition}. \quad (16)$$

3 Results for QED2

Let us consider two-dimensional quantum electrodynamics (QED2) in order to probe the concept of the “improved” local chiral orientation parameter X_{imp} . In QED2 the gauge field is (anti)self-dual and one should therefore observe peaks in the chiral orientation near plus and minus one.

In order to test the LR definition, Eq. (14), we perform an analysis on a 24^2 lattice for the β -values 6.0, 2.0 and 1.0. We determined the complex eigenmodes in the physical branch with $|\text{Im } \lambda| < 0.3$. The resulting histograms for 500 quenched gauge field configurations for each β are shown in

²I. Horváth, private communication

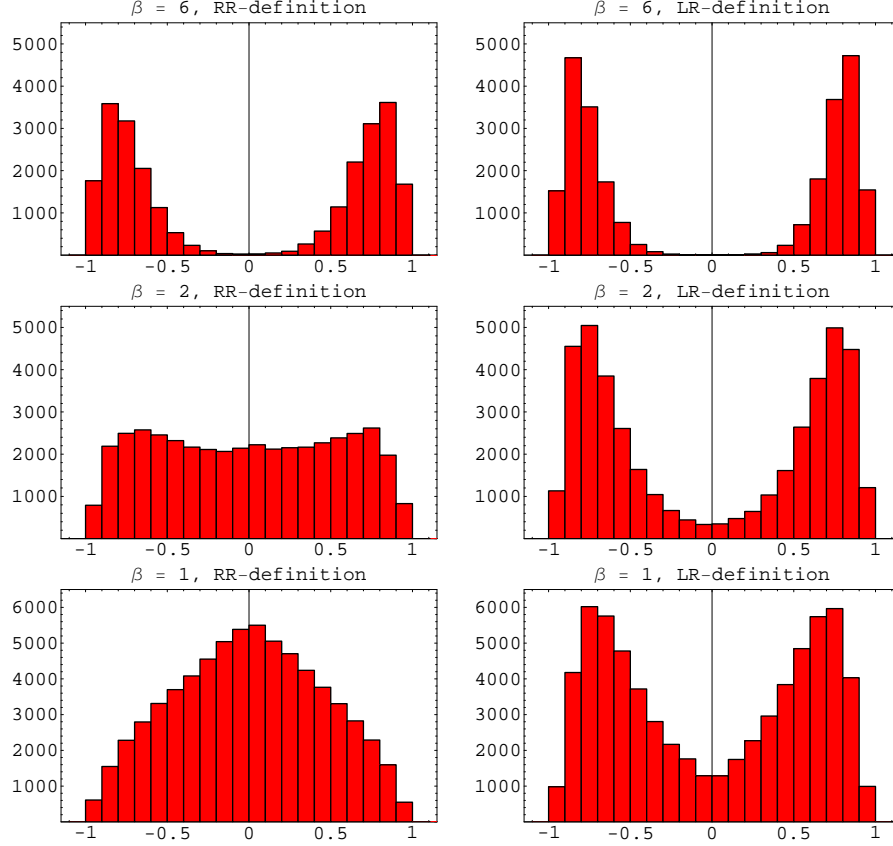


Figure 1: Histograms of the local chiral orientation parameter for (complex) near-zero eigenvectors in QED2 for standard Wilson-Dirac fermions. The two definitions, RR and LR , have been used at $\beta = 6.0, 2.0$ and 1.0 .

the right column of Fig. 1. Throughout our β -range we are able to observe a very clear double peak structure. For comparison, we also included the histograms based on the RR definition, Eq. (12), for the same sets of configurations. We find that the signal becomes almost invisible at $\beta = 2.0$ and vanishes below. We emphasize that the results from both definitions tend to coincide for small lattice spacings, as they should. On the other hand, one can easily be misled by using the RR definition at too coarse lattice spacings.

