# PERFORMANCE OPTIMIZATION OF PM BRUSHLESS DC MOTOR DRIVE WITH REFERENCE MODEL AND SIGNAL ADAPTATION CONTROLLER

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*Abstract* **-** Model reference adaptive control (MRAC) in general form and modified signal adaptation algorithms are presented. Model reference adaptive control with modified signal adaptation algorithm has been applied as outer speed controller in a permanent magnet brushless DC motor (PMBDCM) drive. Dynamic simulation results obtained by optimization of main PI controller and outer MRAC with signal adaptation speed controller show significant reduction in the error caused by parameter variations and load torque.

*Keywords* – Adaptive control, adjustable speed drives, brushless drives, control, design.

## 1. INTRODUCTION

The influence of minor parameter changes on a drive dynamic behaviour may be satisfactorily compensated using standard control algorithms, to ensure the required quality and accuracy of the system response. However, the effects of substantial parameter variations can no longer be effectively compensated by standard control algorithms. In such cases the control algorithm should allow for adaptation (adjustment) to changed parameter values in order to ensure approximately the same system dynamic behaviour irrespective of the magnitude of parameter variations.
In electrical drives with significant parameter variations, or pronounced nonlinear characteristics, it is not possible to obtain satisfactory behaviour by applying only standard PI or PID controllers. The variations of drive parameters can be compensated using: a gain scheduling system [1], [6], self-tuning controller [1] and model reference adaptive control (MRAC) [1], [2], [6].
Model reference adaptive control with parameter adaptation [1], [6] contains proportional and integral parts of an adaptation algorithm and it needs more iteration for optimal tuning of adaptation algorithm coefficients and new tuning of controller parameters for changed plant parameters. The main advantage of model reference adaptive control with signal adaptation is that it does not contain integral parts and hence it does not need tuning of controller parameters for changed plant parameters [2], [3].

## 2. MODEL REFERENCE SIGNAL ADAPTATION ALGORITHM

The signal adaptation algorithm generates additional control signal $u_A$ which minimizes the difference between reference model $y_M$ and system output $y$ (Fig. 1). The adaptation signal $u_A$ acts on the system input so that the adaptation mechanism forms an outer control loop, while the adjustable system with main PI controller forms an inner control loop.
A *Single Input Single Output* (SISO) (Fig. 1) linear time invariant system can be described in state space form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t), \tag{1}$$

where: $\mathbf{A}$ – system matrix ($n \times n$),
$\mathbf{b}$ – system input vector ($n \times 1$),
$\mathbf{x}$ – system state space variables vector ($n \times 1$),
$u$ – system control signal (scalar).
Likewise, reference model is described as follows:

$$\dot{\mathbf{x}}_\mathbf{M}(t) = \mathbf{A}_\mathbf{M}\mathbf{x}_\mathbf{M}(t) + \mathbf{b}_\mathbf{M}u_r(t), \tag{2}$$

where: $\mathbf{A_M}$ – reference model matrix ($n \times n$),
$\mathbf{b_M}$ – reference model input vector ($n \times 1$),
$\mathbf{x_M}$ – reference model state space variables vector ($n \times 1$),
$u_r$ – reference signal (scalar).
The control signal (Fig. 1) is given by:

$$u_s(t) = u_r(t) + u_A(t), \tag{3}$$

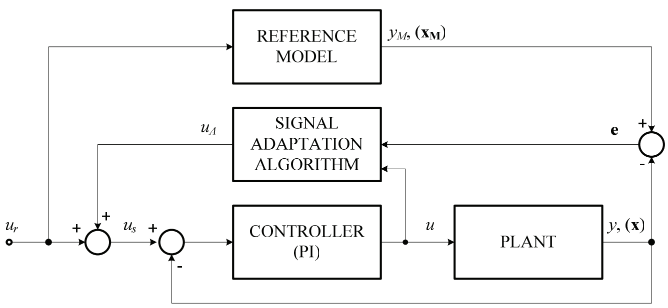


Fig. 1. Adaptive control system with reference model and signal adaptation controllers.

