Absorption and Emission of Radiation by an Atomic Oscillator

Milan Perkovac

Abstract

The theory of absorption and emission of electromagnetic radiation by an oscillator consisting of the atomic nucleus and one electrically charged particle is deduced using classical electrodynamics. In the steady state of an atom, emission and absorption of electromagnetic radiation are equal, so the atom is stable. In order to include reactive effects of electromagnetic radiation in the motion equations, the Newton equation is modified by adding the radiative reaction force. This paper is an introduction to the derivation of the basic assumptions of quantum mechanics.

Key words: absorption, atoms, classical electrodynamics, electromagnetic radiation, emission, oscillators, stability, steady state

1. INTRODUCTION

In 1904 J.J. Thompson (1857–1939) proposed a static model of the atom, and in 1911 E. Rutherford (1871–1937) proposed a dynamic model of the atom. It was hoped that any atomic phenomena could be explained by Newton's mechanics and Maxwell's electrodynamics.

However, two big problems remained unsolved. According to Maxwell's electrodynamics, each accelerated charged particle (such as an electron in Rutherford's model of the atom) inevitably emits electromagnetic radiation and therefore collapses into the nucleus.

The other problem was the atom's discrete spectrum, which was proved experimentally. The theories of Newton and Maxwell did not provide for such discontinuity. Theories that cannot explain experiments are rejected, and rightly so.

Among other scientists, even the founders of modern physics, M. Planck(1,2),1 and A. Einstein,(3) tried to find satisfactory answers within the classical continuity theories, but their attempts were futile.

It was obvious that the classical theories did not contain a principal limitation that would prevent their application down to the level of the atom. One of the last efforts to apply the classical theories to the model of the atom was made by J.H. Jeans in the early 1900s. But all presented arguments could not prevent modern physics from developing in some other direction.

However, the application of classical theories to the model of the atom is gaining ground again.(4–6) In this paper the problem of electromagnetic radiation and the atom's instability is approached in terms of Maxwell's electrodynamics and Newton's and Coulomb's laws. The problem of the atom's discrete spectrum requires two more classical laws, the law of charge and the law of momentum conservation. The problem of the atom's discrete spectrum is elaborated in another paper by the same author.(7)

2. EMISSION OF ELECTROMAGNETIC RADIATION

Emission of electromagnetic radiation results from electromagnetic fields emitted by accelerated electric charges(8) or generally from dynamic electric and magnetic fields (Poynting vector). We view the atom as a system consisting of a nucleus, with *charge* Q, and one particle, with *charge* q and *mass* m, moving in a circular orbit within a *radius* r at an *angular velocity* Ω. This is Rutherford's "planetary" model of the atom(9,10) (see Fig. 1). The distance of a particle from point $x = 0$ is

$$x(\Omega, t) = r\cos(\Omega t + \varphi_{0m}) = \hat{X}\cos(\Omega t + \varphi_{0m}), \qquad (1)$$

and the distance of a particle from point $y = 0$ is

$$y(\Omega, t) = r\sin(\Omega t + \varphi_{0m}) = \hat{Y}\sin(\Omega t + \varphi_{0m}), \qquad (2)$$

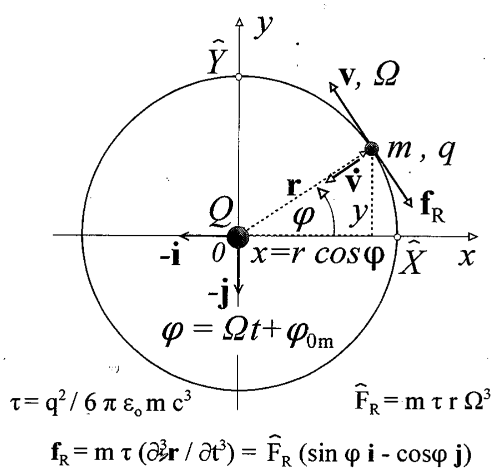

Figure 1. The particle of mass m and charge q rotating in a circle of radius r with velocity v and angular frequency Ω in the field of central charge Q (Rutherford's "planetary" model of the atom). The radiative reaction force f_R is, according to Wheeler and Feynman, 90° out of phase with the acceleration $\dot{\mathbf{v}}$, which means that it is perpendicular to the radius vector r and opposite to the velocity v. This force contributes to the absorption of the radiant energy and does not allow an electron to be more accelerated and thus contributes to the stability of the atom.

where

$$\hat{X} + \hat{Y} = r \tag{3}$$

is the amplitude of forced harmonic motion and φ_{0m} is a phase angle (*which explains that at the moment* $t = -\varphi_{0m}/\Omega$ *prior to the beginning of observation* $t = 0$ *the distance x has reached its maximum value*). The *radius vector* **r** of the electron moving in the plane is

$$\mathbf{r} = r[\cos(\Omega t + \varphi_{0m})\mathbf{i} + \sin(\Omega t + \varphi_{0m})\mathbf{j}]. \tag{4}$$

The total emitted power of electromagnetic radiation, $p_E(t)$, of an electron in the atom is the sum of the momentary power of electromagnetic radiation $p_{Ex}(t)$ and $p_{Ey}(t)$ of two dipoles[11,12] at right angles to each other, along the x and y axes of Cartesian coordinates; i.e.,

$$p_{Ex}(t) = \frac{q^2 r^2 \Omega^4}{6\pi\varepsilon_0 c^3}\cos^2(\Omega t + \varphi_{0m}), \tag{5}$$

$$p_{Ey}(t) = \frac{q^2 r^2 \Omega^4}{6\pi\varepsilon_0 c^3}\sin^2(\Omega t + \varphi_{0m}), \tag{6}$$

$$p_E(t) = p_{Ex}(t) + p_{Ey}(t) = \frac{q^2 r^2 \Omega^4}{6\pi\varepsilon_0 c^3}. \tag{7}$$

The result (7) is known as the Larmor formula for radiated power,[13] an invariant of the Lorentz transformation.[14] The average power of one dipole in the x or y axis, $\overline{P}_{Ex}$ or $\overline{P}_{Ey}$, emitted in one electron rotation cycle, is[15,16]

$$\overline{P}_{Ex} = \overline{P}_{Ey} = \frac{q^2 r^2 \Omega^4}{12\pi\varepsilon_0 c^3}. \tag{8}$$

The total average power $\overline{P}_E$ is the sum of $\overline{P}_{Ex}$ and $\overline{P}_{Ey}$:

$$\overline{P}_E = \overline{P}_{Ex} + \overline{P}_{Ey} = \frac{q^2 r^2 \Omega^4}{6\pi\varepsilon_0 c^3}. \tag{9}$$

