ABSTRACT
In this paper, applications of theoretical, numerical (FEM) and experimental methods to investigation of fracture of laminated composite plane structures with hole are described. The ultimate loading capacities until fracture for the strips with central holes made from laminated composite material are obtained. Especially attention is given to the stress distribution in the region beside the point of initiated crack on contour of holes in structure. Results of numerical (FEM) and experimental stress analysis (photoelastic method) in plane structures with holes subjected to uniaxial loading are presented in tables and graphically in figures and photos.

Keywords: failure, stress concentration, laminated composite structure

1. INTRODUCTION
The laminated composite materials in many engineering products are used, including aerospace, building structures, autos, rail vehicles, marine, ships, robots, prosthetic devices, sporting equipments etc. [4,6]. Compared to conventional engineering materials, composites can be designed to produce exceptional strength and stiffness with minimum weight [5]. Because of the heterogeneous and anisotropic nature of multilayered composites, the problem of stress analysis those structures with geometrical discontinuities are more complicated. One of great problems to safely designing of structures is stress concentration existing. Usual analytical methods for stress analysis of laminated composites results in having to solve the systems of equations with a large number of unknown constants. As the alternative efficient method to the stress analysis of composite structures, a numerical method as the finite element method and an experimental photoelastic method as birefringent coating method [2] has been developed. These two methods of stress analysis by investigated models with central elliptical holes made from layered composites, in past presented works [1,2,3] were explained. In this paper, for these models the analysis of properties and the stresses by tensile loading until fracture has been continued. Discrepancies among several the failure criteria for anisotropic composites were discussed, too. Several results of provided analyses in figures and photos are shown.

2. THE FAILURE CRITERIA AND THE STRENGTH OF THE MATERIAL MODELS
In practice, various failure criteria for strength of orthotropic composite material were developed, e.g. as the polynomial criteria in contracted notation [5]:

1. Ashkenazi, Fisher and Tsai-Hill criteria: 
   \[ F_{ij} \sigma_i \sigma_j \leq 1, \quad i, j = 1, 2, 6 \]  

2. Covin, Hoffman, Malmeister, Marin, Tsai-Wu criteria: 
   \[ F_i \sigma_i + F_{ij} \sigma_i \sigma_j \leq 1, \quad i, j = 1, 2, 6 \]  

where the strength tensor components \( F_i \) and \( F_{ij} \) are different between itself.

In this paper, the failure criteria in polynomial tensor form for plane case of stresses was used [1,3]:
the Gol’denblat-Kopnov failure criterion: 
\[ \Pi_{ij} \sigma_{ij} + \left( \Pi_{ijk}\sigma_{ijk} \right)^{1/2} \leq 1, \quad i,j,k,m = 1,2, \]  

where the components of strength tensors were calculated from the engineering strength properties of used composite materials [1], Table 1.

Table 1.  The strength values of the materials "A" and "B", in MPa.

<table>
<thead>
<tr>
<th>Strength</th>
<th>Mat. &quot;A&quot;</th>
<th>Mat. &quot;B&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_1^T )</td>
<td>1200</td>
<td>515</td>
</tr>
<tr>
<td>( \sigma_2^T )</td>
<td>31</td>
<td>515</td>
</tr>
<tr>
<td>( \sigma_{12}^{T(45)} )</td>
<td>62</td>
<td>151</td>
</tr>
<tr>
<td>( \sigma_1^C )</td>
<td>620</td>
<td>515</td>
</tr>
<tr>
<td>( \sigma_2^C )</td>
<td>140</td>
<td>515</td>
</tr>
<tr>
<td>( \sigma_{12}^{C(45)} )</td>
<td>252</td>
<td>151</td>
</tr>
<tr>
<td>( \tau_{12}^F )</td>
<td>76</td>
<td>72</td>
</tr>
</tbody>
</table>

The strength values can be determined in simple failure tests: T – tension and C – compression in principal axes \( x_1 \) and \( x_2 \), respectively; tension and compression in axis that is 45° with the principal axes \( (x_1, x_2) \); \( \tau_{12}^F \) – the failure shear strength [1]. For investigated models, two the orthotropic glass fiber reinforced laminated composite materials consisting from the unidirectional layers (E-glass/epoxy composite, \( V_f = 0,55 \)) were used: material "A" - an angle-ply laminate with fiber directions \((\pm 5^\circ/-5^\circ)_{17S}\) in respect to longitudinal axis \( x_1 \); material "B" - a cross-ply laminate \((0^\circ/90^\circ)_{16S}\), Fig. 1. The strength values of the composites were used from the data prospect and for the principal material coordinate system \( (x_1, x_2) \) of laminates are given in Table 1.

Comparison between the applications of three the failure criteria for calculations the tensile strength values of the laminates, in the direction the principal axis \( x_1 \) of the materials "A" and "B", respectively, as functions of angle \( \phi \) (Fig. 1) are shown graphically on diagrams, Fig. 2.a) and b).

Figure 2.  Comparisons of three the failure criteria by calculation of the tensile strength values in the direction the x-axis of the material "A" (Fig. a) and "B" (Fig. b), as functions of angle \( \phi \) (Fig. 1).