where: $u_A$ – adaptation signal.
Tracking error vector (Fig. 1) equals:

$$\mathbf{e}(t) = \mathbf{x_M}(t) - \mathbf{x}(t). \tag{4}$$

From the mathematical descriptions of a SISO linear time-invariant system (1) and reference model (2) for the reference model, tracking error derivative is obtained [2], [3]:

$$\dot{\mathbf{e}}(t) = \dot{\mathbf{x}}_\mathbf{M}(t) - \dot{\mathbf{x}}(t) = \mathbf{A_M}\mathbf{e}(t) + \boldsymbol{\sigma}(t) + \mathbf{b}u_A(t), \tag{5}$$

where:

$$\boldsymbol{\sigma}(t) = (\mathbf{A_M} - \mathbf{A})\mathbf{x}(t) + (\mathbf{b_M} - \mathbf{b})u_r(t). \tag{6}$$

Vector $\boldsymbol{\sigma}$ is determined by variations of system (plant) parameters from reference model parameters.
Stability of chosen signal adaptation controller is tested using Lyapunov stability criterion. Suitable Lyapunov positive-definite function [2], [3] is given by:

$$V = \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e}, \tag{7}$$

where $\mathbf{P}$ is positive-definite matrix given by:

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}, \tag{8}$$

where $\mathbf{Q}$ is positive-definite matrix.
Lyapunov function (7) derivative equals:

$$\dot{V} = \dot{\mathbf{e}}^T\mathbf{P}\mathbf{e} + \mathbf{e}^T\mathbf{P}\dot{\mathbf{e}}. \tag{9}$$

Replacing $\dot{\mathbf{e}}$ in (9) with (5), follows:

$$\dot{V} = -\mathbf{e}^T\mathbf{Q}\mathbf{e} + 2\mathbf{e}^T\mathbf{P}\boldsymbol{\sigma} - 2\mathbf{e}^T\mathbf{P}\mathbf{b}u_A. \tag{10}$$

It is shown that relation (10) is satisfied with following signal adaptation form [2], [3]:

$$u_A(t) = h \cdot \mathrm{sign}(\nu(t)), \tag{11}$$

$$\nu(t) = \mathbf{d}^T\mathbf{e}(t), \quad \mathbf{d}^T = \mathbf{b}^T\mathbf{P}, \tag{12}$$

where: $h$ is an adaptation coefficient, $\nu(t)$ is a generalized error, $\mathbf{d}^T$ is an error weighting coefficient vector.
Coefficients of matrix $\mathbf{P}$ can be determined by solving Lyapunov equation (8), with given coefficients of matrix $\mathbf{Q}$. Therefore, error weighting coefficient vector $\mathbf{d}^T$ can be determined. However, obtained coefficients may not give best adaptation or smallest transient error and hence they are not optimal. Because of that, error weighting coefficient vector is obtained by optimization using such program packages as *Matlab – Optimization Toolbox* [7], [8].
The signal adaptation algorithm (3) produces a sliding mode of operation [2], [3] and generates a high frequency adaptation signal, which can not be directly applied in electrical drives. To avoid high frequency oscillation in the drive, a sliding mode of operation can be organized in the reference model and/or observer (state variable estimator) [2], [3]. Another way is to replace signum function by a saturation function [2]:

$$u_A(t) = \mathrm{sat}(\nu(t), h) = \\ = \begin{cases} h, & \text{for } \nu(t) > \nu_s \\ K_\nu \nu(t), & \text{for } |\nu(t)| \le \nu_s \\ -h, & \text{for } \nu(t) < -\nu_s \end{cases}, \tag{13}$$

where: $\nu_s$ is a region where the saturation function is linear, $K_\nu$ is the gain coefficient of generalized error $\nu(t)$, $h$ is the value of saturation.
To illustrate the method, signal adaptation algorithm was applied for parameter changes and load torque compensation of a PMBDCM drive and results are discussed in the succeeding section.

