Narrow BP Active-RC Filters with Reduced Power and Sensitivity

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Abstract − **This paper presents the low-sensitivity and low-power realization of narrow high-order band-pass (BP) active resistance-capacitance (RC) filters. It is known that filters using multiple-feedback structures have reduced sensitivities to passive component tolerances. We present a filter realization using cascaded "two-amplifier biquartic" BP active filter blocks (CBQ), with reduced numbers of operational amplifiers. The "impedance tapering" and the gain-sensitivity-product minimization decrease both passive and active sensitivities of biquartic blocks. As an example, the design procedure and the Schoeffler's sensitivity analysis, were performed for an 8th-order Chebyshev narrow-BP filter.**

1   INTRODUCTION

The realization of narrow band-pass filters yields very high pole Q-factor values, resulting from the application of the low-pass-to-band-pass (LP-BP) transformation to an all-pole low-pass function. By increasing the filter order, the pole Q-factors increase even more. High pole Q-factors produce high filter-magnitude sensitivity to the variation of passive components. In this paper, we compare a cascade (CAS) structure [Fig. 1(a)], to the filters using a cascaded biquart (CBQ) structure [1] [Fig. 1(b)] in order to reduce high passive component sensitivity. The idea of using cascaded biquarts to reduce pole Qs and, hence, sensitivities, was first proposed in [2] and sensitivity was examined in details in [3].
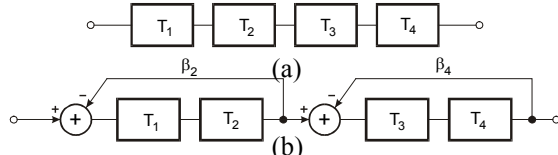

Figure 1: (a) CAS structure. (b) CBQ structure.

The power consumption is reduced by minimization of the overall number of operational amplifiers (opamps). We replace the commonly used multi-amplifier realization of biquads by 1-opamp biquads. Additionally, biquartic sections with two amplifiers

instead of three-amplifier sections are proposed. The filter magnitude sensitivity to the passive components (which we refer to as *passive* sensitivity) is further reduced by the use of low-sensitivity biquads of the CBQ, which are designed using the "impedance tapering" design method [4]. Additionally, the gain-sensitivity-product (GSP [5]) of those biquads is minimized, reducing a filter's magnitude sensitivity to the open-loop gain (*A*) variation of the active component. We refer to this as *active* sensitivity.

2   THE HIGH-ORDER NARROW BP FILTER REALIZATION

The 2nd-order building blocks (biquads) of the structures in Fig. 1, are commonly realized using "general purpose building blocks" (e.g. GP-2) [5]. The GP-2 section has low sensitivity to passive component variations (always less than unity) and it belongs to the "high-Q" circuits according to the classification in [5]. Its main disadvantage is that it requires a large number of opamps. For example, the 8th-order BP filter realized by the CBQ structure requires 3 opamps per each GP-2 biquad and 2 more for the summing amplifiers, i.e. a total of 14 opamps!

2.1 Low-power band-pass 1-opamp biquads

In order to reduce the power consumption, we use single-opamp biquads to build the 2nd-order BP blocks, and leave out the summing amplifiers in the CBQ. This reduces the opamp number in an 8th-order BP filter from 14 to 4. We use Single-amplifier biquad (SAB) and Sallen-and-Key (SAK) biquads (with general impedance scaling factors r and ρ) as shown in Figs. 2, and 3, respectively.
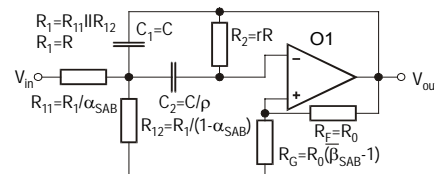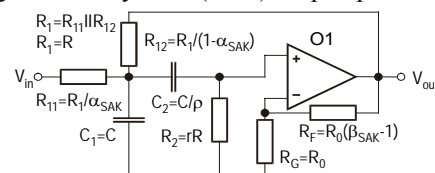

Figure 2: Deliyannis (SAB) 1-opamp-section.

Figure 3: Sallen & Key (SAK) 1-opamp-section.

