Transient Performance Based Design Optimization of PM Brushless DC Motor Drive Speed Controller

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Abstract – Using standard integral criteria for optimization of speed controller parameters in an electric motor drive result in relatively high overshoot in speed response. Besides that, the speed controller's integral time constant is much higher than maximum drive time constant, which is unfavorable for load torque compensation. In this paper the speed controller parameters of a permanent magnet (PM) brushless direct current (DC) motor drive are obtained in terms of the overshoot in speed response. A dependence of overshoot in speed on speed controller's gain coefficient and integral time constant is derived and graphically presented for easier adoption of the derived results in industrial and research laboratory settings. It is further demonstrated that faster and better load torque compensation is achieved with smaller values of integral time constant and larger values of speed controller gain coefficient. A desired speed overshoot is achieved by adding a filter at drive input. Responses on both reference and load torque variations of PM brushless DC motor drive validates the proposed design approach. The results are obtained using *Matlab* program package for simulation and optimization of the PM brushless DC motor drive.

I. INTRODUCTION

The most commonly used methods for design of the industrial speed controllers are [1, 2, 8]: experimental, root locus, frequency and optimization methods. The pros and cons of these methods are summarized in the following.

Experimental methods of controller design are based on system behavior approximation with a transfer function of the first order and delay time or they are based on critical gain and period of oscillation on the margin of stability when only proportional controller is used. Applying these methods for determining controller parameters of electric motor drives with insignificant delay time result in relatively high drive overshoot.

Root locus of closed system transfer function is a suitable design method for second order systems because it provides determination of controller parameters, which result in desired pole placement and hence in quality indices of transient response. Applying root locus method for real systems of higher order to place poles at desired locations is quite possible but that invariably requires computer access. Furthermore, simulating the system on a computer is necessary for determination of transient response indices, because the analytic procedure for transient response indices determination is relatively complicated for higher order systems.

The most suitable frequency domain method for design of controller parameters is Bodé's line approximation of magnitude and phase-frequency characteristics. The approximate relations are derived, which connects closed loop system overshoot and open loop phase margin, and first maximum response time and crossover frequency for second and third order systems [3]. Thereby, controller parameters are determined using maximum system time constant compensation and open loop symmetrical frequency characteristics. These relations can be applied for system synthesis, i.e. determination of controller parameters based on desired system overshoot and system maximum response time. The procedure of synthesizing controller parameters of electric motor drive is developed using Bodé plot, which result in better and faster load torque compensation than standard controller design and desired system overshoot for change in a reference value [3].

Different optimization methods can be applied for electric drive controller design and they are: gradient, simplex and Hooke-Jeves. Program package *Matlab* [5] uses gradient and simplex methods. Thereby, different optimization criteria can be used, i.e., integral error criteria and response quality indices. When standard integral error criteria are used for optimization of controller parameters of electric motor drive (ISE, ITSE, IAE, ITAE) in relation to ideal system response, they result approximately in 20% system overshoot. Very small values up to zero overshoot response can be achieved using integral square error criterion and weighted square derivative error. However, in such a case controller integral time constant is much greater than maximum time constant of system, which is not favorable for load torque compensation.

To achieve controller integral time constant smaller than maximum time constant of system when using optimization of electric motor drive with integral criteria, it is necessary to apply reference model for generation of system behavior [4]. A design procedure of electric motor drive controller parameters using reference model and integral square error criterion has been developed in cited literature. The procedure results in better and faster load torque compensation than standard (traditional) controller design and desired system overshoot for a change in reference value [4].

Controller parameter optimization can be achieved also based on desired control quality indices, i.e. system overshoot and the fastest possible response or minimum time of response's first maximum. In that case, the controller integral time constant is much greater than maximum time constant of system. Consequently, stated demands on control quality indices can be accomplished by changing controller gain coefficient, while controller integral time constant practically doesn't have an effect on control quality indices (system overshoot and time of response's maximum). However, it takes great deal of time for errors due to change of reference and load torque to reach zero, because of large value of controller integral time constant.

Therefore, dependence between cascade control system overshoot and controller gain coefficient for different values of controller integral time constants (smaller than maximum time constant of a drive) is determined in this paper. Parameter optimization methods are used for determination of controller gain coefficient as a function of desired values of drive overshoot and controller integral time constant. The developed method is applied to a PM brushless dc motor drive speed controller design and normalized design curves are derived for application in practical settings. The design method is validated with extensive dynamic simulation. It is believed that the approach will be put to practical use in industrial settings because of its simplicity, robustness and superior results compared to controllers designed using other methods.

