# Parameters Optimization of Analytic Fuzzy Controllers for Robot Manipulators

J. Kasac, B. Novakovic, D. Majetic & D. Brezak
*Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Croatia.*

## Abstract

An open question in fuzzy logic control of robot manipulators is how to modify the fuzzy controller parameters to guarantee appropriate performance specifications. In this paper a new approach to performance tuning of analytic fuzzy controllers for robot manipulators is presented. The analytic fuzzy control is a nonconventional approach that uses an analytic function for output determination, instead of a fuzzy rule base. The proposed approach is based on construction of a parameter dependent Lyapunov function. With the appropriate choice of the free parameter an estimation of integral performance index is obtained. The estimated performance index depends on controller parameters and few parameters which characterize the robot dynamics. The optimal values of the controller gains are obtained by minimization of the performance index. An example is given to demonstrate the obtained results.
*Keywords: parameters optimization, fuzzy control, robot control, performance evaluation, global stability, Lyapunov stability.*

## 1 Introduction

A significant problem in the conventional fuzzy logic control (FLC) is the exponential growth in rules as the number of variables increases. Consequently, the application of the conventional FLC to the multivariable systems like robots, in the process of the real-time control, becomes difficult. It becomes necessary to suggest ways to cope with a vexing problem in fuzzy logic: the exponential growth in rules as the number of variables increases [1].

These problems have been avoided in [2, 3] by introducing a new, nonconventional analytic method for synthesis of the fuzzy robot control. For this purpose a new analytic function is defined that determines the positions of the centers of output fuzzy sets, instead of the definition of a fuzzy rule base. In this way, the number of input and output variables and the number of fuzzy sets of FLC system have not been limited, because there are no fuzzy rules. In this way, the vexing problem in fuzzy logic has been solved.

Further, the analytic representation of proposed controller provide more elegant stability analysis [4] then conventional approach that uses Takagi-Sugenos's fuzzy model. The basic idea of the Takagi-Sugenos's fuzzy model is to obtain representation of nonlinear plant model by linearized plant models valid around different operating points. In the case of robot dynamics such linearisation disable opportunity of using robot's energy function as a part of the closed-loop system Lyapunov's function.

In this paper a new approach to performance tuning of analytic fuzzy controllers for robot manipulators is presented. The proposed approach is based on construction of a parameter dependent Lyapunov function using passivity properties of Euler-Lagrange systems [5, 6]. With the appropriate choice of the free parameter, which is not included in stability conditions, an estimation of integral performance index is obtained. The optimal values of the controller parameters are obtained by minimization of the estimated performance index.

## 2 Fuzzy Control Without a Fuzzy Rule Base

A conventional FLC system is composed of four principal elements: fuzzy rule base, fuzzification interface, fuzzy inference machine, and defuzzification interface. In order to create a new fuzzification interface process, we first define a new type of the fuzzy membership functions $s_{ji}(x_j)$, $j = 1, ..., m$, $i = 1, ..., N_j$, where $x_j$ are input variables, $m$ is the number of input variables and $N_j$ is the number of fuzzy sets belonging to the $j$-th input variables.

The function $s_{ji}(x_j)$ is positive, bounded, $0 < s_{ji}(x_j) \leq 1$, and symmetric, $s_{ji}(x_j) = s_{ji}(-x_j)$. A possible choice of the fuzzy membership functions $s_{ji}(x_j)$ is

$$s_{ji}(x_j) = \gamma_{ji} + \bar{\gamma}_{ji} \exp(-\alpha_{ji} x_j^2 - \beta_{ji}|x_j|), \tag{1}$$

where $\bar{\gamma}_i^j = 1 - \gamma_i^j$ and $0 < \gamma_i^j < 1$.

In order to create a new analytic inference algorithm, in this approach max/min operators are replaced by sum/product operators. The activation function $\omega_j$ of the $j$-th output fuzzy set can be computed by an analytic form

$$\omega_j(x_j) = \sum_{i=1}^{N_j} s_{ji}(x_j), \quad j = 1, ..., m. \tag{2}$$

The activation function $\omega_j$ denotes the grade of membership of input $x_j$ to all of the input fuzzy sets.