The transfer function of the SAB (class-3; negative feedback) band-pass section in Fig. 2 is given by:

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$$T(s) = \frac{V_{out}}{V_{in}} = \frac{-\alpha_{SAB}\overline{\beta}_{SAB} \cdot n(s)}{d(s) - \overline{\beta}_{SAB}\alpha_{SAB}n(s)} = \frac{K \cdot (\omega_p/q_p) \cdot s}{s^2 + (\omega_p/q_p) \cdot s + \omega_p^2} , (1)$$

and the transfer function of the SAK (class-4; positive feedback) in Fig. 3 is given by:

$$T(s) = \frac{V_{out}}{V_{in}} = \frac{\alpha_{SAK}\beta_{SAK} \cdot n(s)}{d(s) - \beta_{SAK}(1-\alpha_{SAK})n(s)} . \quad (2)$$

The filters in Figs. 2 and 3 have the same passive RC network with the numerator and denominator given by:

$$n(s) = k_0\omega_0 s/q_0 ; \quad d(s) = s^2 + \omega_0 s/q_0 + \omega_0^2 , (3)$$

where $k_0 = \dfrac{R_2 C_2}{R_1(C_1+C_2)+R_2 C_2}$; $q_0 = \dfrac{\sqrt{R_1 C_1 R_2 C_2}}{R_1(C_1+C_2)+R_2 C_2}$;

$$\omega_0 = (R_1 C_1 R_2 C_2)^{-1/2} ; \text{ and } R_1 = R_{11} \| R_{12} . \quad (4)$$

The gains $\overline{\beta}_{SAB} = 1 + R_G/R_F$ and $\beta_{SAK} = 1 + R_F/R_G$ represent the positive-feedback gain factors in class-3 and class-4 circuits, respectively. Note that the gains are related by $1/\overline{\beta}_{SAB} + 1/\beta_{SAK} = 1$. Both circuits are related by the complementary transformation [6], and the optimization conditions for the two, in terms of sensitivity, are identical. The attenuations α_{SAB} and α_{SAK} in both circuits are given by $\alpha = R_{12}/(R_{11}+R_{12})$. Comparing (1) and (2) we obtain:

$$\alpha_{SAB} = \alpha_{SAK}/(1-\alpha_{SAK}) ; \quad \overline{\beta}_{SAB} = \beta_{SAK}(1-\alpha_{SAK}) . \quad (5)$$

Consequently, if all passive elements of the SAK filter are known, we can use the shortcut way to calculate the passive elements of the SAB filter, and vice versa. We just need to calculate the values of the new α_{SAB} and $\overline{\beta}_{SAB}$ from the known α_{SAK} and β_{SAK} using (5), or vice versa. In the new filter, passive elements such as R_1, R_2, C_1 and C_2 have the same values, whereas R_{11}, R_{12}, R_G and R_F readily follow from the new α and β.

2.2 Sensitivity comparison of 1-and 3-amp biquads

The disadvantage of the filters in Figs. 2 and 3 in comparison to the GP-2 section is that their magnitude sensitivity to passive components is proportional to the pole-Q factors. The sensitivity of the overall filter transfer function $F(s)$ to the component x_{ij} variation is given by:

$$S_{x_{ij}}^{F(s)} = S_{T_i(s)}^{F(s)} \cdot S_{a_k}^{T_i(s)} \cdot S_{x_{ij}}^{a_k} , \quad (6)$$

where $T_i(s)$ represents the transfer function of the ith filter's 2nd-order sub-block and a_k (k=0, 1) represents its coefficients [where $a_0 = \omega_p^2$, $a_1 = \omega_p/q_p$; ω_p and q_p are pole parameters defined in (1)]. The first factor in (6) is called *structure-to-block sensitivity* and it is dependent on the filter structure. Using multiple feedback structures this factor can be reduced, whereas in the cascade structure it equals unity. The second factor is called *coefficient sensitivity* and it depends exclusively on the generic filter type (e.g. Butterworth, Chebyshev, etc.) or, in other words, on the filter pole-Q values. Finally, the third factor

represents the *coefficient-to-component sensitivity* and it depends on the way in which the filters were designed (e.g. "impedance tapering" design of the "medium-Q" filters). For the GP-2 sections the third sensitivity factor in (6) is always less than unity.