We view the atom as an electromechanical oscillatory system. We assume that the atom has at least one stable state. Suppose the x and y components of the *driving force* **f** acting on the electron in an atom oscillate sinusoidally with amplitude $\hat{F}$ at particular frequency ω:

$$f_x(\omega,t) = \hat{F}\cos(\omega t + \varphi_{0f}), \tag{10}$$

$$f_y(\omega,t) = \hat{F}\sin(\omega t + \varphi_{0f}), \tag{11}$$

$$\mathbf{f}(\omega,t) = f_x(\omega,t)\mathbf{i} + f_y(\omega,t)\mathbf{j}, \tag{12}$$

$$|\mathbf{f}(\omega,t)| = \sqrt{f_x^2 + f_y^2} = \hat{F}, \tag{13}$$

where φ_{0f} is a phase angle (*which explains that at the moment* $t = -\varphi_{0f}/\omega$ *prior to the beginning of observation* $t = 0$ *the force* f_x *reached its maximum value*). A correct calculation must include the reaction of the electromagnetic radiation on the motion of the source.[17] So, besides the Coulomb force ($qQ/4\pi\varepsilon_0 r^2$), which is actually the centripetal force (mv^2/r), and the other external forces, there is another force acting on an electron, i.e., the *radiative reaction force*.[18–20] According to J.A. Wheeler and R.A. Feynman, who take up the proposition put long ago (1922) by Tetrode[21] that the act of emission should be somehow associated with the presence of an absorber, this force is

$$\mathbf{f}_R = m\tau \frac{\partial^3 \mathbf{r}}{\partial t^3}, \tag{14}$$

where τ is the *characteristic time*,[(22),2] which will be defined later. Wheeler and Feynman made a rather general derivation of the law of radiative reaction. Consequently, expression (14) is generally accepted as correct for a slowly moving particle subjected to arbitrary acceleration. Hence the total force acting on the electron is the sum of the Coulomb force, other external forces, and the radiative reaction force.

The mechanism of electromagnetic radiation is complex[(23)] and not sufficiently explained.[(24)] In this article we consider only one atom, so no statistical mechanics[(25,26)] can be applied. However, a question arises now of how to include the radiative reaction force in the equation of motion. There are two possibilities.

a) The sum of the radiative reaction force $\mathbf{f}_R$ and the external force $\mathbf{f}_{ext}$ is a single driving force $\mathbf{f} = \mathbf{f}_R + \mathbf{f}_{ext}$, and the equation of motion is

$$m\dot{\mathbf{v}} = \mathbf{f}_R + \mathbf{f}_{ext}. \tag{14a}$$

b) The driving force is only the external force $\mathbf{f}_{ext}$, and the equation of motion is the *Abraham–Lorentz equation of motion*

$$m(\dot{\mathbf{v}} - \tau\ddot{\mathbf{v}}) = \mathbf{f}_{ext}. \tag{14b}$$

As is well known, the Abraham–Lorentz equation generated so-called runaway solutions,[(27)] so we opt for the first possibility. So the equation of motion based on Newton's second law of motion, which includes the resistive force and the restoring force,[(28–31)] is

$$m\frac{\partial^2 x}{\partial t^2} + b\frac{\partial x}{\partial t} + kx = \hat{F}\cos(\omega t + \varphi_{0f}), \tag{15}$$

or, after dividing by m and rearranging,

$$\frac{\partial^2 x}{\partial t^2} + \Gamma\frac{\partial x}{\partial t} + \Omega_0^2 x = \frac{\hat{F}}{m}\cos(\omega t + \varphi_{0f}), \tag{16}$$

where b is the *damping constant*,[(32)] k is the *spring constant*,[(33)]

$$\Gamma = \frac{b}{m} \tag{17}$$

is the *decay constant*,[(34)] also called the *half-width* or *line breadth*,[(35)] and

$$\Omega_0 = \sqrt{\frac{k}{m}} \tag{18}$$

is the angular frequency at which the simple harmonic oscillator oscillates, also called its *natural frequency*[(36)] (to distinguish it from the angular frequency ω at which it might be forced to oscillate in steady state by a driving force $\mathbf{f}$).

Using (1) and (2), we get

$$\frac{\partial x}{\partial t} = -\hat{X}\Omega\sin(\Omega t + \varphi_{0m}), \quad \frac{\partial y}{\partial t} = \hat{Y}\Omega\cos(\Omega t + \varphi_{0m}), \tag{19}$$

$$\begin{aligned} \frac{\partial^2 x}{\partial t^2} &= -\hat{X}\Omega^2\cos(\Omega t + \varphi_{0m}), \\ \frac{\partial^2 y}{\partial t^2} &= -\hat{Y}\Omega^2\sin(\Omega t + \varphi_{0m}), \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{\partial^3 x}{\partial t^3} &= \hat{X}\Omega^3\sin(\Omega t + \varphi_{0m}), \\ \frac{\partial^3 y}{\partial t^3} &= -\hat{Y}\Omega^3\cos(\Omega t + \varphi_{0m}). \end{aligned} \tag{21}$$

Using (3), (4), (14), and (21), we get

$$\mathbf{f}_R = m\tau r\Omega^3[\sin(\Omega t + \varphi_{0m})\mathbf{i} - \cos(\Omega t + \varphi_{0m})\mathbf{j}], \tag{22}$$

where the amplitude of the radiative reaction force is

$$\hat{F}_R = m\tau r\Omega^3. \tag{23}$$

The solution of differential equation (15) in the steady state is (1). So, from (1), (16), (19), and (20), we get

$$\begin{aligned} &\hat{X}[(\Omega^2 - \Omega_0^2)\sin\varphi_{0m} - \Gamma\Omega\cos\varphi_{0m}]\sin\Omega t \\ &-\hat{X}[(\Omega^2 - \Omega_0^2)\cos\varphi_{0m} + \Gamma\Omega\sin\varphi_{0m}]\cos\Omega t \\ &+\left(\frac{\hat{F}}{m}\sin\varphi_{0f}\right)\sin\omega t - \left(\frac{\hat{F}}{m}\cos\varphi_{0f}\right)\cos\omega t = 0. \end{aligned} \tag{24}$$

The electron oscillates in the steady state of the atom at a particular frequency ω of the driving force $\mathbf{f}$, even if this frequency is different from the natural frequency Ω_0 of the undamped oscillations.[37] So, in the case of steady state,

$$\Omega = \omega, \tag{25}$$

and (24) becomes

$$\begin{aligned} &\left(\hat{X}[(\omega^2 - \Omega_0^2)\sin\varphi_{0m} - \Gamma\omega\cos\varphi_{0m}] \right. \\ &\qquad \left. + \frac{\hat{F}}{m}\sin\varphi_{0f} \right)\sin\omega t \\ &- \left(\hat{X}[(\omega^2 - \Omega_0^2)\cos\varphi_{0m} + \Gamma\omega\sin\varphi_{0m}] \right. \\ &\qquad \left. + \frac{\hat{F}}{m}\cos\varphi_{0f} \right)\cos\omega t = 0. \end{aligned} \tag{26}$$