Instead of using fuzzy rules, the function for an analytic determination of the centers of output fuzzy sets has been defined using following intuitive reasoning. If the membership of the input variable is smaller, then the distance $x_j$ to zero is bigger. Consequently, if the control error is bigger, then the amplitude of the control variable should be bigger.

Thus, amplitudes of normalized positions of output fuzzy sets centers can be computed by the equation

$$y_{Cj}(x_j) = K_{Cj}\mu_j \left(1 - \frac{\omega_j(x_j)}{N_j}\right) \mathrm{sign}(x_j). \tag{3}$$

where $K_{Cj}$ is a gain of the output fuzzy set center position and $\mu_j = 1/(1 - \sum_{i=1}^{N_j} \gamma_{ji}/N_j)$ is a normalization factor which ensure that $-K_{Cj} \leq y_{Cj}(x_j) \leq K_{Cj}$.

In order to generate a non fuzzy output (crisp value) of the FLC system the centroid defuzzification method [7] is employed

$$u(x_1, ..., x_m) = \left(\sum_{j=1}^{m} \omega_j(x_j) y_{Cj}(x_j) I_j\right) / \left(\sum_{j=1}^{m} \omega_j(x_j) I_j\right) \tag{4}$$

where $I_j$ is area of $j$-th output fuzzy set. More details about fuzzy control without fuzzy rule base is available in [2, 3].

## 3 System Description

We consider a nonlinear mechanical system with $n$-degrees of freedom in closed loop with a analytic fuzzy PD plus saturated PID controller. The saturated PID controller [8, 9] ensures global asymptotic stability of the closed loop systems and analytic fuzzy PD controller provides performance tuning.

### 3.1 Dynamics of Rigid Robot

The model of $n$-link rigid-body robotic manipulator, in the absence of friction and disturbances, is represented by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u, \tag{5}$$

where $q$ is the $n \times 1$ vector of robot joint coordinates, $\dot{q}$ is the $n \times 1$ vector of joint velocities, $u$ is the $n \times 1$ vector of applied joint torques, $M(q)$ is

$n \times n$ inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of centrifugal and Coriolis torques, and $g(q)$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $U(q)$. The following well known properties of the robot dynamics, [6, 5], are important for stability analysis.

*Property 1.* The inertia matrix $M(q)$ is a positive definite symmetric matrix which satisfies

$$\lambda_m\{M\}\|\dot{q}\|^2 \leq \dot{q}^T M(q)\dot{q} \leq \lambda_M\{M\}\|\dot{q}\|^2, \tag{6}$$

where $\lambda_m\{M\}$ and $\lambda_M\{M\}$ denotes strictly positive minimum and maximum eigenvalues of $M(q)$, respectively.

*Property 2.* The matrix $S(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e.,

$$z^T S(q,\dot{q})z = 0, \quad \forall z \in \mathbb{R}^n. \tag{7}$$

This implies $\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T$.

*Property 3.* The Coriolis and centrifugal terms $C(q,\dot{q})\dot{q}$ satisfies

$$\|C(q,\dot{q})\dot{q}\| \leq k_c\|\dot{q}\|^2, \tag{8}$$

for some bounded constant $k_c > 0$.

*Property 4.* There exists some positive constant $k_g$ such that gravity vector satisfies

$$\|g(x) - g(y)\| \leq k_g\|x - y\|, \quad \forall\, x, y \in \mathbb{R}^n. \tag{9}$$

### 3.2 Analytic Fuzzy PD plus saturated PID controller

The analytic fuzzy PD plus saturated PID control law is given by

$$u = -\Psi_P(\tilde{q},\dot{q})\varphi_P(\tilde{q}) - \Psi_D(\tilde{q},\dot{q})\varphi_D(\dot{q}) - K_P\tilde{q} - K_D\dot{q} - K_I\nu, \tag{10}$$

$$\dot{\nu} = \varphi_P(\tilde{q}). \tag{11}$$

where $\tilde{q} = q - q_d$ is the joint position error, $K_P$, $K_D$ and $K_I$ are constant positive-definite diagonal matrix, $\Psi_j(\tilde{q},\dot{q})$, $j = P, D$, are positive definite diagonal matrix functions which can be written in the following form