3 2-OPAMP BIQUARTIC SECTIONS IN CBQ REALIZATION OF LOW-POWER BP FILTERS

In [7][8] the two-amplifier version of biquart was first presented, in that feedback resistors replaced the summing opamps. The realization of two-amplifier biquarts using SAB and SAK sections are shown in Figs. 4 and 5, respectively.
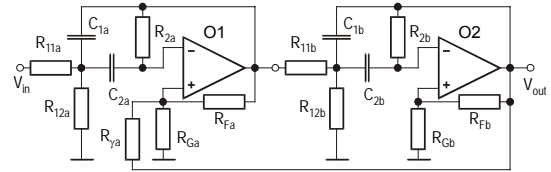

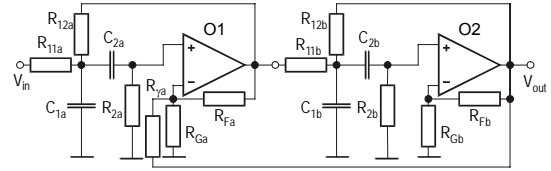
Figure 4: Biquart with SABs and feedback resistor.

Figure 5: Biquart with SAKs and feedback resistor.

Each high-order (i.e. n^{th}-order; $n>2$) symmetrical BP transfer function can be written as a product of 4th-order factors if $n=4k$; k=1,2,… and eventually one 2nd-order factor if $n=2(2k+1)$; k=1,2,…. The 4th-order factors have the form:

$$F_4(s) = T_1(s) \cdot T_2(s) = \prod_{i=1}^{2} \frac{k_i\omega_{0i}s/q_i}{s^2 + \omega_{0i}s/q_i + \omega_{0i}^2} , (7)$$

where $\omega_{01}\omega_{02} = \omega_0$, $q_1 = q_2 = q$, and ω_0 is the center frequency of a BP filter. Equation (7) represents the cascade realization of a 4th-order BP filter.

It is well known that the overall sensitivity of a BP filter is minimal if each 4th-order factor defined by (7) is realized by a biquartic structure (in CBQ) with *identical* 2nd-order blocks [1]. The transfer function of a biquartic BP structure is:

$$F_4(s) = \frac{T_{1B}T_{2B}}{1+bT_{1B}T_{2B}} ; \quad T_{iB}(s) = \frac{k_{iB}\omega_0 s/q_B}{s^2 + \omega_0 s/q_B + \omega_0^2} , \quad (8)$$

where $T_{iB}(s)$ (i=1,2) represent the transfer function of each 2nd-order BP sub-network, and b represents a negative feedback factor. The design parameters for realizing the 4th-order biquartic structure from a given 4th-order cascade structure follows from equating (7) and (8). The design parameters are:

$$q_B = 2q\omega_0/(\omega_{01}+\omega_{02}) ; \quad k_{1B}k_{2B} = k_1k_2q_B^2/q^2 ; \quad (9a,b)$$

$$bk_{1B}k_{2B} = (4q^2-1) \cdot (\omega_{01}-\omega_{02})^2 \cdot (\omega_{01}+\omega_{02})^{-2} . \quad (9c)$$

In [7][8] it is shown that if we leave out a summing amplifier and introduce a feedback resistor R_γ to

realize the two-amplifier biquart, the resulting network has two identical 2nd-order blocks with higher Q-factors, q_N, that readily follow from:

$$(q_N/q_B)^2 - 2\frac{q_B}{2q_B - Xq_0}(q_N/q_B) + \frac{q_0}{2q_B - Xq_0} = 0 \,, (10)$$

where $X = 1 + bk_{1B}k_{2B}$ is defined by (9c), and q_0 is the passive pole Q defined by (4). Finally, with a known value of q_N, the feedback coefficient γ_{1N} follows from

$$\gamma_{1N} = 2(q_N/q_B - 1)/(\beta_1 k_{2B}) \,, \qquad (11)$$

where β_1 represents the gain β of the 1st biquad section in the biquart, and k_{2B} is the pass-band gain of the 2nd biquad section. Eqs. (10) and (11) hold for both SAB and SAK biquarts. Note that, because $q_N > q_B$ the resistance ratio γ_{1N} in (11) is always positive.