The reason behind this remarkable difference between the two schemes can be traced to the loss of orthonormality of the right eigenvectors. For if the left and hermitian conjugate right eigenmodes coincide, the following “cosine”

$$\{L_i, R_i\} \equiv \frac{\langle L_i | R_i \rangle}{\sqrt{\langle L_i | L_i \rangle} \sqrt{\langle R_i | R_i \rangle}} \quad (17)$$

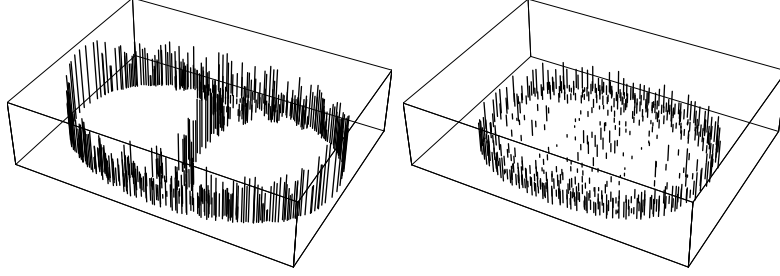


Figure 2: Needle plots of the quantity $\{L_i, R_i\}$ (Eq. (17)) computed for entire spectra on 16^2 lattices at $\beta = 6.0$ (left) and 1.0 (right). The lengths of the vertical needles represent the values of $\{L_i, R_i\}$ for eigenmode i , while their positions are given in the complex λ_i plane.

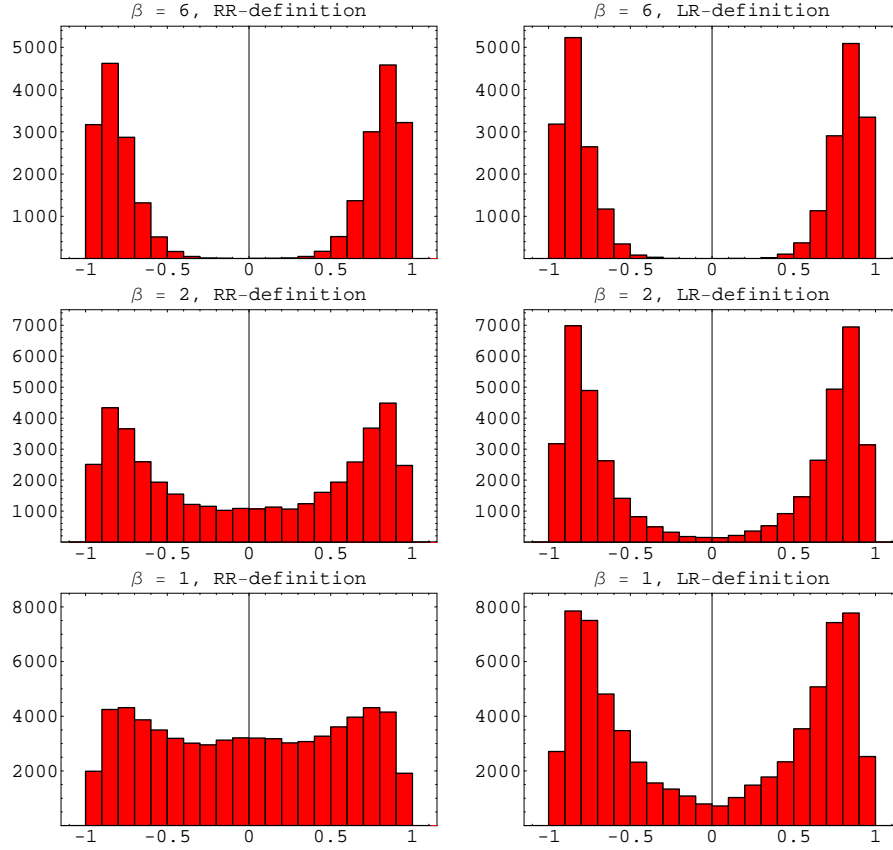


Figure 3: Same as Fig. 1 but with clover improved Wilson-Dirac fermions.

would be equal to *one*. In Fig. 2 we show this quantity in form of needle plots for the entire spectrum of eigenmodes for a 16^2 lattice at $\beta = 6.0$ (left) and 1.0 (right) in form of vertical lines positioned at the complex eigenvalues. For a normal operator, $\{L_i, R_i\}$ is identical to 1 (for all i). Hence deviations from *one* give us a measure for non-normality.