Equation (26) is valid for every value of t only when the coefficients of the linearly independent time-functions $\sin \omega t$ and $\cos \omega t$ each equal zero. So, for *amplitude* $\hat{X}$, we get

$$\hat{X} = \frac{\hat{F}}{m\sqrt{(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2}}, \tag{27}$$

and, for a phase $(\varphi_{0m} - \varphi_{0f})$,[38,39]

$$\tan(\varphi_{0m} - \varphi_{0f}) = \frac{\Gamma\omega}{\omega^2 - \Omega_0^2}, \tag{28}$$

where $(\varphi_{0m} - \varphi_{0f})$ is the phase angle between the driving force $f_x(\omega, t)$ and the distance $x(\omega, t)$ [or the phase angle between the driving force $f_y(\omega, t)$ and the distance $y(\omega, t)$], i.e., the phase angle between the driving force **f** and the radius vector **r**.

If we substitute r in (8) for $\hat{X}$ from (27), then the average power (8) of one dipole in an x or y axis, $\overline{P}_{Ex}$ or $\overline{P}_{Ey}$, emitted in one cycle of electron rotation in steady state ($\Omega = \omega$), is (see Figs. 2 and 3)

$$\overline{P}_{Ex} = \frac{q^2}{12\pi\varepsilon_0 c^3 m^2}\frac{\omega^4\hat{F}^2}{(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2}, \tag{29}$$

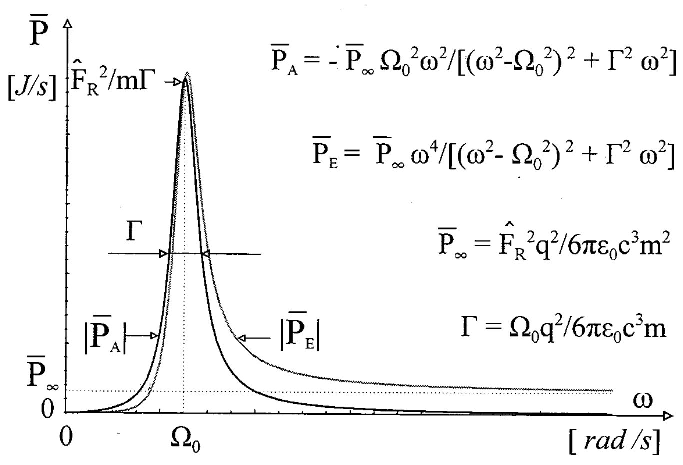

Figure 2. The total average power $\overline{P}_E$ emitted in one cycle of particle rotation and the total average power $\overline{P}_A$ absorbed in one cycle of particle rotation, versus the frequency of external force ω; the natural frequency of a particle is Ω_0 and the width of an oscillator is Γ.

$$\overline{P}_{Ey} = \frac{q^2}{12\pi\varepsilon_0 c^3 m^2}\frac{\omega^4\hat{F}^2}{(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2}, \tag{30}$$

and the total average emitted power is

$$\begin{aligned} \overline{P}_E &= \overline{P}_{Ex} + \overline{P}_{Ey} \\ &= \frac{q^2}{6\pi\varepsilon_0 m^2 c^3}\frac{\omega^4\hat{F}^2}{(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2} \\ &= \overline{P}_{E\infty}\frac{\omega^4}{(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2}, \end{aligned} \tag{31}$$

where

$$\overline{P}_{E\infty} = \frac{\hat{F}^2 q^2}{6\pi\varepsilon_0 m^2 c^3} \tag{32}$$

is the average power emitted in one cycle of electron rotation if the frequency ω of the external force is approaching ∞, i.e., $\omega \to \infty$.

3. ABSORPTION OF ELECTROMAGNETIC RADIATION

Absorption of electromagnetic radiation[40] of one dipole in the x axis results from the work of $f_x dx = f_x v_x dt = f_x \partial x/\partial t dt = p_x dt$ done on the charge q by the

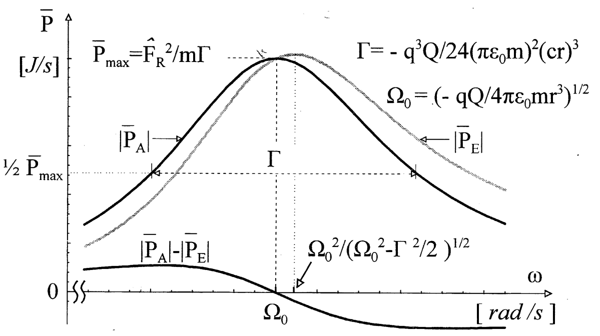

Figure 3. Detail of Fig. 2.

driving force f_x:

$$\int_{t_1}^{t_2} p_x dt = \int_{t_1}^{t_2} f_x \frac{\partial x}{\partial t} dt = \int_{t_1}^{t_2} [\hat{F}\cos(\omega t + \varphi_{0f})][-\hat{X}\Omega\sin(\Omega t + \varphi_{0m})]dt. \qquad (33)$$

So, in the same system of an atom as the one mentioned above, the average power $\overline{P}_{Ax}$ absorbed in one cycle of driving force f_x with frequency ω [because of (25) we can also take the integral to $t_2 = 2\pi/\Omega$ instead of $t_2 = 2\pi/\omega$], using (27) and (33), in steady state ($\Omega = \omega$), is

$$\overline{P}_{Ax} = -\frac{\omega}{2\pi}\int_{t_1=0}^{t_2=2\pi/\omega} \frac{\hat{F}^2\omega\cos(\omega t+\varphi_{0f})\sin(\omega t+\varphi_{0m})}{m\sqrt{(\omega^2-\Omega_0^2)^2+\Gamma^2\omega^2}}dt = -\frac{\hat{F}^2\omega^2}{2\pi m\sqrt{(\omega^2-\Omega_0^2)^2+\Gamma^2\omega^2}} \times \int_{t_1=0}^{t_2=2\pi/\omega}\cos(\omega t+\varphi_{0f})\sin(\omega t+\varphi_{0m})dt. \qquad (34)$$

The solution[3] of this equation is

$$\overline{P}_{Ax} = -\frac{\hat{F}^2\omega}{2m\sqrt{(\omega^2-\Omega_0^2)^2+\Gamma^2\omega^2}}\sin(\varphi_{0m}-\varphi_{0f}). \qquad (35)$$

