Computer 3D Spectral Analysis of Human Movements

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ABSTRACT

In the world of digital human figures moving through the virtual space there is a little mathematical information about dynamic properties. In the aim to overcome this limitations, 3D human body virtual model consisting of 14 segments has been developed and validated against camera motion capturing of the real human model. The dynamic mass properties of this virtual model have been expressed via tensor of inertia, and model motion has been correlated to the space-time transformation of this tensor. As this tensor has 9 components, there is a huge amount of data that should be processed when doing transformations. By using evolution operators and their spectral properties, we were able to express 3D human body movements with metadata that fit into the realm of 2D computer interface and are suitable for the analysis.

Keywords

virtual 3D human, tensor of inertia, operators, spectral analysis.

INTRODUCTION

Biomechanical analysis of the mechanism of human motion is a very complex and demanding task, regarding which the authors have proposed the new approach of computer visualization and scientific analysis of virtual 3D characters. They have developed virtual 3D computer biomechanical models that describe the kinematic and locomotory systems of human being. These models are based on digitalized data and very realistically reflect human static and dynamic characteristic. Based on the cognition of the structure and functions of inner kinematic system of human body, computer kinematic model has been made, which serves as an originating base for the construction and animated analysis of virtual 3D human [1].

Fundamental measures in the movement description are anthropological measures. Practically, in the so called "biological anthropology" we consider only static anthropometry. That means we analyze data obtained only from the linear measures like the distance between two emphasized points on the body. Also in this group of data there are angles of the relative motion of body parts, which we can call kinematical data. Under the term "biomechanical anthropometry" we do not understand only the linear measures like the distances mentioned above. We also include the segmental masses and their distribution with reference to the chosen coordinate system, dynamical moments of inertia reduced to the centers of gravity of each segment, and then reduced to the center of gravity of the whole body. In addition, it is important to know that emphasized points for the same segmental part are not of the same value. In dynamical anthropometry the emphasized points are centers of instantaneous relative rotation inbetween two neighbouring members. In biomechanical anthropometry is also of the particular importance to know time functions of these quantities [2].

BIOMECHANICAL ANTHROPOMETRY

The determination of anthropometric measures for each individual, employing conventional methods, is a complex and time-consuming job. New computerized methods offer fact and accurate determination of all the key body measures. A software application "*ErSABA",* has been developed, which, using the input data on body height, weight and gender, can determine 22 characteristic anthropometric measures [3].

The results of the biomechanical anthropometric analysis have shown that the anthropometrical magnitudes are the functions of the human standing height. From these results it followed the idea to use the harmonic circle for the definitions of the ratios between lengths of the various body segments.

A Greek canon stating that a human standing height is a sum of the eight head lengths has been used in the anthropometric analysis mentioned. If this canon is joined to the harmonic circle, it is possible to construct a net that outlines the boundaries of the human's contour, as shown in the figure 1. In this way a canon of eight head lengths is joined with harmonic circle.

By taking into account these values and positions of the knee, hip, shoulder and elbow joints, it was possible to draw characteristic points A, B, C etc. into a grid from figure 1.

By further drawing of the joining lengths it was possible to draw the auxiliary skeleton of the human, the so-called wired model.

Further step is related to the harmonic analysis of the external segmental body parts mass distribution and their lengths compared to the whole body mass and body height. From this analysis comes out that the segmental masses and segmental lengths for normally developed adult males and females follow harmonic numbers with relatively great accuracy [4].

3D BIOMECHANICAL MODEL

Figure 2 (a) shows real human model by recording its biomechanical movements with SABA*Lab* motion Capture System "*VatoSABA*" for Computerized Movement Analysis. This system digitizes recorded real human movements and transforms it into corresponding virtual 3D character behaviour. Figure 2 (b) shows a section of a computerized virtual human stick biomechanical figure animation in (X,Y,Z) coordinate system, based on the recorded data.