## 3. MODEL OF PM BRUSHLESS DC MOTOR DRIVE

This model is based on the PM brushless DC motor drive discussed and given in [6, 7]. For the sake of easy reference, the model is derived in brief and given in the following. During two phase conduction, the entire dc voltage is applied to the two phases and the transfer function for the stator current is given by (Fig. 2):

$$\frac{I_{as}(s)}{V_{is}(s) - E(s)} = \frac{K_a}{1 + T_a s}, \tag{14}$$

where: $K_a = 1/R_a$, $T_a = L_a/R_a$, $R_a = 2R_s$, $L_a = 2(L - M)$, $R_s$ is the stator resistance per phase, $L$ is the self inductance per phase, $M$ is the mutual inductance per phase, $E$ is the induced emf and $s$ is the Laplace operator.
The induced electromagnetic force (emf) $E$ is proportional to rotor speed $\Omega_m$:

$$E = K_b \Omega_m, \tag{15}$$

where:

$$K_b = 2\lambda_p, \tag{16}$$

$\lambda_p$ is the flux linkages per phase (volt/rad/sec).
Note that the electromagnetic torque for two phases combined is given by:

$$T_e = 2\lambda_p I_{as} = K_b I_{as}. \tag{17}$$

The load is assumed to be proportional to speed:

$$T_l = B_l \Omega_m. \tag{18}$$

With that included in the feedback path, the speed to air gap torque transfer function can be evaluated as (Fig. 2):

$$\frac{\Omega_m(s)}{T_e(s)} = \frac{K_t}{1 + T_t s}, \tag{19}$$

where: $K_t = 1/B_t$, $T_t = J/B_t$, $B_t = B_1 + B_2$, where $B_1$ is the friction coefficient of the motor, $B_2$ is proportion coefficient between load torque and speed and $J$ is the inertia of the machine.

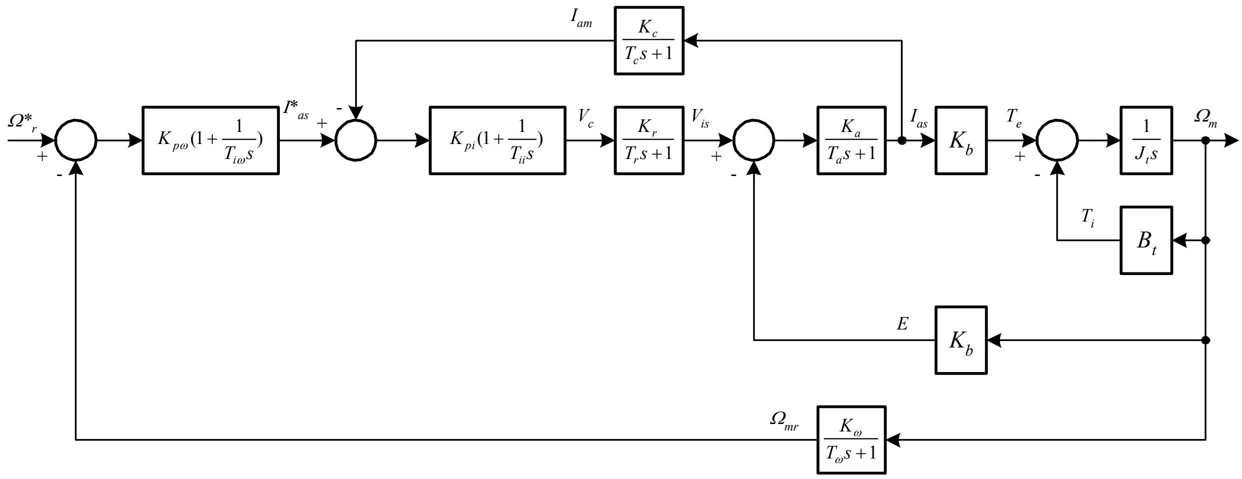


Fig. 2. Block schematic of cascade speed control system of PM brushless DC motor drive.