Using (28) and $\sin(\varphi_{0m} - \varphi_{0f}) = \tan(\varphi_{0m} - \varphi_{0f})/[1 + \tan^2(\varphi_{0m} - \varphi_{0f})]^{1/2}$, we get

$$\overline{P}_{Ax} = -\frac{\hat{F}^2}{2m}\frac{\Gamma\omega^2}{(\omega^2-\Omega_0^2)^2+\Gamma^2\omega^2}. \qquad (36)$$

Also, the average power $\overline{P}_{Ay}$ absorbed in one cycle of the driving force f_y is like $\overline{P}_{Ax}$ in (36):

$$\overline{P}_{Ay} = -\frac{\hat{F}^2}{2m}\frac{\Gamma\omega^2}{(\omega^2-\Omega_0^2)^2+\Gamma^2\omega^2}. \qquad (37)$$

The total average absorbed power in one cycle of electron rotation is the sum of $\overline{P}_{Ax}$ and $\overline{P}_{Ay}$:[41]

$$\overline{P}_A = \overline{P}_{Ax} + \overline{P}_{Ay} = -\frac{\hat{F}^2}{m}\frac{\Gamma\omega^2}{(\omega^2-\Omega_0^2)^2+\Gamma^2\omega^2}. \qquad (38)$$

The sum of the total emitted and absorbed average powers, $\overline{P}$ (overall electromagnetic spectrum, if not only ω but other components of frequency in the Fourier series of driving forces are present), in the steady state ($\omega = \Omega$), is zero (see Fig. 3):

$$\overline{P} = \overline{P}_A + \overline{P}_E = 0 \qquad (39)$$

(by the steady state, i.e., by $\omega = \Omega$). By using (31), (38), and (39), we get

$$\Gamma = \frac{\Omega^2 q^2}{6\pi\varepsilon_0 m c^3}, \qquad (40)$$

and, by using (32) and (40), (38) is

$$\overline{P}_A = -\overline{P}_{E\infty}\frac{\Omega^2\omega^2}{(\omega^2-\Omega_0^2)^2+\Gamma^2\omega^2}. \qquad (41)$$

4. PARAMETERS OF AN ATOMIC OSCILLATOR

We observe an electron on one stationary circular orbit. The centrifugal and centripetal forces of the electron in this orbit are in equilibrium. If there is any small disturbance in such a system, the electron becomes exposed to additional radial oscillations with a small amplitude near the stationary circular orbit, i.e., near the equilibrium position. Such an amplitude is smaller than the radius of an electron orbit. So there is one electromechanical oscillator with a restoring force and parameters such as the spring constant (force constant), natural frequency, damping constant, half-width, and characteristic time. Although we do not know the size of the electron, we will determine all these parameters. In this article it is not necessary to know the size of the electron nor the size of the nucleus. We assume that the size of the electron and the size of the nucleus are much smaller than the distance between them.

(a) Spring constant and natural frequency

The next three relations (all resulting from the same equation $v = \Omega r$) are valid for the uniform circular motions of an electron on the radius r with angular frequency Ω and linear velocity v:

$$r(\Omega, v) = \frac{v}{\Omega}, \tag{42}$$

$$\Omega(v, r) = \frac{v}{r}, \tag{43}$$

$$v(\Omega, r) = \Omega r. \tag{44}$$

We assume that f_Δ is the sum of all radial forces acting on the electron near the equilibrium position. The equilibrium position is on the circle of radius r. If the electron is moved either to one side or to the other side away from the equilibrium position, the force f_Δ returns it to the equilibrium position. This force is called the *restoring force*.[42] The small magnitude of the restoring force df_Δ is found to be directly proportional to the distance dr (dr being the dislocation of the electron from the equilibrium position on the radius r):

$$df_\Delta = -k dr. \tag{45}$$

We assume that in a near-equilibrium position three radial forces are acting on the electron:

$$F_d = \frac{mv^2}{r} = \text{centripetal (radial) force}, \tag{46}$$

$$F_C = \frac{qQ}{4\pi\varepsilon_0 r^2} = \text{Coulomb's law (force)}, \tag{47}$$

and the restoring force f_Δ. Thus we can write

$$f_\Delta + \frac{mv^2}{r} + \frac{qQ}{4\pi\varepsilon_0 r^2} = 0, \tag{48}$$

or, using (44),

$$f_\Delta + m\Omega^2 r + \frac{qQ}{4\pi\varepsilon_0 r^2} = 0. \tag{49}$$

The differential of the force $f_\Delta(\Omega, r)$ is

$$d[f_\Delta(\Omega, r)] = \left(\frac{\partial f_\Delta}{\partial r}\right) dr + \left(\frac{\partial f_\Delta}{\partial \Omega}\right) d\Omega, \tag{50}$$

i.e., using (49)

$$d[f_\Delta(\Omega, r)] = \left(-m\Omega^2 + \frac{qQ}{2\pi\varepsilon_0 r^3}\right) dr + \left(-2m\Omega r\right) d\Omega. \tag{51}$$

The differential $d\Omega$ of angular frequency Ω according to (43) is

$$d\Omega(v, r) = \left(\frac{\partial \Omega}{\partial v}\right) dv + \left(\frac{\partial \Omega}{\partial r}\right) dr = \frac{1}{r} dv - \frac{v}{r^2} dr. \tag{52}$$

The absolute value of the linear velocity v in steady state is constant, so

$$dv = 0. \tag{53}$$

From (51), by using (43), (52), and (53), we get

$$d[f_\Delta(\Omega, r)] = \left(-m\Omega^2 + \frac{qQ}{2\pi\varepsilon_0 r^3} + 2m\Omega r \frac{v}{r^2}\right) dr, \tag{54}$$

i.e.,

$$d[f_\Delta(\Omega, r)] = \left(m\Omega^2 + \frac{qQ}{2\pi\varepsilon_0 r^3}\right) dr. \tag{55}$$

In compliance with (45), the restoring force is $df_\Delta = -kdr$ and, if we set this equal to (55), we get the spring constant k:

$$k = -m\Omega^2 - \frac{qQ}{2\pi\varepsilon_0 r^3}. \tag{56}$$

So the natural frequency, (18), of an electron moving in circular atomic orbits is

$$\Omega_0 = \sqrt{\frac{k}{m}} = \sqrt{-\Omega^2 - \frac{qQ}{2\pi\varepsilon_0 mr^3}}. \quad (57)$$

In the equilibrium state $f_\Delta = 0$ and, according to (48), we have

$$\frac{mv^2}{r} = -\frac{qQ}{4\pi\varepsilon_0 r^2}. \quad (58)$$

Because of (43) (i.e., $v/r = \Omega$), from (58) it follows that

$$\Omega^2 = -\frac{qQ}{4\pi\varepsilon_0 mr^3}. \quad (59)$$

So, from (57) and using (59), we get

$$\Omega_0 = \sqrt{-\Omega^2 - \frac{qQ}{2\pi\varepsilon_0 mr^3}} = \sqrt{-\frac{qQ}{4\pi\varepsilon_0 mr^3}} = \Omega. \quad (60)$$