$$\psi_{ji}(\tilde{q}_i,\dot{q}_i) = \frac{I_{ji}\omega_{ji}(\chi_{ji})}{I_{Pi}\omega_{Pi}(\tilde{q}_i) + I_{Di}\omega_{Di}(\dot{q}_i)}, \tag{12}$$

and $\varphi_j(\chi_j)$, $j = P, D$, $(\chi_P = \tilde{q}, \chi_D = \dot{q})$, is vector function

$$\varphi_{ji}(\chi_{ji}) = y_{Cji}(\chi_{ji}) = K_{Cji}\mu_{ji}\left(1 - \frac{\omega_{ji}(\chi_{ji})}{N_{ji}}\right)\mathrm{sign}(\chi_{ji}), \tag{13}$$

where $i = 1, ..., n$.

## 4 Construction of Lyapunov function

The stationary state of the system (5), (10), (11) is $\tilde{q} = 0$, $\dot{q} = 0$, $\nu = \nu^*$, where $\nu^*$ satisfies $g(q_d) = -K_I \nu^*$. If a new variable $z = \nu - \nu^*$ is introduced, then system (5), (10), (11) becomes

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - g(q_d) = u \tag{14}$$

$$u = -\Psi_P(\tilde{q},\dot{q})\varphi_P(\tilde{q}) - \Psi_D(\tilde{q},\dot{q})\varphi_D(\dot{q}) - K_P\tilde{q} - K_D\dot{q} - K_I z, \tag{15}$$

$$\dot{z} = \varphi_P(\tilde{q}). \tag{16}$$

Further, an output variable $y = \dot{q} + \alpha\varphi_P(\tilde{q})$ with parameter $\alpha > 0$ is introduced, and inner product between (14) and $y$ is made, resulting in a nonlinear differential form which can be separated in the following way

$$\frac{dV(\tilde{q},\dot{q},z;\alpha)}{dt} = -W(\tilde{q},\dot{q};\alpha), \tag{17}$$

where $V(\tilde{q},\dot{q},z;\alpha)$ is the Lyapunov function candidate parameterized by the positive parameter $\alpha$. For easier determination of conditions for positive-definiteness of function $V$ and $W$, the following decompositions are made: $V = V_1 + V_2$ and $W = W_1 + W_2$, where

$$\begin{aligned} V_1 &= \frac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha\varphi_P(\tilde{q})^T M(q)\dot{q} + \alpha\sum_{i=1}^{n} K_{Di}\int_0^{\tilde{q}_i}\varphi_{Pi}(\xi)d\xi + \\ &+ \alpha\sum_{i=1}^{n}\int_0^{\tilde{q}_i}\psi_{Di}(\xi,\dot{q}_i(\xi))\phi_{Di}(\dot{q}_i(\xi))\varphi_{Pi}(\xi)d\xi, \end{aligned} \tag{18}$$

$$\begin{aligned} V_2 &= \sum_{i=1}^{n}\int_0^{\tilde{q}_i}\psi_{Pi}(\xi,\dot{q}_i(\xi))\varphi_{Pi}(\xi)d\xi + U(q) - U(q_d) - \tilde{q}^T g(q_d) + \\ &+ \frac{1}{2}\tilde{q}^T K_P\tilde{q} + \tilde{q}^T K_I z + \frac{1}{2}\alpha z^T K_I z, \end{aligned} \tag{19}$$

$$\begin{aligned} W_1 &= \dot{q}^T\Psi_D(\tilde{q},\dot{q})\varphi_D(\dot{q}) + \dot{q}^T K_D(\tilde{q})\dot{q} - \alpha\dot{q}^T\varphi_{P,\tilde{q}}(\tilde{q})^T M(q)\dot{q} + \\ &+ \alpha\varphi_P(\tilde{q})^T[\dot{M}(q) - C(q,\dot{q})]\dot{q}, \end{aligned} \tag{20}$$