Transistor chopper transfer function is given by:

$$\frac{V_{is}(s)}{V_c(s)} = \frac{K_r}{1+T_r s}, \tag{20}$$

where:

$$T_r = \frac{T_{ch}}{2} = \frac{1}{2f_{ch}}, \tag{21}$$

$f_{ch}$ is chopper frequency.
The current and speed feedbacks have low pass filters with transfer functions (Fig. 2):

$$\frac{I_{am}(s)}{I_{as}(s)} = \frac{K_c}{1+T_c s}, \tag{22}$$

$$\frac{\Omega_{mr}(s)}{\Omega_m(s)} = \frac{K_\omega}{1+T_\omega s}. \tag{23}$$

Numerical value of the drive parameters are:
Base speed, $n_b$ = 4000 rev/min, Base power, $P_b$ = 373 W, Base current, $I_b$ = 17.35 A, Base voltage, $V_b$ = 40 V, Base torque, $T_b$ = 0.89 Nm, Supply voltage, $V_s$ = 160 V, Maximum phase current, $I_{max} = 2I_b$ = 34.7 A, Maximum torque, $T_{max} = 2T_b$ = 1.78 Nm, Gain of the inverter, $K_r$ = 16 V/V, Time constant of the converter, $T_r$ = 50 µs, Phase resistance, $R_a$ = 1.4 Ω, Phase inductance $L_a$ = 2.44 mH, Phase time constant, $T_a = L_a/R_a$ = 1.743 ms, $K_a = 1/R_a$ = 0.71428 A/V, Emf constant, $K_b$ = 0.051297 Vs, Total friction coefficient, $B_t$ = 0.002125 Nm/rad/sec, Inertia, $J$ = 0.0002 kgm$^2$, $K_t$ = $1/B_t$ = 41.89, Motor and load time constant, $T_t = J/B_t$ = 94.1 ms, Current feedback gain $K_c$ = 0.288 V/A, Current feedback time constant, $T_c$ = 0.159 ms, Speed feedback gain, $K_\omega$ = 0.02387 Vs/rad, Speed feedback time constant, $T_\omega$ = 1 ms.
Integral time constant of the current controller is usually chosen to be equal to the armature time constant (compensates maximum time constant in the current loop): $T_{ii} = T_a$ = 1.743 ms. For the overshoot $M_{pi}$ = 5% current controller gain coefficient determined from the Bode plot and simulation is $K_{pi}$ = 1.267.

## 4. MAIN PI SPEED CONTROLLER DESIGN

Main PI speed controller parameters are determined using transient performance based design optimization of PMBDCM [9].
In case of classic (standard) speed controller design, which compensates maximum time constant of drive, speed controller integral time constant equals: $T_{i\omega} = T_t$ = 94.1 ms. Controller gain coefficient for drive overshoot $M_{p\omega mr}$ = 10% gives $K_{p\omega}$ = 24.8, as seen from Fig. 3. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current $\Delta i_{as}$ on step change of reference value $\Delta\omega^*_r(t) = 0.1S(t)$, with $T_{i\omega} = T_t$ = 94.1 ms and $K_{p\omega}$ = 24.8, are shown on Fig. 4.
For faster and better load torque compensation it is necessary that controller integral time constant be as small as possible and controller gain coefficient as large as possible. Therefore, controller integral time constant is picked as: $T_{i\omega} = 0.125T_t$ = 11.76 ms (Fig. 3) and controller gain coefficient is chosen for speed feedback signal overshoot $M_{p\omega mr}$ = 40%: $K_{p\omega}$ = 44.9 (Fig. 3). To achieve system overshoot $M_{p\omega mr}$ = 10%, first order filter with time constant $T_f$ = 1.96 ms has been added to the drive input.