(b) A half-width and damping constant

Relation (60) means that absorption and emission are equal in case of the condition $\Omega = \Omega_0$. Any circular motion satisfies this condition. So, according to (40), Γ is also

$$\Gamma = \frac{\Omega_0^2 q^2}{6\pi\varepsilon_0 mc^3}, \quad (61)$$

and, according to (41), $\overline{P}_A$ is also

$$\overline{P}_A = -\overline{P}_{E\infty} \frac{\Omega_0^2 \omega^2}{(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2}. \quad (62)$$

Finally, in the equilibrium state ($\Omega = \Omega_0$), from (56) and (60), we get

$$k = -\frac{qQ}{4\pi\varepsilon_0 r^3}, \quad (63)$$

or the *force constant*[43] is also

$$k = m\Omega^2 = m\Omega_0^2. \quad (64)$$

In the steady state ($\omega = \Omega = \Omega_0$, $\omega \neq 0$, $\Gamma \neq 0$), according to (28), $\tan(\varphi_{0m} - \varphi_{0f}) = \infty$, i.e.,

$$\varphi_{0m} - \varphi_{0f} = \frac{\pi}{2}. \quad (65)$$

The radius vector **r** and the driving force **f** are at right angles to each other (see Fig. 1). Using (10), (11), and (12), and by $\omega = \Omega$ and $\varphi_{0f} = \varphi_{0m} - \pi/2$, the driving force **f** is

$$\mathbf{f} = \hat{\mathrm{F}}[\sin(\Omega t + \varphi_{0m})\mathbf{i} - \cos(\Omega t + \varphi_{0m})\mathbf{j}]. \quad (66)$$

A comparison of the radiative reaction force (22) and the driving force (66) shows that the driving force **f** and the radiative reaction force $\mathbf{f}_R$ in the steady state are two parallel forces. If there are no other external forces, the radiative reaction force is the only acting force.[44] So we get

$$\hat{F} = \hat{F}_R = m\tau r\Omega^3. \quad (67)$$

Using (3), (27), and $\omega = \Omega = \Omega_0$, we get[45]

$$\Gamma = \tau\Omega^2, \quad (68)$$

and, using (40) and (68),

$$\tau = \frac{q^2}{6\pi\varepsilon_0 mc^3}. \quad (69)$$

According to (17) and (68),

$$b = m\tau\Omega^2. \quad (70)$$

Using (60), we can show (61) in the steady state as

$$\Gamma = -\left(\frac{Q}{6}\right)\left(\frac{1}{2\pi\varepsilon_0 m}\right)^2\left(\frac{q}{cr}\right)^3. \quad (71)$$

(c) Oscillator energy and frequency

The total energy of a harmonic oscillator is[46,47]

$$E = \frac{1}{2}m\Omega^2\hat{X}^2 = \frac{1}{2}m\Omega^2\hat{Y}^2 = \frac{1}{2}kr^2, \quad (72)$$

and, using (58) and (63), we get

$$E = \frac{1}{2}mv^2 = -\frac{qQ}{8\pi\varepsilon_0 r}. \tag{73}$$

Now we have all the parameters of the atom as an electromechanical oscillator. We can now discuss other interesting relations.

We can quite freely select any of the states in an atom as the *state of reference*. All the variables in that state will be written with an underlined symbol. The period of one cycle of an electron is

$$T = \frac{2\pi}{\Omega} = \frac{1}{\nu}. \tag{74}$$

According to (60) and (74),

$$\frac{\nu}{\underline{\nu}} = \sqrt{\frac{1}{(r/\underline{r})^3}}. \tag{75}$$

The product of the period T and energy E of an oscillator we call the *mechanical action* and denote as $\mathring{a} = ET$. According to (60), (72), (73), and (74), we get

$$\mathring{a} = ET = \pi r^2 m\Omega = \sqrt{\frac{-qQ\pi m r}{4\varepsilon_0}} = \pi r m v. \tag{76}$$

If we divide (76) by mv, we get $\pi r = \mathring{a}/mv$, the *action/momentum of the particle*. It resembles *de Broglie's basic postulate for the matter wave,*[48] $\lambda = h/mv$, where h is Planck's constant. Since an electron moves in circles, according to de Broglie's hypothesis, the electron wave must be a circular standing wave with $2\pi r = n\lambda$, and $\mathring{a}$ should be $nh/2$. But for our consideration here we do not need the matter wave or Planck's constant.

For the state of reference we have, from (76),

$$\underline{\mathring{a}} = \underline{ET} = \sqrt{\frac{-qQ\pi m\underline{r}}{4\varepsilon_0}} = \pi \underline{r} m \underline{v}. \tag{77}$$

If we select the state of reference in one atom, then $\underline{\mathring{a}}$ is the *referent mechanical action* for that atom. It is the value of the fixed amount; i.e., it is a constant for that atom.

We denote the *quotient of two mechanical actions* as $\kappa_{\mathring{a}}$ and, according to (76) and (77), it is

$$\kappa_{\mathring{a}} = \frac{\mathring{a}}{\underline{\mathring{a}}} = \frac{ET}{\underline{ET}} = \sqrt{\frac{r}{\underline{r}}} \tag{78}$$

or

$$\frac{r}{\underline{r}} = \kappa_{\mathring{a}}^2, \tag{79}$$

where $\kappa_{\mathring{a}}$ is any *positive real number*. Obviously in any state of reference $\underline{\kappa_{\mathring{a}}} = (\underline{r}/\underline{r})^{1/2} = 1$. This means that $\underline{\kappa_{\mathring{a}}}$ is a natural number and always equals one. From (75) and (79) we get

$$\nu = \frac{\underline{\nu}}{\kappa_{\mathring{a}}^3}, \tag{80}$$

and, from (78),

$$ET = \mathring{a} = \kappa_{\mathring{a}}\underline{\mathring{a}}, \tag{81}$$

and (76) becomes

$$\pi r m v = \kappa_{\mathring{a}}\underline{\mathring{a}}. \tag{82}$$

Expression (82) resembles *Bohr's quantum condition*[49] ($2\pi rmv = nh$). $\kappa_{\mathring{a}}$ is a real number that can also be any natural number n. The referent mechanical action $\underline{\mathring{a}}$ is a constant, as h is in Bohr's condition. But we cannot affirm now that $\kappa_{\mathring{a}}$ is a natural number. This affirmation is confirmed in Ref. 7, where electromagnetic properties of the atom are included.