In the design process of 2-amplifier biquarts in Figs. 4 and 5, we use the impedance scaling factors (defined in the SAK and SAB above) and given by:

$$R_2 = rR_1; \quad C_2 = C_1/\rho \,. \qquad (12)$$

For both SAK and SAB biquads with known k_{1B}, ω_0 and q_N, the step-by step design procedure follows: (*i*) Choose capacitor C, and impedance scaling factors $\rho = 1$, and $r > 1$ and then calculate R:

$$R = (\omega_0 C)^{-1}\sqrt{\rho/r} \,. \qquad (13)$$

(*ii*) Calculate the values

$$q_0 = \sqrt{\rho r}/(1 + \rho + r) \,, \quad k_0 = r/(1 + \rho + r) \,, \quad (14)$$

and from (1)-(5) above calculate the αs and βs from:

$$\overline{\beta}_{SAB} = 1 + (1 + \rho)/r - q_N^{-1}\sqrt{\rho/r} \,; \qquad (15)$$

$$\beta_{SAK} = \beta_{SAK}\alpha_{SAK} + \overline{\beta}_{SAB} = k_{1B}q_0/(k_0 q_N) + \overline{\beta}_{SAB} \,; \quad (16)$$

$$\alpha = k_{1B}q_0/(k_0 q_N\beta) \,. \qquad (17)$$

The above design equations are readily obtained by introducing (12) into (4). Note that when we write α and β without subscripts they represent the gain and attenuation of both the SAB and SAK filter sections. (*iii*) The component values follow from:

$$R_1 = R; R_{11} = R_1/\alpha; R_{12} = R_1/(1-\alpha); C_1 = C; C_1 = C/\rho \,.(18)$$

(*iv*) Finally, choose R_G and using feedback-factor γ_{1N}, from (11), compute R_F and R_γ using:

$$R_F = R_G[\beta_{SAK}(1 - \gamma_{1N}) - 1]; \quad R_\gamma = R_F/(\beta_{SAK}\gamma_{1N}) \,, \quad (19)$$

where, for SAB filter design, we have:

$$\beta_{SAK} = \overline{\beta}_{SAB}/(\overline{\beta}_{SAB} - 1) \,. \qquad (20)$$

To design the circuit with min. GSP according to [5], (for both SAB and SAK) we modify the step (*i*) into: (*i'*) choose $r > 1$ and then calculate ρ using:

$$\rho = r/(36q_N^2) \cdot [\sqrt{1 + 12q_N^2(1 + 1/r)} + 1]^2 \,. \quad (21)$$

By this we reduce both *passive* (by choosing $r > 1$) and *active* sensitivities. We can calculate the GSP:

$$\Gamma_{SAK} = q_p\beta_{SAK}^2(1 - \alpha_{SAK})\sqrt{r/\rho}; \quad \Gamma_{SAB} = q_p\overline{\beta}_{SAB}^2\sqrt{r/\rho} \,. (22)$$

The q_0 in (10) has to be known in advance. If it were calculated according to the min. GSP criterion for SAB and SAK circuit, this would complicate the

procedure for finding q_N, because the optimum q_0 depends on the value of q_N as well. The problem is easily solved using a simple computer program. This problem does not arise (e.g. in the "impedance tapering" case) if we choose r and ρ in advance and they do not change their values until the end of the calculation.

4   EXAMPLE

As an example consider an 8th-order Chebyshev narrow-band BP filter with normalized bandwidth $B = 0.1$, center frequency $\omega_0 = 1$, and reflection coefficient $\rho = 10\%$, (the pass-band ripple $R_p = 0.044$ dB). Its transfer function magnitude $\alpha(\omega) = 20\log |F(j\omega)|$ [dB] is shown in Fig. 6(a). The normalized transfer function parameters of the structures in Fig. 1 and their 2nd-order blocks $T_i(s)$ are given in Table 1. Note that the gain values k_i and feedback values β_i ($i = 1,\ldots,4$) are optimized by the method given in [9], to provide max. dynamic range. The frequency responses at all opamp outputs are shown in Fig. 6(b).