The paper is organized on the following lines. Second section of this paper describes a cascade speed control system of PM brushless DC motor drive, for which speed controller parameter optimization based on control quality indices is developed. The results of dependence between system overshoot and controller gain coefficient, and optimal gain coefficient on desired system overshoot and integral time constant are derived and developed in the third section of the paper. The conclusion and references are given in fourth and fifth sections, respectively.

II. Model of a PM Brushless DC Motor Drive

This model is based on the PM brushless DC motor drive discussed and given in [6, 7]. For the sake of easy reference, the model is derived in brief and given in the following. During two phase conduction, the entire dc voltage is applied to the two phases and the transfer function for the stator current is given by (Fig. 1),

$$\frac{I_{as}(s)}{V_{is}(s)-E(s)}=\frac{K_a}{1+T_a s}, \tag{1}$$

where $K_a = 1/R_a$, $T_a = L_a/R_a$, $R_a = 2R_s$, $L_a = 2(L - M)$, R_s is the stator resistance per phase, L is the self inductance per phase, M is the mutual inductance per phase, E is the induced emf and s is the Laplace operator.

The induced emf E is proportional to rotor speed Ω_m,

$$E = K_b \Omega_m, \tag{2}$$

where

$$K_b = 2\lambda_p, \tag{3}$$

λ_p is the flux linkages per phase (volt/rad/sec).

Note that the electromagnetic torque for two phases combined is given by,

$$T_e = 2\lambda_p I_{as} = K_b I_{as}. \tag{4}$$

The load is assumed to be proportional to speed,

$$T_l = B_t \Omega_m. \tag{5}$$

With that included in the feedback path, the speed to air gap torque transfer function can be evaluated as (Fig. 1),

$$\frac{\Omega_m(s)}{T_e(s)}=\frac{K_t}{1+T_t s}, \tag{6}$$

where: $K_t=1/B_t$, $T_t= J/B_t$, $B_t=B_1+B_2$, where B_1 is the friction coefficient of the motor and J is the inertia of the machine.

Transistor chopper transfer function is given by,

$$\frac{V_{is}(s)}{V_c(s)}=\frac{K_r}{1+T_r s}, \tag{7}$$

where

$$T_r=\frac{T_{ch}}{2}=\frac{1}{2f_{ch}}, \tag{8}$$

f_{ch} is chopper frequency.

The current and speed feedbacks have low pass filters with transfer functions (Fig. 1),