Equations (74), (80), and (81) give

$$E = \kappa_{\mathring{a}}\underline{\mathring{a}}\nu = \frac{\underline{\mathring{a}}\,\underline{\nu}}{\kappa_{\mathring{a}}^2}. \tag{83}$$

The relation (83), $E = \kappa_{\mathring{a}}\underline{\mathring{a}}\nu$, resembles *Planck's quantum hypothesis*[50] ($E = nh\nu$), which is very important for quantum physics. There was a lot of discussion on how to explain in terms of physics the meaning of some quantities in the relation $E = nh\nu$. However, in our relation (83) all of the quantities are completely clear in terms of physics. The basic physical difference between Planck's relation $E = nh\nu$ and (83) is the meaning of frequency ν. According to Planck's relation $E = nh\nu$, the frequency ν means the frequency of an electromagnetic wave ν_{em}. In relation (83), $E = \kappa_{\mathring{a}}\underline{\mathring{a}}\nu$, the frequency ν means the frequency of mechanical rotation of an electron in the atomic orbit.

At the state of reference (where $\kappa_{\mathring{a}} = 1$), according to (83), we have

$$\underline{E} = \underline{\mathring{a}}\,\underline{\nu}. \tag{84}$$

Notwithstanding the similarity between (84) and Einstein's photon equation[51] ($E = h\nu_{em}$), we note that $\underline{\nu}$ is not the frequency of electromagnetic oscillations in a vacuum ν_{em} (the frequency of light) but the frequency of mechanical rotation of an electron in the atom's state of reference, and this is a significant difference. The physical connection between these two frequencies demands a detailed electromagnetic analysis, which is made in Ref. 7.

E^3/ν^2, according to (60), (63), (72), and (74), can be shown as

$$\frac{E^3}{\nu^2} = \frac{m}{2}\left(\frac{qQ}{4\varepsilon_0}\right)^2. \tag{85}$$

For a definite system (where q and Q are determinate and fixed) E^3/ν^2 is constant. We can show (85) as

$$E = \sqrt[3]{\frac{m}{2}\left(\frac{qQ}{4\varepsilon_0}\nu\right)^2}. \tag{86}$$

Setting $E = \kappa_{\mathring{a}}\underline{\mathring{a}}\nu$ from (83) equal to (86), we get

$$\nu = \frac{m}{2\kappa_{\mathring{a}}^3\underline{\mathring{a}}^3}\left(\frac{qQ}{4\varepsilon_0}\right)^2 = \frac{mq^2Q^2}{32\varepsilon_0^2\kappa_{\mathring{a}}^3\underline{\mathring{a}}^3}. \tag{87}$$

If we put (87) in (86), we get

$$E = \frac{mq^2Q^2}{32\varepsilon_0^2\kappa_{\mathring{a}}^2\underline{\mathring{a}}^2}. \tag{88}$$

The greatest ionization potential of hydrogen[52,53] (^1_1H) is V_i = 13.5978 V. We select that state as the state of reference of the hydrogen atom. The total energy of a harmonic oscillator in that state is $\underline{E} = eV_i$ = 2.1786 × 10^{-18} J. So, according to (88), we can now calculate the mechanical action $\underline{\mathring{a}}$ in the state of reference ($\kappa_{\mathring{a}} = 1$, $|q| = |Q| = e$):

$$\underline{\mathring{a}} = \frac{|qQ|}{4\varepsilon_0}\sqrt{\frac{m}{2\underline{E}}} = 3.3139\times10^{-34}\ \text{J}\cdot\text{s}. \tag{89}$$

The greatest ionization potentials of $^4_2He^{+}$, $^7_3Li^{++}$, $^9_4Be^{+++}$, and $^{11}_5B^{++++}$ are 54.41 V, 122.414 V, 217.605 V, and 339.965 V, respectively. If we select these states as the states of reference and calculate the mechanical actions according to (89), we get the same result for all of the elements: $\underline{\mathring{a}}$ = 3.3139 × 10^{-34} J · s (see Table I). It seems that the mechanical action in such states of reference is constant for all elements. Still it does not mean that $\underline{\mathring{a}}$ is a fundamental constant, because $\underline{\mathring{a}}$ depends on our free choice. In different states of reference $\underline{\mathring{a}}$ has a different value.

Setting $E = mv^2/2$ and $E = -qQ/8\pi\varepsilon_0 r$, both expressed from (73), separately equal to (88), we get

$$v = \frac{-qQ}{4\varepsilon_0\kappa_{\mathring{a}}\underline{\mathring{a}}} = \frac{\underline{v}}{\kappa_{\mathring{a}}} \tag{90}$$

and

$$r = -\frac{4\varepsilon_0\kappa_{\mathring{a}}^2\underline{\mathring{a}}^2}{\pi mqQ} = \kappa_{\mathring{a}}^2\underline{r}. \tag{91}$$

Setting (83) equal to (88) and using (84), we get

$$E = \underline{\mathring{a}}\sqrt[3]{\underline{\nu}\nu^2}. \tag{92}$$

Using (87) and (88) for the frequency difference and the energy difference of any two stationary states characterized by $\kappa_{\mathring{a}}$ and $\kappa_{\mathring{a}}'$ ($\kappa_{\mathring{a}}' > \kappa_{\mathring{a}}$), we get

$$\Delta\nu = \nu - \nu' = \frac{mq^2Q^2}{32\varepsilon_0^2\underline{\mathring{a}}^3}\left(\frac{1}{\kappa_{\mathring{a}}^3} - \frac{1}{\kappa_{\mathring{a}}'^3}\right) = \underline{\nu}\left(\frac{1}{\kappa_{\mathring{a}}^3} - \frac{1}{\kappa_{\mathring{a}}'^3}\right) \tag{93}$$

and

$$\Delta E = E' - E = \frac{mq^2Q^2}{32\varepsilon_0^2\underline{\mathring{a}}^2}\left(\frac{1}{\kappa_{\mathring{a}}^2} - \frac{1}{\kappa_{\mathring{a}}'^2}\right) = \underline{E}\left(\frac{1}{\kappa_{\mathring{a}}^2} - \frac{1}{\kappa_{\mathring{a}}'^2}\right). \tag{94}$$

Equations (93) and (94) remind us of the well-known Bohr expression of the atomic radiant frequency[54] and the radiant energy.[55] But there is a fundamental difference between the frequencies ν of the circular motion of the electron and the frequency of radiated electromagnetic energy ν_{em} by Bohr. The explanation of this difference is the subject of another article.[7]