	i	q_{pi}	ω_{pi}	k_i	β_i
CAS	1	13.202	0.9755	1.0000	
	2	13.202	1.0251	1.4274	
	3	31.919	0.9420	2.1733	
	4	31.919	1.0615	7.1188	
CBQ	1	13.198	1.0000	1.3767	
	2	13.198	1.0000	1.0363	0.2991
	3	31.862	1.0000	4.4167	
	4	31.862	1.0000	3.4903	0.9402

Table 1: Parameters of filter structures in Fig. 1.

	i	q_N	ω_p	k_{iB}	γ_{iN}
1)	1	13.2714	1.0000	1.3899	0.00353542
$r=1$	2	13.2714	1.0000	1.0379	
$\rho=1$	3	34.5044	1.0000	4.8090	0.0141851
	4	34.5044	1.0000	3.7593	

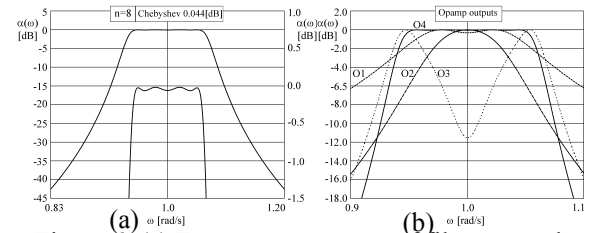Table 2: Parameters of filter biquarts as in Fig. 5.

Figure 6: (a) Frequency response of filter example. (b) Frequency responses at section outputs.

In what follows we design the 8th-order BP filter using 2-opamp cascaded biquarts with SAK as in Fig. 5. To design the filters we apply various ways of impedance tapering to the SAK biquads, in the CAS and CBQ structures, i.e.: 1) $r = 1$, $\rho = 1$; 2) $r = 4$, $\rho = 4$; 3) $r = 4$, $\rho = 1$; 4) $r = 1$, $\rho = 4$; and 5) $r = 4$, ρ for min. GSP. After the transformation from 3- to 2-opamp biquarts a new optimization is required to provide maximum dynamic range, i.e. we have to calculate new gain values k_{iB} ($i = 1,\ldots,4$), and new values of negative feedback factors γ_{iN} ($i = 1,3$) using (11). The new design parameters in the case 1) are in Table 2.
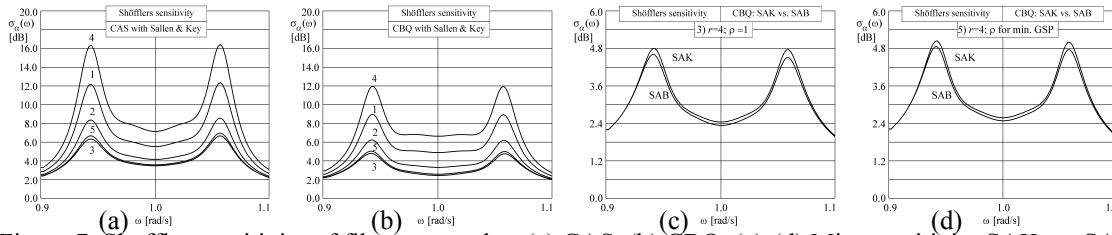
Figure 7: Shöffler sensitivity of filters examples. (a) CAS. (b) CBQ. (c)-(d) Min. sensitivity SAK vs. SAB.

In a sensitivity analysis we assume the relative changes of the biquad resistors and capacitors to be uncorrelated normal random variables, with zero-mean and 1% standard deviation. The corresponding standard deviation $\sigma_\alpha(\omega)$ [dB] (related to the Shöfflers sensitivities) of the variation of the log gain $\Delta\alpha=8.68588 \ \Delta|F(j\omega)|/|F(j\omega)|$, with respect to passive elements, for CAS and CBQ structures for all design cases from 1) to 5) are in Figs. 7(a) and (b), respectively. The influence of feedback resistors on sensitivity was not considered.