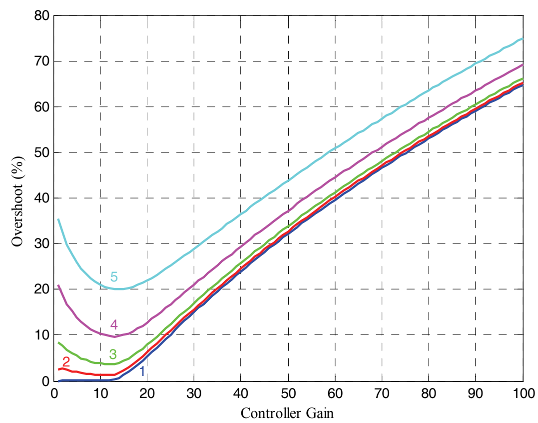


Fig. 3. Dependence of speed feedback signal overshoot $M_{p\omega mr}$ and controller gain coefficient $K_{p\omega}$ for different values of controller integral time constant:
1. $T_{i\omega} = T_t$; 2. $T_{i\omega} = 0.75T_t$; 3. $T_{i\omega} = 0.5T_t$; 4. $T_{i\omega} = 0.25T_t$; 5. $T_{i\omega} = 0.125T_t$.

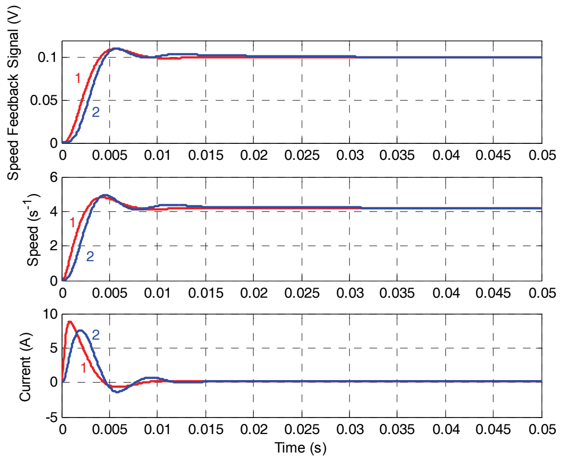


Fig. 4. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current $\Delta i_{as}$ for a change in reference speed $\Delta\omega^*_r(t)$ = 0.1S($t$) with speed controller parameters:
1. $K_{p\omega} = 24.8$, $T_{i\omega} = 94.1$ ms, $T_f = 0$;
2. $K_{p\omega} = 44.9$, $T_{i\omega} = 11.76$ ms, $T_f = 1.96$ ms.

Responses in Fig. 4 (curves 2) show that speed feedback signal $\Delta\omega_{mr}$ and speed $\Delta\omega_m$ have approximately the same time of response at maximum, while maximum value of armature current is a little bit less than in the case of maximum time constant compensation (curves 1).

Responses on Fig. 5 show that influence of load torque on speed is significantly faster (8 times) and better (2 times) compensated in case of controller parameters determined for integral time constant $T_{i\omega} = 11.76$ ms (curves 2) than for integral time constant $T_{i\omega} = 94.1$ ms (curves 1).

## 5. IMPLEMENTATION OF REFERENCE MODEL AND SIGNAL ADAPTATION ALGORITHM

This paper covers the application of modified signal adaptation algorithm (13) with three state space variables and third order reference model. First state space variable is speed feedback signal, while other two are its first and second