$$\begin{aligned} W_2 &= \alpha\varphi_P(\tilde{q})^T\Psi_P(\tilde{q},\dot{q})\varphi_P(\tilde{q}) + \alpha\varphi_P(\tilde{q})^T K_P\tilde{q} - \varphi_P(\tilde{q})^T K_I\tilde{q} + \\ &+ \alpha\varphi_P(\tilde{q})^T[g(q) - g(q_d)]. \end{aligned} \tag{21}$$

where $\phi_{Di}(\dot{q}_i) = \varphi_{Di}(\dot{q}_i)/\dot{q}_i > 0$, $\varphi_{P,\tilde{q}}(\tilde{q})$ is positive diagonal matrix of partial derivatives $\varphi_{P,\tilde{q}}(\tilde{q}) = \mathrm{diag}\{\varphi_{P1,\tilde{q}_1}(\tilde{q}_1), ..., \varphi_{Pn,\tilde{q}_n}(\tilde{q}_n)\}$, and $\dot{q}_i(\tilde{q}_i)$ denotes explicit dependance variable $\dot{q}_i$ on variable $\tilde{q}_i$. Although we cannot find explicit analytic expression for function $\psi_{Pi}(\tilde{q}_i, \dot{q}_i(\tilde{q}_i))$ we can find its upper and lower bounds.

The following step is determination of conditions for positive-definiteness of functions $V$ and $W$. First, we consider function $V$ which can be rearranged to be of the form

$$\begin{aligned} V \geq \alpha \sum_{i=1}^{n} K_{Di} \int_0^{\tilde{q}_i} \varphi_{Pi}(\xi) d\xi - \frac{1}{2}\alpha^2 \lambda_M\{M\} \|\varphi_P(\tilde{q})\|^2 + \\ + \frac{1}{2}\left(k_1 - \frac{1}{\alpha}\lambda_M\{K_I\}\right) \|\tilde{q}\|^2 \geq 0, \end{aligned} \tag{22}$$

where $k_1 = \lambda_m\{K_P\} - k_g > 0$, that is positive-definite function if

$$k_1 \lambda_m\{K_D\} > \lambda_M\{K_I\}\lambda_M\{M\}\lambda_M\{\varphi_{P,\tilde{q}}\}. \tag{23}$$

Further, we consider condition which ensure that time derivative of Lyapunov function is negative definite function, i.e., $W \geq 0$. Applying properties (6) and (8) we get

$$\begin{aligned} W \geq \lambda_m\{K_D\}\|\dot{q}\|^2 - \alpha \lambda_M\{M\}\|\dot{q}\|^2 - \alpha k_c s_M \|\dot{q}\|^2 + \\ + (\alpha k_1 - \lambda_M\{K_I\})\tilde{q}^T \varphi_P(\tilde{q}) \geq 0, \end{aligned} \tag{24}$$

that is positive-definite if the following condition is satisfied

$$k_1 \lambda_m\{K_D\} > \lambda_M\{K_I\}(\lambda_M\{M\}\lambda_M\{\varphi_{P,\tilde{q}}\} + k_c s_M). \tag{25}$$

where $s_M = \lambda_M\{K_{CP}\}$. Notice that the condition (23) is trivially implied by the condition (25). So, the condition (25) is the final stability condition which guaranty global stability. Finally, invoking the LaSalle's invariance principle we conclude asymptotic stability. More details about stability analysis of analytic fuzzy control and saturated PID control of robot manipulators can be found in [4].

## 5 Performance Optimization

The Lyapunov function $V$ and its time derivative $\dot{V} = -W$ contain free parameter $\alpha > 0$ which is not included in stability condition. This fact can be employed for the evaluation of the following performance index

$$I = I_1 + \tau^2 I_2, \tag{26}$$

where the constant $\tau^2$ is the weighting factor, and

$$I_1 = \int_0^\infty \tilde{q}^T \varphi_P(\tilde{q})dt, \qquad I_2 = \int_0^\infty \|\dot{q}\|^2 dt. \tag{27}$$