$$\frac{I_{am}(s)}{I_{as}(s)}=\frac{K_c}{1+T_c s}, \tag{9}$$

$$\frac{\Omega_{mr}(s)}{\Omega_m(s)}=\frac{K_\omega}{1+T_\omega s}. \tag{10}$$

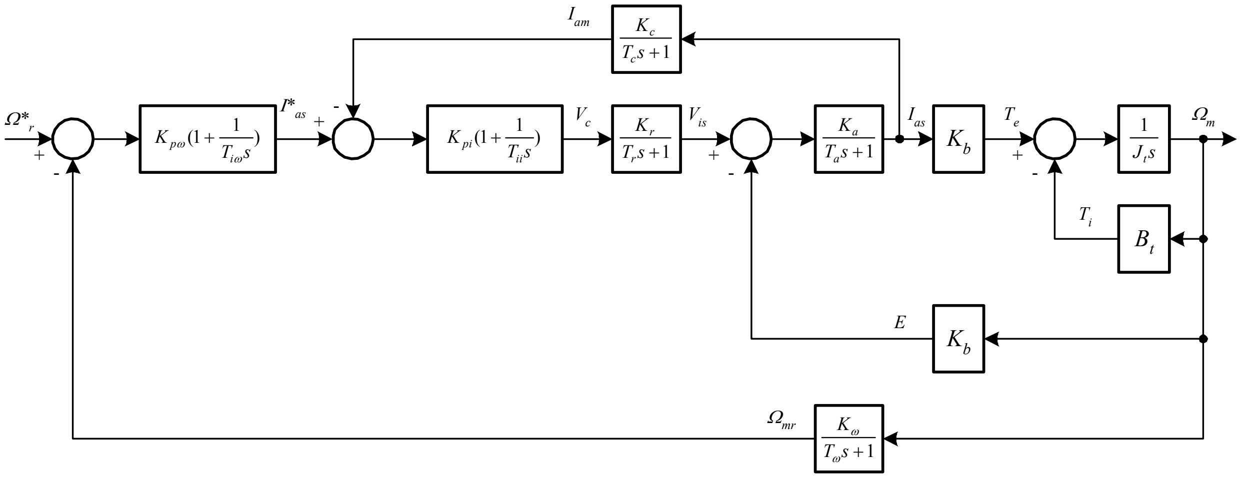

Fig. 1. Block schematic of cascade speed control system of PM brushless DC motor drive.

Numerical value of the drive parameters are:

Base speed, $n_b = 4000$ rev/min, Base power, $P_b = 373$ W, Base current, $I_b = 17.35$ A, Base voltage, $V_b = 40$ V, Base torque, $T_b = 0.89$Nm., Supply voltage, $V_s = 160$V, Maximum phase current, $I_{max} = 2I_b = 34.7$ A, Maximum torque, $T_{max} = 2T_b = 1.78$Nm., Gain of the inverter, $K_r = 16$V/V, Time constant of the converter, $T_r = 50\mu$s, Phase resistance, $R_a = 1.4\Omega$, Phase inductance $L_a = 2.44$mH, Phase time constant, $T_a = L_a/R_a = 1.743$ ms, $K_a = 1/R_a = 0.71428$ A/V, Emf constant, $K_b = 0.051297$ Vs, Total friction coefficient, $B_t = 0.002125$ Nm/rad/sec, Inertia, $J = 0.0002$ kgm^2, $K_t = 1/B_t = 41.89$, Motor and load time constant, $T_t = J/B_t = 94.1$ ms, Current feedback gain $K_c = 0.288$ V/A, Current feedback time constant, $T_c = 0.159$ ms, Speed feedback gain, $K_\omega = 0.02387$ Vs/rad, Speed feedback time constant, $T_\omega = 1$ ms.

Integral time constant of the current controller is usually chosen to be equal to the armature time constant (compensates maximum time constant in the current loop): $T_{ii} = T_a = 1.743$ ms. For the overshoot $M_{pi} = 5\%$ current controller gain coefficient determined from the Bode plot and simulation is $K_{pi} = 1.267$.

III. Results of Controller Parameters Determination Based on Desired System Overshoot

Dependence of overshoot in speed feedback signal response $M_{p\omega mr}$ and controller gain coefficient $K_{p\omega}$, for different values of controller integral time constant $T_{i\omega}$, is determined by simulation of cascade speed control system (Fig. 1) on computer, using program package *Matlab*. There are five values of controller integral time constant which are used in relation to the maximum time constant of the system ($T_t = J_t/B_t = 94{,}1$ ms): $T_{i\omega} = T_t$; $0{,}75T_t$; $0{,}5T_t$; $0{,}25T_t$ and $0{,}125T_t$ (Fig. 2).

Fig. 2 shows that lowering integral time constant $T_{i\omega}$, with constant value of controller gain coefficient $K_{p\omega}$, increases drive response overshoot $M_{p\omega mr}$. In explanation, lowering PI controller integral time constant, with constant value of controller gain coefficient, increases overall open loop gain coefficient which means higher drive overshoot.

Besides that, curves shown in Fig. 2 have a minimum in case when controller integral time constant is smaller than maximum time constant of the drive: $T_{i\omega} < T_t$. This means that for a certain value of controller integral time constant, it wouldn't be possible to achieve drive overshoot smaller than minimum values determined from curves shown in Fig. 2. Consequently, to achieve smaller drive overshoot than 20 (10) %, integral time constant should be higher than 0.125 (0.25) T_t.

Likewise, desired drive overshoot, for certain value of controller integral time constant, can be achieved with two different values of controller gain coefficient (Fig. 2). This means that optimization of controller gain coefficient, for desired value of drive overshoot, can result in completely different values of gain.