The values of $\{L_i, R_i\}$ are found to be significantly closer to *one* for the larger β -value. This holds in particular in the physical branch (near the left edge of the box). In fact, at the lower β -value, we are too far away from *one* as to still rely on the orthogonality of the right eigenvectors alone.

As another check of the consistency of this picture we have examined to what extent an improved action helps to enhance the signal. To this end we have computed the chiral orientation for the same set of quenched configurations but with clover improved Wilson-Dirac fermions. The results are gathered in Fig. 3 which is to be compared directly to Fig. 1. We find that the improvement on the signal is fully consistent with the idea of accelerated convergence $\langle R_i | \rightarrow \langle L_i |$.

4 Results for QCD

Encouraged by our results in QED2 we shall now turn to quenched QCD. To compare with the results of Ref. [3], we computed 50 near-zero (complex) eigenmodes on 30 configurations of size $12^3 \times 24$ at the same β -value, $\beta = 5.7$. The resulting chirality distributions are plotted in Fig. 4, for both the RR and the LR definitions. While the RR -result shows only a plateau very similar to the one seen in Ref. [3], we do find a slight indication of a double peak structure in the LR case. As a next step, we proceed to finer

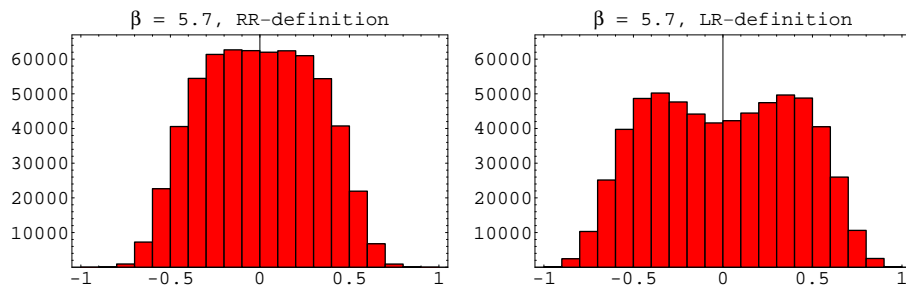


Figure 4: The histograms for X for the two definitions at $\beta = 5.7$ for QCD for near-zero eigenvectors of the Wilson-Dirac matrix, D_W .

lattices, corresponding to $\beta = 5.8, 6.0$ and 6.1 , on $16^3 \times 32$ configurations. The results from the RR and LR definitions are confronted in Fig. 5. We see unambiguous evidence for a deepening of the valley between two clear peaks, which appear to move outwards, towards ± 1 . This is confirmed by

the results from the RR definition, though with quite some delay when increasing β . In order to achieve the same signal quality one has to work with $\beta = 6.1$ (RR), instead of $\beta = 5.8$ (LR).

To complete the study we repeated the computation with clover improvement, at $\beta = 6.0$. The histograms are shown in Fig. 6. As in the QED2 case we observe a markedly better separation of the peaks in the RR situation. There is also some further signal improvement in the LR case.