Also, in this section, because of compactness, following shortened notation is introduced: $k_{jm} = \lambda_m\{K_j\},\ k_{jM} = \lambda_M\{K_j\},\ \bar{m} = \lambda_M\{M\}\lambda_M\{\varphi_{P,\tilde{q}}\} + k_c s_M,\ \mu_j = \lambda_M\{K_j\}/\lambda_m\{K_j\}$, where $j = P, I, D$. The performance index (26) can be evaluated using Lyapunov function (22) and its time derivative. From the equation (17) we can get

$$V(0) \geq \int_0^\infty W(\tilde{q}(s), \dot{q}(s))ds, \tag{28}$$

where we used $V(\infty) = 0$. Putting (24) in (28) we get

$$V(0) \geq (k_{Dm} - \alpha\bar{m})I_2 + (\alpha k_1 - k_{IM})I_1. \tag{29}$$

The next step is the estimation of the upper bounds on V(0). We have $\tilde{q}(0) = -q_d$, $\dot{q}(0) = 0$, $z(0) = -\nu^* = K_I^{-1}g(q_d)$, so that $V(0)$ satisfies the following expression

$$\begin{aligned} V(0) = {} & \frac{1}{2}q_d^T K_P q_d + \frac{1}{2}\alpha g(q_d)^T K_I^{-1} g(q_d) + \alpha\sum_{i=1}^n K_{Di}\int_0^{-q_{di}} \varphi_{Pi}(\xi)d\xi + \\ & + U(0) - U(q_d) + \alpha\sum_{i=1}^n \int_0^{-q_{di}} \psi_{Di}(\xi, \dot{q}_i(\xi))\phi_{Di}(\dot{q}_i(\xi))\varphi_{Pi}(\xi)d\xi + \\ & + \sum_{i=1}^n \int_0^{-q_{di}} \psi_{Pi}(\xi, \dot{q}_i(\xi))\varphi_{Pi}(\xi)d\xi. \end{aligned} \tag{30}$$

So, we can estimate the upper bounds

$$V(0) \leq w_2\left(k_{PM} + \bar{k}_{PM}(\alpha) + \alpha\frac{k_g^2}{k_{Im}} + \alpha\varrho_{PM}k_{DM}\right), \tag{31}$$

where $w_2 = \frac{1}{2}\|q_d\|^2$, $\varrho_{PM} = \lambda_M\{\Phi_P\}$ and

$$\bar{k}_{PM}(\alpha) = [\alpha\lambda_M\{\Psi_D\}\lambda_M\{\Phi_D\} + \lambda_M\{\Psi_P\}]\,\lambda_M\{\Phi_P\}. \tag{32}$$

Finally, comparing (29) and (31) we have

$$(k_{Dm} - \alpha\bar{m})I_2 + (\alpha k_1 - k_{IM})I_1 \leq w_2\alpha\varrho_{PM}k_{DM} +$$

$$+ w_2 \left( k_{PM} + \bar{k}_{PM}(\alpha) + \alpha \frac{k_g^2}{k_{Im}} \right) . \tag{33}$$

From the above mentioned expression we can get integral terms $I_1$ and $I_2$ in the following way. Because the choice of the free parameter $\alpha$ is not limited by stability conditions (25), we can put $\alpha = \alpha_1 = k_{Dm}/\bar{m}$ in expression (33) so that

$$I_1 \leq \frac{w_2}{S_M} \left[ (k_{PM} + \bar{k}_{PM}(\alpha_1))\bar{m} + k_g^2 \frac{k_{Dm}}{k_{Im}} + \varrho_{PM} k_{Dm} k_{DM} \right] , \tag{34}$$

where $S_M = k_1 k_{Dm} - k_{IM}\bar{m} > 0$.