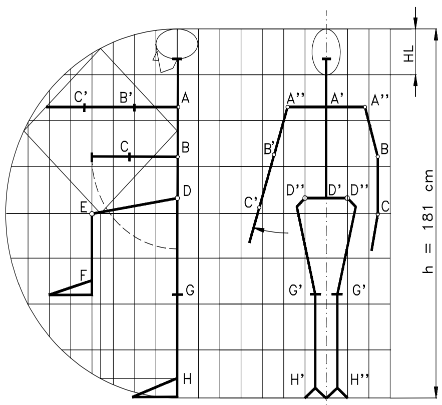
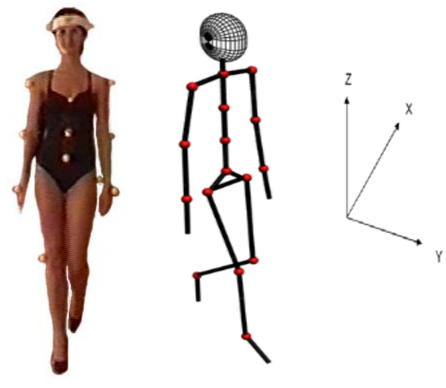


Figure 1: Harmonic circle with a planar model of the human geometrical skeleton



a) b)

Figure 2: Fashion model studio recording with VatoSABA motion capture system (a) and section of a computerized virtual character animation (b)

Video capture of the walking female model has been converted to 31 frames that comprise two steps of her walk. Positions of 14 joints related to the reference coordinate system (X,Y,Z) have been determined for each frame. The joints we analyzed were pelvis, neck, and right and left shoulder, elbow, wrist, hip, knee and ankle. Mathematical body segments have been modeled with geometrical shapes as follows: head with sphere, right and left wrist and foot with parallelepipeds, and torso, right and left forearm, upper arm, lower leg and upper leg with cylinders.

The inertial properties of the virtual body shapes have been determined according to the estimated segmental masses and measurements of the several body magnitudes.

The estimation of the segmental masses has been done according to [5]:

$$m_i = B0_i + B1_i \cdot M + B2_i \cdot h \tag{1}$$

where m_i stands for i-th segment mass, M is a measured whole body mass, h is a measured height of the fashion model, while $B0_i$, $B1_i$ and $B2_i$ represent regression coefficients for a body segments. Segmental densities were as well obtained from [5]: In addition, torso, upper arm, forearm, fist, upper leg, lower leg and foot lengths, as well as the foot and fist width were measured on the real fashion model. From this parameters, head, torso, upper arm, forearm, upper leg and lower leg radius, as well as fist and foot height, were computed. In order to carry out the computation of the tensor of inertia evolution operator, we wrote a program in C and run it under Unix operating system on Hewlett-Packard PA-RISC rp5430 machine. Segmental volumes were computed from their masses and densities. The symbolic view of the computation process is as follows (the numbers in the brackets denote equations used):

- *Begin computation*
- *compute segmental masses from (1)*
- *compute whole body center of gravity*
- *compute joint rotational angles*
- *compute rotational matrices from (11)*
- *compute transposed rotational matrices*

- *compute reduced tensor of inertia matrices for the body segments from* (9)
- *compute transformations of the reduced tensor of inertia matrices from* (10)
- *compute whole body tensor of inertia matrix from* (12)
- *compute whole body tensor of inertia evolution operator from* (20)
- *End computation*

MATHEMATICAL BACKGROUND

The mathematics behind our virtual model stays on two pillars. First is the tensor of inertia and its transformations. Second is the evolution operator of the tensor of inertia and its spectrum.

Tensor of inertia and its transformations

Tensor of inertia ***I*** relates angular momentum ***L*** of rigid body to his angular velocity $\boldsymbol{\omega}$:

$$\boldsymbol{L} = \boldsymbol{I}\,\omega \tag{2}$$

Tensor of inertia of the rigid body with density $\rho(r)$ in a coordinate system with the given axis of rotation through the origin of this system is given by the volume integral:

$$\boldsymbol{I} \equiv \int_V \rho(r) r_\perp^2 dV \tag{3}$$

where r is a distance between an arbitrary point of the rigid body to the origin of coordinate system, $r_\perp$ is perpendicular distance of this point to the axis of rotation, and integration is given over body volume. In a rectangular coordinated system given with x_1, x_2 i x_3 coordinates, and for a discrete mass distribution case m_i, $i = 1,....,N$, equation (3) can be written in a component form [6] as:

$$I_{jk} = \sum_{i=1}^{N} m_i (r_i^2 \delta_{jk} - x_{i,j} x_{i,k}) \tag{4}$$

where indices are defined with $j,k = 1,2,3$, and δ_{jk} is a Kronecker delta, which in its simplest presentation is a discrete form of Dirac delta function, defined as:

$$\delta_{jk} = \left| \begin{array}{l} 1 \rightarrow j = k \\ 0 \rightarrow j \neq k \end{array} \right. \tag{5}$$