Table I: The Quantity of Oscillators Based on the Hydrogen Atom

Symbol	$V_{i(H)} = 13.5978V^*$ *(Ref. 53) (experimental)*	$^{1}_{1}$H $\kappa_{\mathring{a}} = 1$	$^{1}_{1}$H $\kappa_{\mathring{a}} = 2$	$^{1}_{1}$H $\kappa_{\mathring{a}} = 3$	$^{1}_{1}$H $\kappa_{\mathring{a}} = 4$	$^{4}_{2}He^{+}$ $\kappa_{\mathring{a}} = 1$	$^{7}_{3}Li^{++}$ $\kappa_{\mathring{a}} = 1$	$^{9}_{4}Be^{+++}$ $\kappa_{\mathring{a}} = 1$	$^{1}_{5}Be^{++++}$ $\kappa_{\mathring{a}} = 1$	*Unit*
Z	*Atomic number*	1	1	1	1	2	3	4	5	1
V_i	*Ionization potential* (calculated, ^{1}H experimental)	13.5978*	3.39948	1.510885	0.849873	54.41	122.414	217.605	339.965	V
r	$r = \lvert qQ\rvert/8\pi\varepsilon_0\lvert qV_i\rvert$;[4] $q = -e$; $Q = Ze$	5.29e–11	2.12e–10	4.77e–10	8.47e–10	2.65e–11	1.76e–11	1.32e–11	1.06e–11	m
τ	$\tau = q^2/6\pi\varepsilon_0 mc^3$	6.27e–24	6.27e–24	6.27e–24	6.27e–24	6.27e–24	6.27e–24	6.27e–24	6.27e–24	s/rad
Ω	$\Omega = \Omega_0 = (-qQ/4\pi\varepsilon_0 mr^3)^{1/2}$	4.13e+16	5.16e+15	1.53e+15	6.45e+14	1.65e+17	3.72e+17	6.61e+17	1.03e+18	rad/s
T	$T = 2\pi/\Omega$	1.52e–16	1.22e–15	4.11e–15	9.74e–15	3.80e–17	1.69e–17	9.50e–18	6.06e–18	s
ν	$\nu = 1/T$	6.57e+15	8.22e+14	2.43e+14	1.03e+14	2.63e+16	5.92e+16	1.05e+17	1.64e+17	Hz
Γ	$\Gamma = \tau\Omega^2$	1.07e+10	1.67e+08	1.47e+07	2.61e+06	1.71e+11	8.67e+11	2.74e+12	6.69e+12	rad/s
b	$b = m\tau\Omega^2$	9.74e–21	1.52e–22	1.34e–23	2.38e–24	1.56e–19	7.90e–19	2.50e–18	6.09e–18	kg/s
k	$k = -qQ/4\pi\varepsilon_0 r^3 = m\Omega^2$	1.55e+03	2.43e+01	2.13e+00	3.79e–01	2.49e+04	1.26e+05	3.98e+05	9.73e+05	kg/s^2
E	$E = m\Omega^2 r^2/2 = kr^2/2 = mv^2/2$	2.18e–18	5.45e–19	2.42e–19	1.36e–19	8.72e–18	1.96e–17	3.49e–17	5.45e–17	J
$\hat{F}_R$	$\hat{F}_R = m\tau\Omega^3 r$	2.13e–14	1.66e–16	9.74e–18	1.30e–18	6.83e–13	5.18e–12	2.18e–11	6.67e–11	N
F_C	$F_C = \lvert qQ\rvert/4\pi\varepsilon_0 r^2$	8.23e–08	5.14e–09	1.02e–09	3.21e–10	1.32e–06	6.67e–06	2.11e–05	5.15e–05	N
$\overline{P}_{E\infty}$	$\overline{P}_{E\infty} = \hat{F}_R^2\, q^2/6\pi\varepsilon_0 c^3 m^2$	3.12e–21	1.91e–25	6.53e–28	1.16e–29	3.20e–18	1.85e–16	3.28e–15	3.06e–14	W
$\overline{P}_E$, $\overline{P}_A$	$\overline{P}_E = -\overline{P}_A = \hat{F}_R^2/m\Gamma$	4.66e–08	1.82e–10	7.10e–12	7.11e–13	2.99e–06	3.40e–05	1.91e–04	7.29e–04	W
Γ	$\Gamma = \overline{P}_E/2E$	1.07e+10	1.67e+08	1.47e+07	2.61e+06	1.71e+11	8.67e+11	2.74e+12	6.69e+12	rad/s
p	$p = mv = \hat{F}_R/\Gamma$	1.99e–24	9.96e–25	6.64e–25	4.98e–25	3.99e–24	5.98e–24	7.97e–24	9.96e–24	kg · m/s
$\mathring{a}$	$\mathring{a} = ET = r^2 m\Omega/2 = (-qQ\pi mr/4\varepsilon_0)^{1/2}$	3.31e–34	6.63e–34	9.94e–34	1.33e–33	3.31e–34	3.31e–34	3.31e–34	3.31e–34	J · s
$\underline{\mathring{a}}$	$\underline{\mathring{a}} = \underline{E}\,\underline{T} = (-qQ\pi m\underline{r}/4\varepsilon_0)^{1/2}$	3.31e–34	3.31e–34	3.31e–34	3.31e–34	3.31e–34	3.31e–34	3.31e–34	3.31e–34	J · s
$\kappa_{\mathring{a}}$	$\kappa_{\mathring{a}} = ET/\underline{E}\,\underline{T}$	1.00000	2.00000	3.00000	4.00000	1.00000	1.00000	1.00000	1.00000	1
$\mathring{a}$	$\mathring{a} = \kappa_{\mathring{a}}\underline{E}\,\underline{T} = \pi rmv = \kappa_{\mathring{a}}\underline{\mathring{a}}$	3.31e–34	6.63e–34	9.94e–34	1.33e–33	3.31e–34	3.31e–34	3.31e–34	3.31e–34	J · s
E	$E = \underline{\mathring{a}}(\underline{\nu}\nu^2)^{1/3} = \kappa_{\mathring{a}}\underline{\mathring{a}}\nu = \underline{\mathring{a}}\,\underline{\nu}/\kappa_{\mathring{a}}^2 = mq^2Q^2/(32\varepsilon_0^2\kappa_{\mathring{a}}^2\underline{\mathring{a}}^2)$	2.18e–18	5.45e–19	2.42e–19	1.36e–19	8.72e–18	1.96e–17	3.49e–17	5.45e–17	J
E^3/ν^2	$E^3/\nu^2 = m(qQ/4\varepsilon_0)^2/2$	2.39e–85	2.39e–85	2.39e–85	2.39e–85	9.57e–85	2.15e–84	3.83e–84	5.98e–84	J^3 · s^2
ν	$\nu = mq^2Q^2/(32\varepsilon_0^2\kappa_{\mathring{a}}^3\underline{\mathring{a}}^3) = \underline{\nu}/\kappa_{\mathring{a}}^3$	6.57e+15	8.22e+14	2.43e+14	1.03e+14	2.63e+16	5.92e+16	1.05e+17	1.64e+17	1/s
v	$v = -qQ/(4\varepsilon_0\kappa_{\mathring{a}}\underline{\mathring{a}}) = \underline{v}/\kappa_{\mathring{a}}$	2.19e+06	1.09e+06	7.29e+05	5.47e+05	4.38e+06	6.56e+06	8.75e+06	1.09e+07	m/s
r	$r = -4\varepsilon_0\kappa_{\mathring{a}}^2\underline{\mathring{a}}^2/(\pi mqQ) = \kappa_{\mathring{a}}^2\underline{r}$	5.29e–11	2.12e–10	4.77e–10	8.47e–10	2.65e–11	1.76e–11	1.32e–11	1.06e–11	m