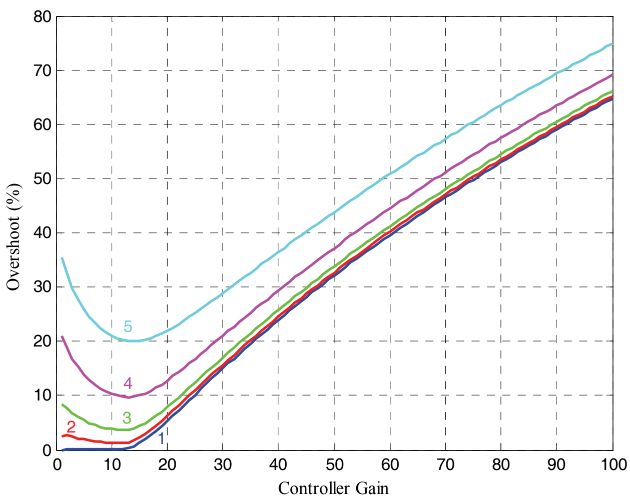

Fig. 2. Dependence of speed feedback signal overshoot $M_{p\omega mr}$ and controller gain coefficient $K_{p\omega}$ for different values of controller integral time constant:
1. $T_{i\omega} = T_t$; 2. $T_{i\omega} = 0.75T_t$; 3. $T_{i\omega} = 0.5T_t$; 4. $T_{i\omega} = 0.25T_t$; 5. $T_{i\omega} = 0.125T_t$.

Consequently, value of controller gain coefficient determined using optimization methods will depend on initial value of gain coefficient. If the initial value of gain coefficient is small (high), it will result in small (high) optimal value.

In case of classic (standard) speed controller design, which compensates maximum time constant of drive, speed controller integral time constant equals: $T_{i\omega} = T_t$ = 94.1 ms. Controller gain coefficient for drive overshoot $M_{p\omega mr}$ = 10% gives $K_{p\omega}$ = 24.8, as seen from Fig. 2. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} on step change of reference value $\Delta\omega^*_r(t)$ = 0,1S(t), with $T_{i\omega} = T_t$ = 94.1 ms and $K_{p\omega}$ = 24.8, are shown on Fig. 3.

For faster and better load torque compensation it is necessary that controller integral time constant be as small as possible and controller gain coefficient as large as possible. Therefore, controller integral time constant is picked as: $T_{i\omega} = 0.125T_t$ = 11.76 ms and controller gain coefficient is chosen for speed feedback signal overshoot $M_{p\omega mr}$ = 40%: $K_{p\omega}$ = 44.9 (Fig. 2). To achieve system overshoot $M_{p\omega mr}$ = 10%, first order filter with time constant T_f = 1.96 ms has been added to the drive input. Responses in Fig. 3 (curves 2) show that speed feedback signal $\Delta\omega_{mr}$ and speed $\Delta\omega_m$ have approximately the same time of response at maximum, while maximum value of armature current is a little bit less than in the case of maximum time constant compensation (curves 1).

Fig. 4 shows responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} on a change of nominal load torque value $M_t(t)$ = 0.89S(t) with controller parameters determined for desired system overshoot in a change of reference value $M_{p\omega mr}$ = 10%. Responses show that influence of load torque on speed is significantly faster (8 times) and better (2 times) compensated in case of controller parameters determined for integral time constant $T_{i\omega}$ = 11.76 ms (curves 2) than for integral time constant $T_{i\omega}$ = 94.1 ms (curves 1).

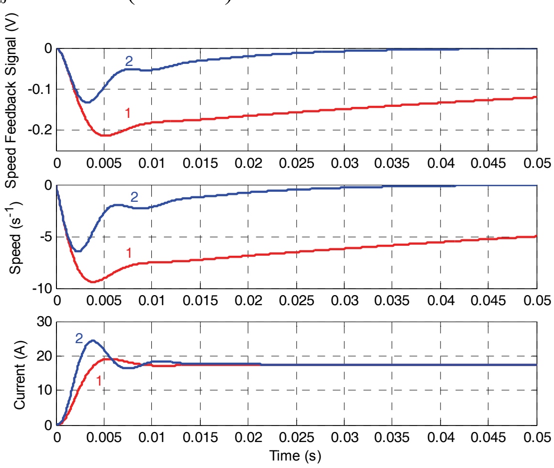

Fig. 3. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} for a change in reference speed $\Delta\omega^*_r(t)$ = 0.1S(t) with speed controller parameters determined for $M_{p\omega mr}$ = 10%:
1 – $K_{p\omega}$ = 24.8, $T_{i\omega}$ = 94.1 ms, T_f = 0;
2 – $K_{p\omega}$ = 44.9, $T_{i\omega}$ = 11.76 ms, T_f = 1.96 ms.