In the same coordinate system, and for a continuous mass distribution case, equation (3) in component form can be expressed with the following volume integral:

$$I_{jk} = \int_V \rho(r)(r^2\delta_{jk} - x_j x_k)dV \tag{6}$$

or in the expanded form with coordinates $x_1 = x$, $x_2 = y$, $x_3 = z$:

$$I = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} = \int_V \rho(x,y,z) \begin{bmatrix} y^2 + z^2 & -xy & -xz \\ -xy & z^2 + x^2 & -yz \\ -xz & -yz & x^2 + y^2 \end{bmatrix} dxdydz \tag{7}$$

The elements on the main matrix diagonal are called the moments of inertia, while the rest of the elements are called the products of inertia. Integrals in (7) can be solved if the spatial body mass distribution is a continuous analityc function. Otherwise, numerical integration methods have to be applied. The sub-integral expressions are much simpler for the rigid body with symmetrical mass distribution, with a condition that the origin of the coordinate system coincides with the center of gravity for this body, and it axis with the axis of the body rotation. In that case elements on the main diagonal remain, all other vanish. The main idea is that one can compute main diagonal elements in such a coordinate system, and then apply transformations to the tensor of inertia to describe it in any other coordinate system.

Parallel axis theorem

Let $I_{j,k}$ be the tensor of inertia of some rigid body, computed in the coordinate system with the origin located in the center of gravity for this body. Then the tensor of inertia of this rigid body with mass *m* computed in some other coordinate system with the origin translated from the center of gravity with vector $\boldsymbol{d}^T$ = (d_x, d_y, d_z) is called the tensor of the central inertial moment, and can be described with the following equation:

$$I'_{jk} = I_{jk} + m\left[d^2\delta_{jk} - d_j d_k\right] \tag{8}$$

where d^2 is a square of the vector ***d*** length. In the matrix form, (8) can be written as:

$$I' = \begin{bmatrix} I_{xx} + m(d_y^2 + d_z^2) & I_{xy} - md_x d_y & I_{xz} - md_x d_z \\ -sym- & I_{yy} + m(d_x^2 + d_z^2) & I_{yz} - md_y d_z \\ -sym- & -sym- & I_{zz} + m(d_x^2 + d_y^2) \end{bmatrix} \tag{9}$$

Tensor of inertia rotation

In the rotating coordinate system tensor of inertia can be described with the following equation:

$$\boldsymbol{I'} = \boldsymbol{A}\ \boldsymbol{I}\ \boldsymbol{A}^T \tag{10}$$

Where **I** is the matrix of the tensor of inertia in the local system, ***I'*** is the matrix of the tensor of inertia described in the system that rotates in respect to the local system, ***A*** is the rotational matrix describing transformation from the local to the rotating system, and $\boldsymbol{A}^T$ its transpose. If we describe *X, Y, Z* axis rotation of the local system with orientation angles φ, ϑ and ζ, rotational matrix can be expressed as:

$$A = \begin{bmatrix} c(\theta)c(\psi) & s(\phi)s(\theta)c(\phi)+c(\phi)s(\psi) & -c(\phi)s(\theta)c(\psi)+s(\phi)s(\psi) \\ -c(\theta)s(\psi) & -s(\phi)s(\theta)s(\psi)+c(\phi)c(\psi) & c(\phi)s(\theta)s(\psi)+s(\phi)c(\psi) \\ s(\theta) & -s(\phi)c(\theta) & c(\phi)c(\theta) \end{bmatrix} \tag{11}$$

Where $c(\arg)$ and $s(\arg)$ stand for sine and cosine trigonometric functions of the orientation angles respectivelly.