Observing the standard deviation $\sigma_\alpha(\omega)$ [dB] in the Fig. 7(a) one can investigate which is the proper impedance tapering design strategy of biquads in the cascade, in order to reduce the overall sensitivity of the 8th-order BP filter. Obviously the "ideally impedance-tapered" biquads (no. 2) and even more the "partially-tapered"[1] biquads with equal capacitors (no. 3), decrease sensitivity with respect to component variations, compared to the non-tapered standard circuit (no. 1). The R-tapered biquads in the case no. 5 with ρ for min. GSP shows somewhat higher passive sensitivities than those for the case no. 3. Further desensitization is obtained by an application of a negative feedback, which has a CBQ structure as in Fig. 1(b). The sensitivity curves of CBQ filters, designed with all five impedance-tapering strategies applied to biquads, are shown in Fig. 7(b). The sensitivity curves in Fig. 7(b) have the same order as those in the CAS case in Fig. 7(a), i.e. 4, 1, 2, 5, 3, starting from the curve with highest and ending with the lowest sensitivity. Note, that all CBQ curves in Fig. 7(b) are lower than those for the CAS structure in Fig. 7(a).

The Schöffler sensitivities of the CBQs with SAB and SAK section for the design strategies 3) and 5) are presented separately in Fig. 7(c) and (d), respectively. In both cases SAB biquads produce slightly lower sensitivity than SAK, although both circuits have the same passive-RC networks. Moreover, SAB has somewhat lower active sensitivity, which is apparent if (5) is substituted into (22). In a future full paper we'll include the effects of variations in the feedback resistors and in the amplifier open-loop gains, reducing the Q using special approximating functions with low Qs, investigate tuning of biquarts, etc.

5 CONCLUSIONS

This paper presents a low-sensitivity and low-power realization of narrow high-order BP-filters. We present the detailed step-by-step design of low-power filters with two-amplifier cascaded biquarts (CBQ). Furthermore, by application of a recently introduced "impedance tapering" design method of biquads, their passive sensitivities are additionally decreased. In this paper, we therefore present a *twofold sensitivity reduction*, one by introduction of negative feedback (application of the CBQ structure) and the other by the proper design of the 2nd-order sub-circuit building blocks in CBQ. *The optimum design strategy that reduces both passive and active sensitivities is tapering the resistors and calculating the capacitor values for minimum GSP.*

By application of the complementary transformation it is demonstrated that the realization of optimized biquarts using Sallen and Key also provides optimized Deliyannis biquads, whereas the latter shows slightly lower passive and active sensitivities.

References

[1]   N. Mijat and G. S. Moschytz, 'Sensitivity of Narrowband Biquartic BP Active Filter Block', In Proc. 5th ISYNT, Sarajevo, 1984, pp.158-163.

[2]   J. Tow, "Some Results on Two-Section Generalized FLF Active Filters," IEEE Tran. on Circuits and Systems, Vol. CAS-25, pp. 181-184, April 1978.

[3]   A. N. Gonuleren, "Sensitivity Analysis and Dynamic Range of Quad Filters and Some Comparative Results," Int. J. Circuit Theory and Applications, Vol. 10, pp. 175-200, July 1982.

[4]   G. S. Moschytz, "Low-sensitivity, low-power, active-RC allpole filters using impedance tapering," IEEE Trans. on CAS—II: vol. CAS-46, no.8, pp.1009-1026, Aug. 1999.

[5]   G. S. Moschytz and P. Horn, Active Filter Design Handbook, Chichester, U.K.: Wiley 1981.

[6]   G. S. Moschytz and P. Horn, "Optimizing two commonly used active-filter building blocks using the complementary transformation," Electronic Circuits and Systems, vol. 1, no. 4, pp. 125-132, July 1977.

[7]   N. Mijat, V. Čosić and M. Vučić, "Design of Two-Amplifier Biquartic BP Active Filter Blocks", In Proc. 37th Intl. Annual Gath. KoREMA, Zagreb, Hr, April 26-29, 1992, pp. 448-451.

[8]   N. Mijat and V. Čosić, "Two-Amplifier Biquartic BP Active Filter Blocks with S&K Sections", In Proc. 38th Intl. Annual Conf. KoREMA, Zagreb, Hr, April 26-28, 1993, pp. 553-556.

[9]   D. J. Perry, "Scaling Transformation of Multiple-Feedback Filters", Proc. IEE, vol.128, pp.176-179, Aug. 1981.

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[1] The terms "ideal tapering" and "partial tapering" are used according to the definition in [4].