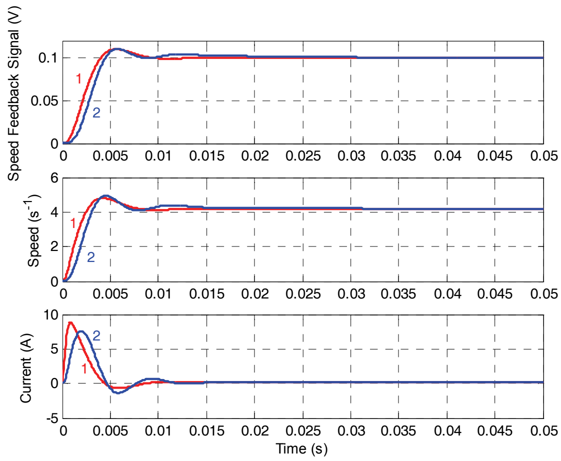

Fig. 4. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} for a change in nominal load torque $M_t(t)$ = 0.89S(t) with speed controller parameters determined for $M_{p\omega mr}$ = 10%:
1 – $K_{p\omega}$ = 24.8, $T_{i\omega}$ = 94.1 ms, T_f = 0;
2 – $K_{p\omega}$ = 44.9, $T_{i\omega}$ = 11.76 ms, T_f = 1.96 ms.

In case when integral time constant is equal to maximum time constant of system $T_{i\omega} = T_t$ = 94.1 ms, controller gain coefficient determined for system overshoot $M_{p\omega mr}$ = 40% equals $K_{p\omega}$ = 60.6 (Fig. 2). In that case a filter with time constant T_f = 1.51 ms is added to drive input to achieve desired system overshoot $M_{p\omega mr}$ = 10%.

Fig. 5 and Fig. 6 show responses comparison for controller parameters determined for desired overshoot $M_{p\omega mr}$ = 10%: 1. $T_{i\omega}$ = 94.1 ms, $K_{p\omega}$ = 60.6, T_f = 1.51 ms and 2. $T_{i\omega}$ = 11.76 ms, $K_{p\omega}$ = 44.9, T_f = 1.96 ms. Response to the reference speed is a little faster for controller integral time constant $T_{i\omega}$ = 94.1 ms (curves 1 in Fig. 5) because of higher value of controller gain coefficient. However, obvious advantage of controller with integral time constant $T_{i\omega}$ = 11.76 ms is in speed of response to load torque compensation (curves 2 in Fig. 6). Speed drop (loss) is smaller for controller integral time constant $T_{i\omega}$ = 94.1 ms because of higher value of gain coefficient, but the difference is not so noticeable.

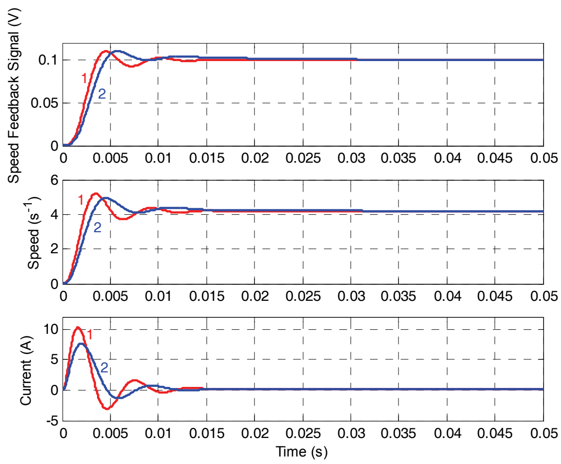

Fig. 5. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} for a change in reference speed $\Delta\omega^*_r(t)$ = 0.1S(t) with speed controller parameters determined for $M_{p\omega mr}$ = 10%:
1 – $K_{p\omega}$ = 60.6, $T_{i\omega}$ = 94.1 ms, T_f = 1.51 ms;
2 – $K_{p\omega}$ = 44.9, $T_{i\omega}$ = 11.76 ms, T_f = 1.96 ms.