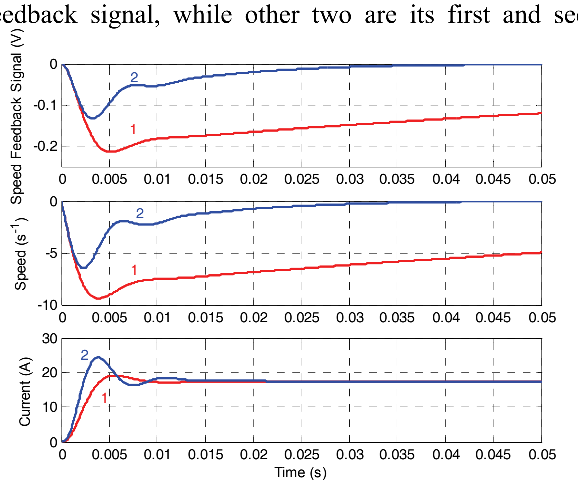


Fig. 5. Responses of speed feedback signal $\Delta\omega_{mr}$, speed $\Delta\omega_m$ and current $\Delta i_{as}$ for a change in nominal load torque $M_t(t)$ = 0.89S($t$) with speed controller parameters:
1. $K_{p\omega} = 24.8$, $T_{i\omega} = 94.1$ ms, $T_f = 0$;
2. $K_{p\omega} = 44.9$, $T_{i\omega} = 11.76$ ms, $T_f = 1.96$ ms.

derivative. Derivations are calculated approximately as follows:

$$G_1(z) = \frac{\dot{\Omega}_{mr}(z)}{\Omega_{mr}(z)} = \frac{z-1}{T_s z}, \qquad (24)$$

$$G_2(z) = \frac{\ddot{\Omega}_{mr}(z)}{\Omega_{mr}(z)} = \frac{z^2-2z+1}{T_s^2 z^2}, \qquad (25)$$

where $T_s$ = 50 µs is sampling time of algorithm.

Reference model is chosen to satisfactorily describe system (plant) behavior with nominal parameters. Its transfer function is given by:

$$G_M(s) = \frac{\Omega_{Mmr}(s)}{U_r(s)} = \frac{1}{(1+T_f s)(1+2\zeta T_n s + T_n^2 s^2)}, \qquad (26)$$

where $\Omega_{Mmr}$ is reference model output, $T_f$ = 1.96 ms is a filter time constant, while $\zeta$ and $T_n$ are determined by optimization and equal: $\zeta = 0.318$, $T_n$ = 1.197 ms.

Discretization of the reference model is made with *zero order hold* (ZOH) blocks in input and output of the reference model. Sample time of ZOH blocks is set to $T_s$ = 50 µs.

Error weighting coefficient vector **d** (size 3×1) is determined by optimization based on *integral square error* (ISE) criterion:

$$I = \int e^2(t)\,dt, \qquad (27)$$

where:

$$e(t) = \omega_{Mmr}(t) - \omega_{mr}(t). \qquad (28)$$

Optimization is carried out on reference step change $\Delta\omega^*_r(t)$ = 0.1S($t$), change of moment inertia to values $J = 0.5J_n$ and $J = 2J_n$, saturation value $h = 0.1$ and gain coefficient $K_v = 1$. Optimization resulted in following:

$$\mathbf{d}^T = \begin{bmatrix} 25 & 0.0059726 & 2.22847\cdot 10^{-6} \end{bmatrix}. \qquad (29)$$

Responses of reference model output, armature current and speed feedback signal error without adaptation on reference step change $\Delta\omega^*_r(t)$ = 0.1S($t$) are shown in Fig. 6. Maximum speed feedback signal error equals $e_m$ = 33.2% for $J = 0.5J_n$ and $e_m$ = 29.7% for $J = 2J_n$.

Responses of speed feedback signal and armature current without adaptation on step change of rated load torque are shown in Fig. 7. Maximum relative speed feedback signal drop equals: $\Delta\omega_{mr}$ = -1.33% for $J = J_n$; $\Delta\omega_{mr}$ = -1.67% for $J = 0.5J_n$; $\Delta\omega_{mr}$ = -1.08% for $J = 2J_n$.

Model reference adaptive controller with modified signal adaptation algorithm (13), optimal error weighting coefficients (29) and sampling time $T_s$ = 50 µs reduces maximum speed feedback signal error to value: $e_m$ = 0.94% for $J = 0.5J_n$ and $e_m$ = 1.83% for $J = 2J_n$ (Fig. 8).

Model reference adaptive controller with modified signal adaptation algorithm (13) and optimal error weighting coefficients (29) reduces maximum speed feedback signal drop to value (Fig. 9): $\Delta\omega_{mr}$ = -0.24% for $J = J_n$; $\Delta\omega_{mr}$ = -0.42% for $J = 0.5J_n$; $\Delta\omega_{mr}$ = -0.14% for $J = 2J_n$.