3. DIMENSIONS OF MODELS

Two series of models in the form of plane strips with elliptical holes made from the laminated composite materials "A" and "B" loaded uniaxial in tension were, Fig. 3. The major axis of the ellipse makes with transverse axis \( x_2 \) of the models angles \( \gamma = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ \). All the results of the
stress distribution on the contours of holes in models were obtained from the analyses of the photos of patterns taken by investigations in reflexion polariscope. For example, the photo of the stress patterns for model A–4 by load $F=20$ kN in Fig. 4. a) is shown.

![Figure 3. Load and geometry of the investigated models](image)

For validation of the statement to form of fracture of laminated structures, three models (A–2, A–4, B–2) were loaded gradually until fracture of these models. The photos of the stress patterns taken immediately before beginning of the fracture of the model A–4 by tensile load $F=65$ kN in Fig. 4. b) is shown. Load-elongation diagram was recorded by increasing of force and is shown in Fig. 5 for model A–4. The numerical calculations of the stress distribution in all models were provided by FEM method and for an example in Fig. 6 the shear stress distribution $\tau_{xy}$ in the model A–4 is shown.

![Figure 4. The photos of the stress patterns in light fields in reflexion polariscope by tensile loading of the model A–4 ($\gamma=60^\circ$): $F=20$ kN (Fig. a) and $F=65$ kN (Fig. b).](image)

![Figure 5. The load-elongation diagram by tensile loading of the model A–4.](image)

![Figure 6. The shear stress distribution $\tau_{xy}$ around of hole in the model A–4 calculated by FEM.](image)
4. THE FRACTURE ANALYSES OF THE MODELS BY TENSILE LOADING

The results of the stress analyses that were obtained by the experiments and numerical (FEM) were used for the calculation of the stress concentration factors $K_O$ and the reduced stress concentration factors $K_T$ from terms:

$$K_O = \max \left| \frac{\sigma(\theta)}{\sigma_O} \right|, \quad K_T = \max \left| \frac{\sigma(\theta)}{\sigma_O} \cdot \frac{\sigma_T^\phi}{\sigma_O^\phi} \right|,$$

where is $\sigma_O$- the boundary stress and $\sigma(\theta)$- the stress on the contour of the elliptical hole in models. For all models with holes by tensile loading, the factor values have been given analytically and graphically in paper [3]. In table 2, the values of these factors only for the broken models are given. The locations of the critical points on the elliptical contour of the models (angles $\theta_1$ and $\theta_2$) are different then the point of the tensile stress maximum (angle $\theta$). The measured angles of the critical points on the contour of elliptical holes on the broken models are given very good agreement with numerical (FEM) obtained the angles for the points of the maximum shear stresses on the contour of elliptical holes, e.g. for model A–4 in Fig. 6. For this reasons by using orthotropic composite materials must be use the polynomial failure criteria for calculation of the strength values and the reduced stress concentration factors for elements of structures with holes.

Table 2. The values of the stress concentration factors and the locations of the critical point on the contour of the elliptical hole in the model A–4 as function from the angle $\theta$ (Fig. 6).

<table>
<thead>
<tr>
<th>Model</th>
<th>$K_O$ exp.</th>
<th>$K_T$ exp.</th>
<th>$(\theta_1)_K_O$ experim.</th>
<th>$(\theta_1)_K_T$ experim.</th>
<th>$(\theta_2)_K_O$ experim.</th>
<th>$(\theta_2)_K_T$ experim.</th>
<th>$(\theta_1)_K_O$ measured</th>
<th>$(\theta_1)_K_T$ measured</th>
<th>$(\theta_2)_K_O$ measured</th>
<th>$(\theta_2)_K_T$ measured</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A – 2</td>
<td>9,75</td>
<td>26,8</td>
<td>163,0°</td>
<td>150,0°</td>
<td>177,5°</td>
<td>152,0°</td>
<td>178,0°</td>
<td>151,5°</td>
<td>173,0°</td>
<td>126,0°</td>
<td>126,0°</td>
</tr>
<tr>
<td>A – 4</td>
<td>5,62</td>
<td>16,3</td>
<td>154,5°</td>
<td>134,0°</td>
<td>190,0°</td>
<td>140,0°</td>
<td>183,0°</td>
<td>141,0°</td>
<td>183,0°</td>
<td>163,0°</td>
<td>163,0°</td>
</tr>
<tr>
<td>B – 2</td>
<td>8,68</td>
<td>12,6</td>
<td>163,0°</td>
<td>155,5°</td>
<td>172,5°</td>
<td>160,0°</td>
<td>160,0°</td>
<td>154,5°</td>
<td>174,0°</td>
<td>126,0°</td>
<td>126,0°</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

In this paper is described the forms of fracture for elements of structures with holes made from orthotropic materials; the set models "A" from angle-ply laminated composite material and the set models "B" from cross-ply laminated composite material. At the diagrams $F = f(\Delta l)$ linear dependence force-elongation could be noticed from the beginning to the fracture of the model. The fracture beginning points at the elliptical hole in models conform well to the points determined by experimental testing and FEM calculations, which confirms the proposed hypothesis of fracture of the elements made from orthotropic materials. Therefore, engineers by design of laminated composite orthotropic structures with stress concentrations must calculate with the reduced stress concentration factor for specific problem that must solve by design.

6. REFERENCES