The experimental value of the ionization potential calculated from spectroscopic data of hydrogen, $^{1}_{1}H_{(\kappa_{\mathring{a}}=1)}$, is[53] V_i = 13.5978 V. The ionization potentials of $^{1}_{1}H_{(\kappa_{\mathring{a}}=2)}$, $^{1}_{1}H_{(\kappa_{\mathring{a}}=3)}$, $^{1}_{1}H_{(\kappa_{\mathring{a}}=4)}$, $^{4}_{2}He^{+}_{(\kappa_{\mathring{a}}=1)}$, $^{7}_{3}Li^{++}_{(\kappa_{\mathring{a}}=1)}$, $^{9}_{4}Be^{+++}_{(\kappa_{\mathring{a}}=1)}$, and $^{11}_{5}Be^{++++}_{(\kappa_{\mathring{a}}=1)}$ are calculated here in such a way that the quotient of mechanical actions of each atom, $\kappa_{\mathring{a}} = ET/\underline{E}\,\underline{T}$, becomes a natural number. Without additional criteria, $\kappa_{\mathring{a}}$ can theoretically be any rational number.

The energy ΔE in (94) and the energy of the electromagnetic wave

$$E_{em(\Delta)} \approx \frac{mc^2}{8\pi^2\gamma^2}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \tag{95}$$

in Ref. 7 are identical (the *atom-structure coefficient* γ is nearly constant; n and n' are positive integers). So $mq^2Q^2/32\varepsilon_0^2\underline{\mathring{a}}^2 = mc^2/8\pi^2\gamma^2$, and we get

$$\mathring{a}_e = \frac{\underline{\mathring{a}}}{\gamma} = \frac{\pi\,\lvert qQ\rvert}{2\varepsilon_0 c} = \frac{1}{2}\pi Z_0\,\lvert qQ\rvert, \tag{96}$$

where $Z_0 = 1/\varepsilon_0 c$ = 376.7303 Ω. We call $\underline{\mathring{a}}/\gamma$ the *elementary action* and denote it as $\mathring{a}_e$. In the case of hydrogen ($\lvert qQ\rvert = e^2$) this action is minimal, $\mathring{a}_e$ = 0.1519

$\times\ 10^{-34}$ J · s. It is 21.81 times less than the referent mechanical action $\underline{\mathring{a}}$ according to (89), and 43.62 times less than the Planck constant h. The elementary action $\mathring{a}_e = {}^1/_2\pi Z_0 e^2$ is a universal physical constant.

5. CONCLUSION

An electric charge emits electromagnetic energy whenever it is accelerating. Thus an electron that rotates around the nucleus, with a constant centripetal acceleration, constantly emits electromagnetic energy. Consequently, its energy should diminish gradually. This would lead to a gradual reduction in the dimensions of its orbit so that the electron would finally fall into the nucleus.(56)

However, at the same time there is a process in the atom working in the opposite direction. Generally, an electron in the atom is also absorbing electromagnetic radiation. Indeed, it is the radiative reaction force that by emission of electromagnetic radiation in a steady state contributes to the absorption of electromagnetic radiation in the atom. This means that in the atom's steady state this absorption is equal to the emission of electromagnetic radiation, and the atom remains stable.

In this article the atom is treated as an electromechanical oscillator. All the parameters of this oscillator are determinate: characteristic time τ, damping constant b, spring constant k, half-width Γ, and natural frequency Ω_0. Emission of electromagnetic radiation in the atom's steady state was a fundamental argument against applying classical electrodynamics to it. According to this article, such objections no longer hold ground.

On the basis of mechanical considerations, this article lays the foundations for a deduction of Planck's quantum hypothesis, Einstein's photon equation, Bohr's quantum condition, and de Broglie's hypothesis, whereas the details of quantization are given by the same author in another article(7) by including the others' electromagnetic consideration.

Received 28 March 2001.

Résumé

La théorie de l'absorption et de l'émission de la radiation électromagnétique d'un oscillateur composé du noyau atomique et d'une particule électriquement chargée est déduite en utilisant l'électrodynamique classique. En état stationnaire d'un atome, l'émission et l'absorption de la radiation électromagnétique sont égales, donc, l'atome est stable. Afin d'intégrer les effets réactifs de la radiation dans l'équation du mouvement, l'équation de Newton est modifiée en ajoutant la force de réaction radiative. L'article présente une introduction à la déduction des propositions de base de la mécanique quantique.

Endnotes

1 "The question as to whether the rays of light are quantized or the quantum effect originates only inside the matter is indeed just the first and toughest dilemma the whole quantum theory is faced with and the answer to that question is still to direct its further development." (In the original: " … In der Tat ist die Frage, ob die Lichtstrahlen selber gequantelt sind, oder ob die Quantenwirkung nur in der Materie stattfindet, wohl das erste und schwerste Dilemma, vor das die ganze Quantentheorie gestellt ist und dessen Beantwortung ihr erst die weitere Entwicklung weisen wird." Lecture "Das Wesen des Lichts," 28 October 1919.)

2 "It is useful to note that the longest characteristic time τ ($\tau = e^2/6\pi\varepsilon_0 mc^3$) for charged particles is for electrons and that its value is $\tau = 6.26 \times 10^{-24}$ sec. This is of the order of time taken for light to travel 10^{-15} m. Only for phenomena involving such distances or times will we expect radiative effects to play a crucial role."(22) This statement shows us that classical analyses are used with systems for which mass m approaches the electron mass and charge q approaches the electron charge e.

3 The results and the figures in the text are generated by Wolfram Research, Mathematica, courtesy of Systemcom, Zagreb, Croatia.

4 The radii of the orbits of hydrogen are computed by means of the ionization potential(57) V_i according to the relativistic formula

$$r = \frac{|qQ|}{4\pi\varepsilon_0 |qV_i|[1 + 1/(1 + |qV_i|/m_0c^2)]}.$$

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Milan Perkovac
Drives-Control
P.O. Box 125
HR-10001 Zagreb, Croatia

e-mail: milan.perkovac@drives.hr