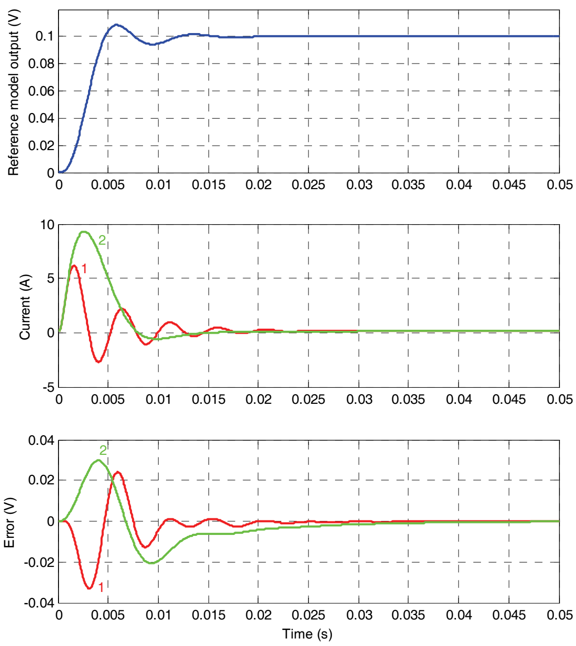


Fig. 6. Reference model output signal $\Delta\omega_{Mmr}$, armature current $\Delta i_{as}$ and error $e$ responses for a step input $\Delta\omega_r^*$=0.1S($t$) and moment of inertia change in the drive without adaptation: $1.\ J = 0.5J_n$, $2.\ J = 2J_n$.

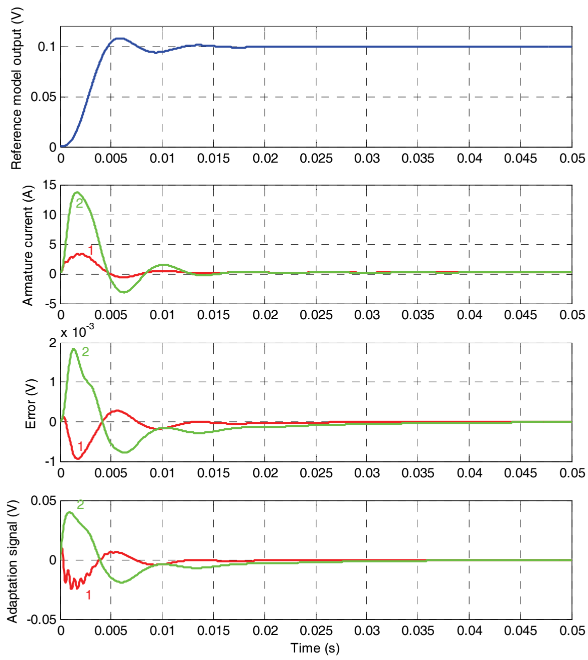


Fig. 8. Reference model output signal $\Delta\omega_{Mmr}$, armature current $\Delta i_{as}$, error $e$ and adaptation signal $u_A$ responses for a step input $\Delta\omega_r^*$=0.1S($t$) and moment of inertia change in the drive with adaptation ($h = \Omega_r = 0.1$, $K_v = 1$): $1.\ J = 0.5J_n$, $2.\ J = 2J_n$.