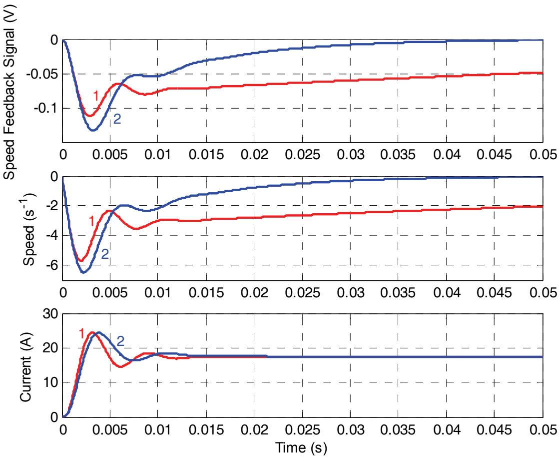

Fig. 6. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} for a change in nominal load torque $M_t(t) = 0.89\mathrm{S}(t)$ with speed controller parameters determined for $M_{p\omega mr} = 10\%$:
1 – $K_{p\omega} = 60.6$, $T_{i\omega} = 94.1$ ms, $T_f = 1.51$ ms;
2 – $K_{p\omega} = 44.9$, $T_{i\omega} = 11.76$ ms, $T_f = 1.96$ ms.

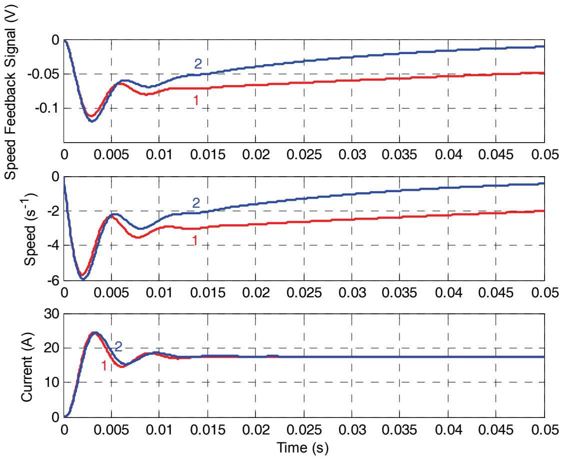

Fig. 8. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} for a change in nominal load torque $M_t(t) = 0.89\mathrm{S}(t)$ with speed controller parameters determined for $M_{p\omega mr} = 10\%$:
1 – $K_{p\omega} = 60.6$, $T_{i\omega} = 94.1$ ms, $T_f = 1.51$ ms;
2 – $K_{p\omega} = 54.5$, $T_{i\omega} = 23.525$ ms, $T_f = 1.66$ ms.

Approximately the same speed of responses to change of reference value (Fig. 7) and to change of nominal load torque (Fig. 8) are achieved in a case of controller parameters determined for desired overshoot $M_{p\omega mr} = 10\%$: 1. $T_{i\omega} = 94.1$ ms, $K_{p\omega} = 60.6$, $T_f = 1.51$ ms and 2. $T_{i\omega} = 23.53$ ms, $K_{p\omega} = 54.5$, $T_f = 1.66$ ms. However, obvious advantage of controller with integral time constant $T_{i\omega} = 23.53$ ms is in speed of load torque compensation (curves 2 on Fig. 8).

From all the foregoing discussion of results, it follows that approximately the same speed of response (time of response at maximum) can be achieved using optimization of controller gain coefficient based on desired drive overshoot for different values of controller integral time constant.

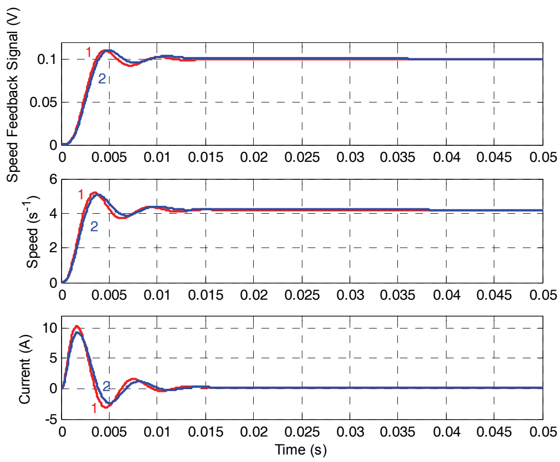

Fig. 7. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current Δi_{as} for a change in reference speed $\Delta\omega^*_r(t) = 0.1\mathrm{S}(t)$ with speed controller parameters determined for $M_{p\omega mr} = 10\%$:
1 – $K_{p\omega} = 60.6$, $T_{i\omega} = 94.1$ ms, $T_f = 1.51$ ms;
2 – $K_{p\omega} = 54.5$, $T_{i\omega} = 23.525$ ms, $T_f = 1.66$ ms.