The positivity of $S_M$ follows from stability conditions (25). Similarly, if we put $\alpha = \alpha_2 = k_{IM}/k_1$ in expression (33) we get

$$I_2 \leq \frac{w_2}{S_M} \left[ (k_{PM} + \bar{k}_{PM}(\alpha_2))k_1 + k_g^2 \mu_I + k_{IM} \varrho_{PM} k_{DM} \right] . \tag{35}$$

Finally, if we put expressions (34) and (35) in (26) we get

$$\hat{I} = \frac{w_2}{S_M} \left[ k_P^* + \mu_D \varrho_{PM} k_{Dm}(k_{Dm} + \tau^2 k_{IM}) + \mu_I k_g^2 \left( \frac{k_{Dm}}{k_{IM}} + \tau^2 \right) \right] , \tag{36}$$

where $\hat{I} \geq I$ is the estimation of the upper bounds of the performance index (26), and $k_P^* = (\bar{m} + \tau^2 k_1)k_{PM} + \bar{k}_{PM}(\alpha_1)\bar{m} + \tau^2 \bar{k}_{PM}(\alpha_2)k_1$. The optimal values of the controller parameters can be found by minimization of expression (36).

## 6 Simulation Example

The manipulator used for simulation is a two revolute jointed robot (planar elbow manipulator) with numerical values of robot parameters which have been taken from [10]. The most simple form of the analytic fuzzy controller is considered

$$\omega_{ji}(\chi_{ji}) = s_{ji}(\chi_{ji}) = \gamma_{ji} + \bar{\gamma}_{ji} \exp(-\beta_{ji}|\chi_{ji}|), \tag{37}$$

$$\varphi_{ji}(\chi_{ji}) = y_{Cji}(\chi_{ji}) = K_{Cji} \left[ 1 - \exp(-\beta_{ji}|\chi_{ji}|) \right] \operatorname{sign}(\chi_{ji}), \tag{38}$$

where $j = P, D$ , $i = 1, ..., n$, $N_j = 1$, $\alpha_j = 0$, ( $\chi_{Pi} = \tilde{q}_i, \chi_{Di} = \dot{q}_i$). In that case we have

$$\lambda_M\{\Phi_P\} = \lambda_M\{\varphi_{P,\tilde{q}}\} = \max_i K_{CPi}\beta_{Pi}, \quad \lambda_M\{\Phi_D\} = \max_i K_{CDi}\beta_{Di}, \tag{39}$$

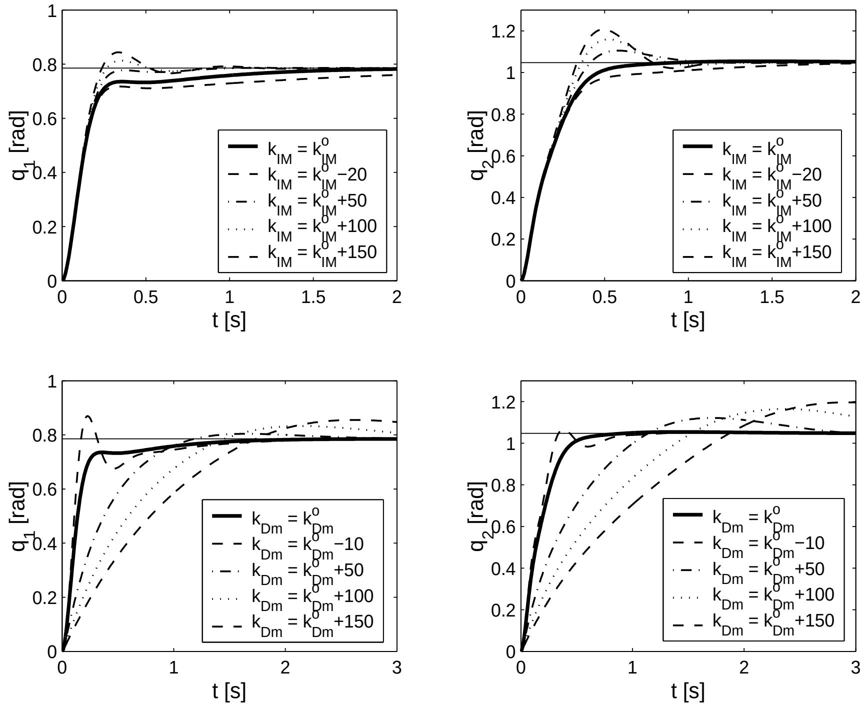


Figure 1: The transient response of the closed loop systems for different values of controller gains $k_{Dm}$ and $k_{IM}$ and for $k_{Pm} = 200$ .