Multibody system tensor of inertia

In computer human body modeling a system of rigid bodies connected with joints that restrict the relative motion between the bodies is usually used, and is called the multibody system. The tensor of inertia of the multibody system in the point-of-time and relative to the origin of one common coordinate system equals the sum of the tensors of inertia for particular bodies, transformed relative to this origin. Let us define the following variables:

N – number of mutually connected bodies in the multibody system

$I_{i,jk}$ - tensor of inertia for i-th body

$I^{'}_{i,jk}$ - tensor of inertia for i-th body relative to the origin of the common coordinate system

Then the tensor of inertia for the multibody system is defined as:

$$\boldsymbol{I} = \sum_{i=1}^{N} I^{'}_{i,jk} \tag{12}$$

By applying the equation (8) to (12), it follows:

$$\boldsymbol{I} = \sum_{i=1}^{N} \left(I_{i,jk} + m_i \left(d_i^2 \delta_{jk} - d_j d_k \right) \right) \tag{13}$$

The evolution operator of the tensor of inertia

The dynamic of a certain system can be presented in a phase space, a set in which each point can be characterized with its co-ordinates. For example, in a Petri nets tokens become points in this phase space, and moving of tokens is replaced with evolution rule f^t that defines where points move after time t. The trajectories of points are parameterized with time t. If we take that t is an integer number $t \in Z$, time becomes discrete and we have an iterated map. Pair (M, f), in which M represents phase space and $f : M \to M$ is an iterated map is called dynamic system [7]. The dimension of the observed system is equal number of points and we shall refer to points $x \in M$ as x_i, where $i = 1,2,3,...., N$. Because the dynamic is deterministic, it suffices to mark the beginning of the points with ξ_i and represent the moving of points along trajectories in a phase space with $x_i(t) = f^t(\xi_i)$, where $\xi_i = x_i(0)$. If we represent sets of points x_i and ξ_i with vectors $\boldsymbol{x}$ and $\boldsymbol{\xi}$, the moving of the points is:

$$\boldsymbol{x}(t) = F^t(\xi) \tag{14}$$

The magnitude F^t is called the evolution operator [7], and we say that it translates vector ξ into vector $\boldsymbol{x}$. The evolution operator will be a linear operator [8], if:

$$\boldsymbol{x}(t) = F^t \xi \tag{15}$$

For the inertia tensor, one can define evolution operator that describes changes of inertia tensors in time-space coordinate system, while virtual 3D body is moving through it. Tensors of inertia for the particular body segments can be defined relative to the center of gravity for the whole body. During the motion of the whole body, changes in the distance and rotation of the segmental centers of gravity relative to the centers of gravity for the whole body will cause the changes in the inertial tensors of these segments. If we approximate the continuous motion of the centers of gravity for the particular segment with N points in space, we can describe its inertial tensor with the series $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_N$. During motion of the segment, its center of gravity periodically traverses all N points in space. Such a tensor of inertia transformation can be simbollically described with:

$$I_1 \to I_2 \to \ldots \to I_N \to I_1 \tag{16}$$

If we define vector $\boldsymbol{V}_n$ $(n = 1, \ldots, N)$ *as an N* - component vector, whose *n*-th element is inertial tensor matrix $\boldsymbol{I}_n$, and all other elements are square matrices of 3rd order with all elements null, that we denote with *0,* we can write:

$$\boldsymbol{V}_n = [0_1 \quad 0_2 \quad \ldots I_n \ldots\ldots \quad 0_N]^T \tag{17}$$

Further on, let's *B* be the linear operator which applied on vector $\boldsymbol{V}_n$ rotates all of its elements in the right direction, by concurrently transforming its element I_n in the following way:

$$
\begin{aligned}
I_n &\to I_{n+1} \quad ; \quad n \neq N \\
I_n &\to I_1 \quad ; \quad n = N
\end{aligned}
\tag{18}
$$

Action of the operator *B* can be described with the following system of equations:

$$
\begin{aligned}
\boldsymbol{V}_{n+1} &= B\boldsymbol{V}_n \quad ; \; n \neq N \\
\boldsymbol{V}_1 &= B\boldsymbol{V}_N \quad ; \; n = N
\end{aligned}
\tag{19}
$$

It's easy to verify that the matrix of the operator B takes the following form:

$$
B = \begin{bmatrix}
0 & . & . & . & 0 & I_1 / I_N \\
I_2 / I_1 & 0 & . & . & . & 0 \\
0 & I_3 / I_2 & 0 & . & . & . \\
. & 0 & . & . & . & . \\
. & . & 0 & I_{N-1} / I_{N-2} & 0 & . \\
0 & . & . & 0 & I_N / I_{N-1} & 0
\end{bmatrix}
\tag{20}
$$

Operator *B* describes changes of the inertial tensor induced by the motion of the particular body segment relative to the centers of gravity for the whole body, thus representing its evolution operator.