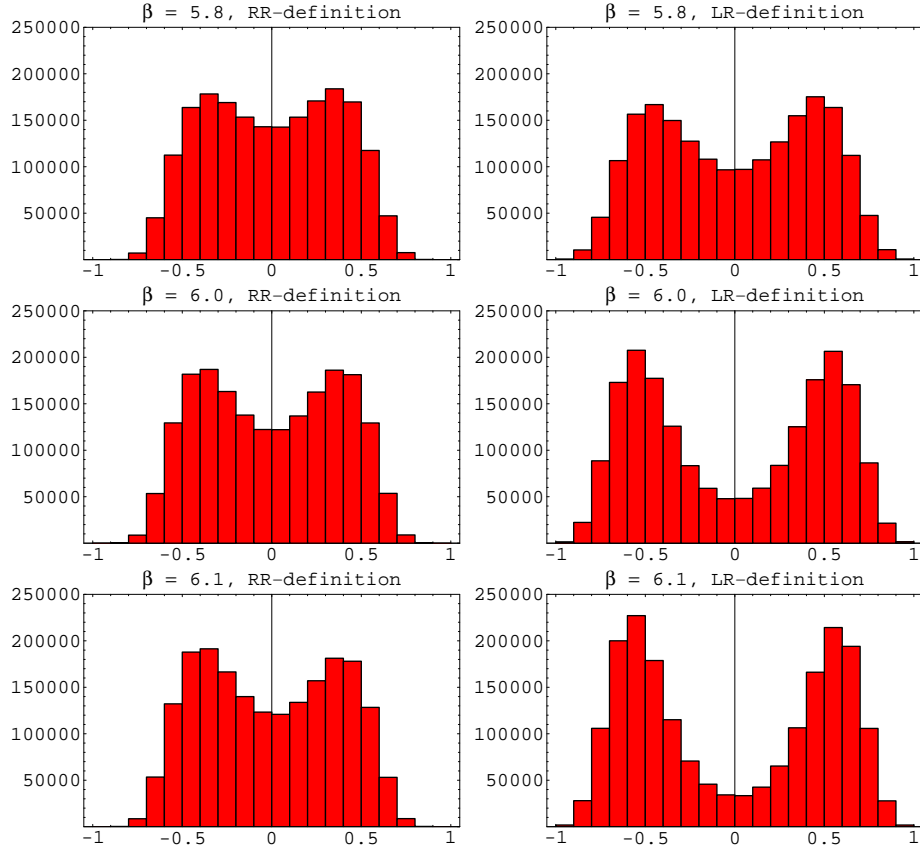


Figure 5: The histograms for X for the two definitions at $\beta = 5.8$, $\beta = 6.0$ and $\beta = 6.1$ for QCD for near-zero eigenvectors of the Wilson-Dirac matrix, D_W .

5 Summary and conclusion

In their debate about the conclusiveness of the local chirality data presented in Ref. [3], the authors of [5] have suspected that the artifacts and the use of a non-chiral lattice action might hide the signal for correlations between

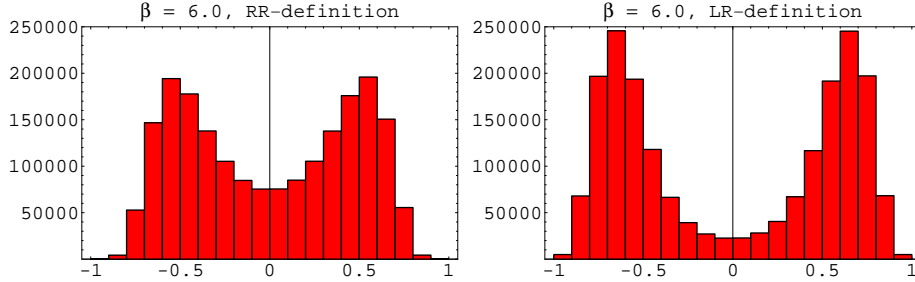


Figure 6: The histograms for X at $\beta = 6.0$ in QCD for the two definitions using the near-zero eigenvectors of the clover-improved Wilson-Dirac operator, D_{CW} .

instanton positions and wave function localizations. This is precisely what we confirm in our analysis.

To clarify the situation, we studied finer lattices and repaired for the loss of orthogonality due to the non-normality of the Wilson-Dirac matrix. As a result we do see a clear double peak structure on lattices with resolutions higher than 0.1 fm ($\beta \geq 6.0$). We found that the lattice artifacts can be considerably reduced by exploiting the biorthogonal system of left and right eigenvectors.

We conclude that the dominance of instantons on topological charge fluctuations is not at all ruled out by local chirality measurements.

In order to consolidate this picture it appears worthwhile to investigate which fermion formulation is actually the most appropriate in this context: although the overlap fermions seem to yield convincing peaks it is not clear whether less local actions might wipe out the fluctuations that one is after. But now the door is open for investigations of the local chiral structure, relying on ultralocal probes. This will help to elucidate the significance of the overlap signal.

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iCE. The computations were done on ALiCE in Wuppertal and CRAY T3E system of ZAM at Research Center Jülich.

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