$$\lambda_M\{\Phi_P\} = \max_i \frac{I_{Pi}}{I_{Pi} + \gamma_{Di} I_{Di}}, \quad \lambda_M\{\Phi_D\} = \max_i \frac{I_{Di}}{I_{Di} + \gamma_{Pi} I_{Pi}}. \tag{40}$$

We can make further simplification: $I_{Pi} = I_{Di} = 1$, $\gamma_{Pi} = \gamma_{Di} = 0$, $\beta_P = \beta_{Pi}$, $\beta_D = \beta_{Di}$, $K_{CP} = K_{CPi} = 1$, $K_{CD} = K_{CDi} = 1$ for $i = 1, 2$, so that $\lambda_M\{\Phi_P\} = \lambda_M\{\Phi_D\} = 1$, $\lambda_M\{\Phi_P\} = \lambda_M\{\varphi_{P,\tilde{q}}\} = K_{CP}\beta_P$, $\lambda_M\{\Phi_D\} = K_{CD}\beta_D$, and $\bar{k}_{PM}(\alpha) = [\alpha K_{CD}\beta_D + 1]K_{CP}\beta_P$.

In the case when $\beta_P \gg 1$ function $\varphi_{Pi}(\tilde{q}_i)$ tend to function $K_{CPi}\mathrm{sign}(\tilde{q}_i)$ what means that performance index $I_1 \to \int_0^\infty \|\tilde{q}\|_1 d\tau$, where $\|\tilde{q}\|_1$ is the $L_1$ norm of the vector $\tilde{q}$. In other words, we have mixed $L_2/L_1$ optimization problem. Note that upper bounds on values of the parameter $\beta_P$ is limited by stability conditions (25). In Fig. 1 we can see the transient response of the closed loop systems for different values of controller gains. The optimal values of controller parameters are $k_{Dm}^o = 26.5$, $k_{IM}^o = 41.9$, $\beta_P = 10$.

## 7 Concluding Remarks

In this paper a new approach to performance tuning of analytic fuzzy controllers for robot manipulators is presented. The proposed tuning rules provide fast transient response without oscillations and large overshoots, overcoming undesirable effect of high control jumps which is characteristic for conventional linear PID controllers. The performance tuning rule involve only few parameters which characterize the robot dynamics.

## References


[1] Kosko, B., *Fuzzy Engineering.* Prentice Hall, Englewood Cliffs, N.J., 1996.

[2] Novakovic, B., Fuzzy logic control synthesis without any rule base. *IEEE Trans Syst, Man, Cyber, B*, **29(3)**, pp. 459–466, 1999.

[3] Novakovic, B., *Fuzzy Theory Systems: Techniques and Application, Vol. 2*, Academic Press, chapter Adaptive fuzzy logic control synthesis without a fuzzy rule base, pp. 781–808, 1999.

[4] Kasac, J., Novakovic, B., Majetic, D. & Brezak, D., Stability analysis of fuzzy robot control without fuzzy rule base. *Proceedings of 2003 International Joint Conference on Neural Networks*, Portland, Oregon, 2003.

[5] Ortega, R., Loria, A., Nicklasson, P. & Sira-Ramirez, H., *Passivity-based control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications.* Springer-Verlag: London, 1998.

[6] Arimoto, S., *Control Theory of Nonlinear Mechanical Systems: A Passivity-Based and Circuit-Theoretic Apprach.* Oxford University Press, 1997.

[7] Pacini, P.J. & Kosko, B., Adaptive fuzzy systems for target tracking. *IEE Intelligent Systems Engineering*, **1(1)**, pp. 3–21, 1992.

[8] Arimoto, S., A class of quasi-natural potentials and hyper-stable PID servo-loops for nonlinear robotic systems. *Trans Soc Instrument Contr Engg*, **30(9)**, pp. 1005–1012, 1994.

[9] Kelly, R., Global positioning of robot manipulators via PD control plus a class of nonlinear integral actions. *IEEE Trans on Autom Control*, **43(7)**, pp. 934–938, 1998.

[10] Kelly, R., A tuning procedure for stable PID control of robot manipulators. *Robotica*, **13**, pp. 141–148, 1995.