Spectrum of the evolution operator

If a number of observation points is finite, evolution operator can be represented with finite matrix. Let's this matrix be ***L***, and ***E*** unary matrix of the same order, defined as the square matrix with all main diagonal elements equal one, other elements equal null. Eigenvalues λ of the matrix ***L*** [9] can be computed from its characteristic determinant $det(\boldsymbol{L} - \lambda \boldsymbol{E}) = 0$ which expanded gives the n-th order polynomial in λ λ, with coefficients $a_1, \ldots, a_n$:

$$
\lambda^n - \alpha_1 \lambda^{n-1} + \alpha_2 \lambda^{n-2} + \ldots\ldots + (-1)^{n-1} \alpha_{n-1} \lambda + (-1)^n \alpha_n = 0
\tag{21}
$$

This is the algebraic equation of the n-th order in λ, with $m \leq n$ different solutions $\lambda_1, \ldots, \lambda_m$. Set of all eigenvalues of some matrix are called spectrum.

Spectral determinant [7], [10] of matrix ***L*** is defined as $S = det(\boldsymbol{E} - \mu \boldsymbol{L})$, where μ *are* inverse eigenvalues of matrix ***L***, and E is a unary matrix. It is easy to verify that the relationship between m-th eigenvalue and its inverse is given by:

$$
\mu_m = \frac{1}{\lambda_m}
\tag{22}
$$

RESULTS

In order to set up the evolution operator matrix it is necessary to compute the position of the whole body center mass first. The program has generated x, y and z coordinates of the whole body center mass for each frame. While x and y coordinates were very close to zero value, z coordinate has shown the variations during the walk of the fashion model. These variations are shown in the figure 3:

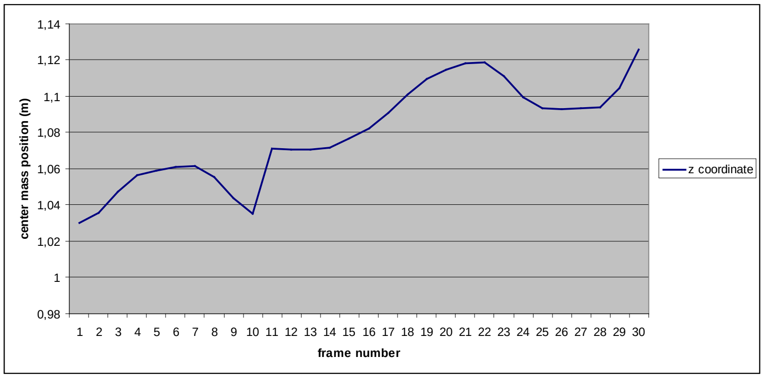


Figure 3: z-coordinate variations of the whole body center mass

According to the equation (9), it was necessary to carry out the computation of the whole body center mass position in order to compute the tensor of the central inertial moment. As this tensor has nine components, for the better reference only moments of inertia are shown in the figure 4:

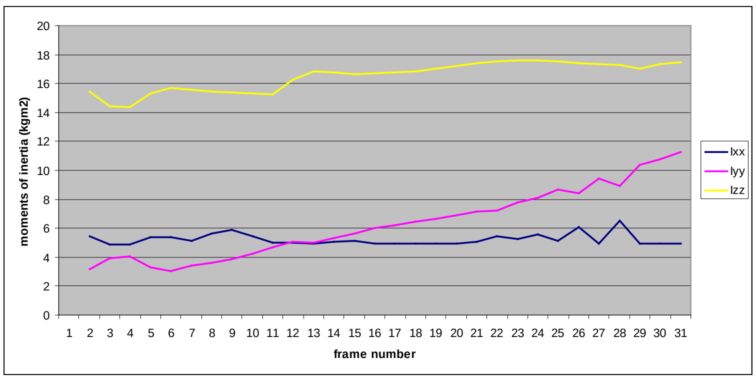


Figure 4: Variations of the whole body moments of inertia

We did a spectrum computation of the tensor of inertia evolution operator by using Scilab 2.7.2 mathematical package that can be downloaded from http://scilabsoft.inria.fr . We run it on personal computer under MS Windows operating system. We carry out the inverse eigenvalues computation by applying equation (22).