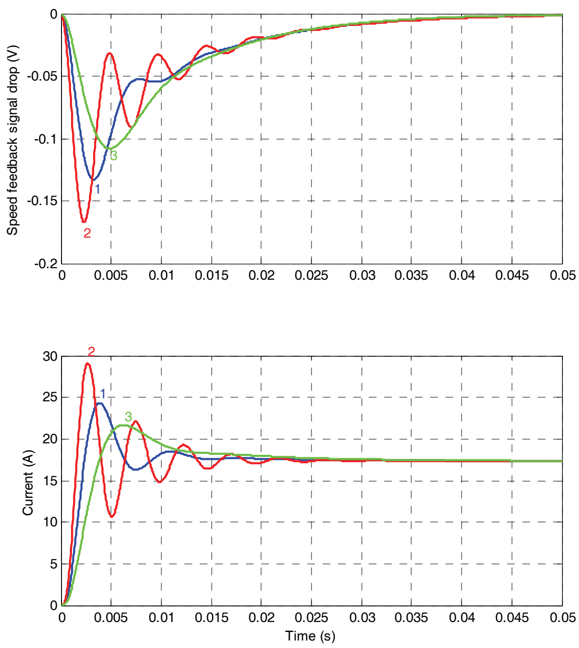


Fig. 7. Speed feedback signal $\Delta\omega_{mr}$ and armature current $\Delta i_{as}$ responses for a step change of the nominal load torque $T_l$= 0.89S($t$) and moment of inertia change in the drive without adaptation: $1.\ J = J_n$, $2.\ J = 0.5J_n$, $3.\ J = 2J_n$.

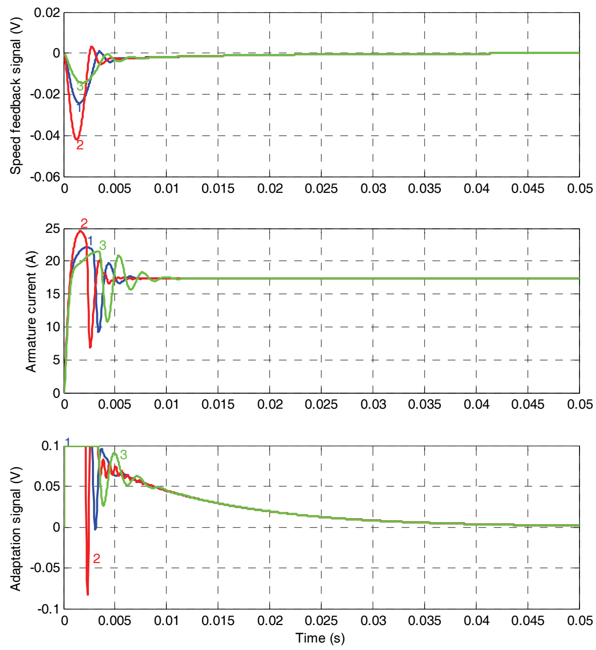


Fig. 9. Speed feedback signal $\Delta\omega_{mr}$, armature current $\Delta i_{as}$ and adaptation signal $u_A$ responses for a step change of the nominal load torque $T_l$= 0.89S($t$) and moment of inertia change in the drive with adaptation ($h = \Omega_r = 0.1$, $K_v = 1$): $1.\ J = J_n$, $2.\ J = 0.5J_n$, $3.\ J = 2J_n$.

## 6. CONCLUSION

The key contributions of the proposed paper are summarized in the following:

(i) It is possible to determine optimal speed controller parameters for faster (4-8 times) and better (2 times) load torque compensation than that obtained using traditional design based on compensation of maximum time constant of system.

(ii) Using a filter at the drive input, the desired speed response overshoot to reference value change is achieved.

(iii) Model reference adaptive control and modified signal adaptation control with optimal coefficients have been applied to minimize the effect of moment of inertia variations on the performance of the PMBDCM drive system. It reduces the error of speed feedback signal from 33.2% (29.7%) to 0.94% (1.83%).

(iv) Model reference adaptive control and modified signal adaptation control with optimal coefficients reduce the influence of load torque on speed feedback signal (4 to 8 times lower speed drop).

The main advantage of model reference adaptive control with signal adaptation is that it does not contain integral parts and hence it does not need tuning of controller parameters for changed plant parameters [2, 3].

Authors are planning to investigate the influence of noise on adaptation algorithm behaviour. Presented choice of state space variables is not suitable because of calculation of derivatives.

Authors are also planning to investigate robustness of optimal main PI controller and MRAC with optimal coefficients of modified signal adaptation algorithm.

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