However, faster compensation of load torque is achieved by determining optimal value of controller gain coefficient based on desired system overshoot for smaller values of controller integral time constant. To achieve smaller value of peak speed drop (loss) $\Delta\omega_{mr}$, due to a change of positive load torque disturbance, it is necessary to determine and set up a higher value of controller gain coefficient. That is possible to achieve by determination of optimal value of controller gain coefficient for drive overshoot calculated for a change of reference value, and then setting it higher than this value. In that case, desired drive overshoot is achieved by adding filter to drive input.

It is necessary to emphasize that all simulation results are obtained for the low value of reference value change $\Delta\omega^*_r(t)$=0.1S(t). Maximum transient value of armature current i_{as} is approximately 10 A (Fig. 3, 5, 7), so that armature current limit is not active. Maximum allowable transient value of armature current is $I_{max}=2I_b=34.7$ A, and it is limited by limitation of speed controller output signal (Fig. 1). Armature current limit will be active for the reference value change $\Delta\omega^*_r(t)>0.34\mathrm{S}(t)$**,** and responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and armature current Δi_{as} on change of reference value will have approximately the same form as in the cases of classic (standard) and optimal speed controller design.

Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and armature current Δi_{as} are obtained by simulation for the change of nominal load torque value $M_t(t) = 0.89\mathrm{S}(t)$ with parameters determined for classic (standard) and optimal speed controller design. Maximum transient value of armature current i_{as} is approximately 25 A (Fig. 4, 6, 8), so that current (speed controller) limit is not active. It follows that it is possible to realize optimal tuning of speed controller parameters for faster and better load torque compensation (Fig. 4).

Authors are planning to experimentally verify described method of transient performance optimization of PM brushless DC motor drive speed controller.

IV. CONCLUSION

Different optimization methods can be applied for determination of controller parameters. Program package *Matlab* uses simplex and gradient methods. Thereby, different integral error criteria and dynamic response quality indices can be used. Optimization of controller parameters based on standard integral error criteria, in relation to ideal system response, results in approximate 20% system overshoot and the value of controller integral time constant is significantly higher than maximum time constant of system, making it unfavorable for load torque compensation.

In this paper a procedure for determination of optimal speed controller parameters of PM brushless DC motor drive based on control quality indices is developed.

Responses of speed feedback signal ω_{mr}, speed ω_m and current i_{as} for step change of speed reference $\omega^*_r(t)$ = 0,1S(t) and step change of load torque T_t = 0,89S(t) for overshoot $M_{p\omega mr}$ = 10% and various values of controller integral time constant are presented in this paper.

Responses to a change of reference speed have the same values of overshoot, while times of response to peak are slightly different. Responses to a change of load torque have approximately the same maximum speed drop (loss) $\Delta\omega_{mr}$, but responses with $T_{i\omega} = 0.125T_t = 11.76$ ms and $T_{i\omega} = 0.25T_t = 23.5$ ms have significantly faster compensation of load torque than response with $T_{i\omega} = T_t = 94.1$ ms.

Presented simulation results show that is possible to realize optimal tuning of speed controller parameters for faster and better load torque compensation

The key contributions of this paper are in:

(i) Determination and graphical presentation of dependence between drive overshoot and controller gain coefficient, as a function of integral time constant of the speed controller,

(ii) Development of a procedure for the determination of controller integral time constant that is smaller than maximum time constant of system ($T_{i\omega} < T_t$), and that provides for faster (4 to 8 times) compensation of load torque's effects on speed,

(iii) Derivation of a procedure for the determination of optimal value of speed controller gain coefficient for a given system overshoot, that is higher than the desired value, and that which provides for better (2 times) load torque compensation on speed, and

(iv) Embedding a procedure for the determination of the filter time constant in drive input that provides desired system overshoot for a change in reference value.

Authors are planning to experimentally verify described method of transient performance optimization of PM brushless DC motor drive speed controller.

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