The example of the Scilab program which computes the inverse phase of the evolution operator matrix spectrum is shown next:

```
M=fscanfMat(’\directory\spectral_matrix.txt’) / matrix scan
A=spec(M)                                     / spectrum computation
u=file('open', '\directory\eigenvalues.txt)   / file open
c=1
for i=1:90
```

```
c=atan(imag(1/A(i))/real(1/A(i)))            / compute inverse
                                               eigenvalue phase
write(u,c)
end
file('close',u)                              / file close
```

The inverse spectrum of the tensor of inertia evolution operator we got shows the following form on the figure 5:

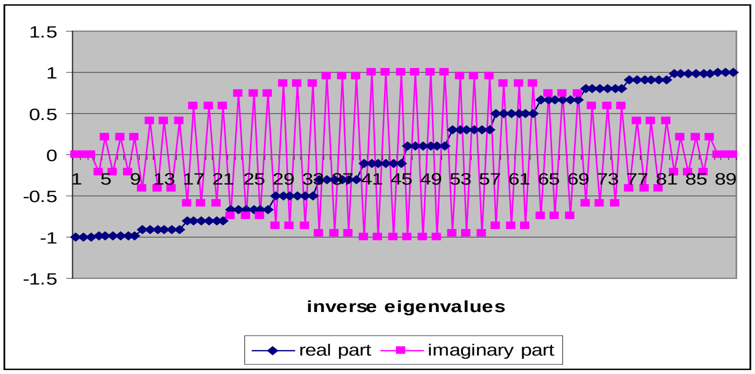


Figure 5: Real and imaginary parts of the evolution operator spectrum

As already observed in [11] for the particular segments, real part of the whole body evolution operator spectrum starts from -1 value, takes 0 value in the middle and ends with +1 value

Let's denote the 1st matrix in the tensor of inertia evolution operator matrix expressed in (20) with $(1,1)$, according to its position (row=1, column=1). Further on, let's denote the rank of the evolution operator matrix with $D = 3d$, where $d = 30$ equals number of frames we have analysed. Now we can take the real part of the evolution operator spectrum under the loop by taking the 1st element to be (m,m), where $m > 1$, and by reducing the rank of the matrix to some value $D = 3d$, where $d < 30$.

For example, if we want to take under the loop frames in the range $[10-20]$, $1st$ element will be $(10,10)$, number of frames to take under the loop will be $d = 10$, and the matrix rank will be $D = 30$. The real part spectrum obtained in such a way has a pretty high-degree of similarity with the real time spectrum shown in figure 6, thus expressing a fractal nature [12] of the spectrum curve.

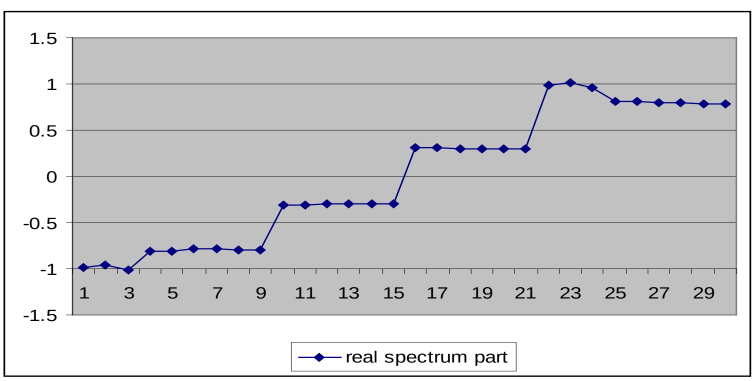


Figure 6: Real part of the spectral curve with 1st element (10,10) and D=30

It has been shown in [11] that the oscillation of the spectrum curve depends on the way the model is walking, thus giving us the possibility to compare various styles of walk for the same model, and similar styles of walk for various models. In addition the possibility of choosing the frame from which we start the spectrum computation gives us the possibility to analyse the various parts of the captured sequence more in depth.

POSSIBLE APPLICATIONS

Because even the same person never walks in the same way, the success of such a comparisons largely depends on the implementation of the self-learning expert system with a large database containing spectral data computed from the captured sequences. For example, if this system is given the task to recognize and authorise person from his usual walking style, e.g when entering a company building, the more spectral data this system has, the greater is its chance to statistically derive the best possible reference spectral curve that reflects the way she or he is walking.

The optimisation of the human locomotion from the spectral curves can be founded on the well developed mathematical field of the semidefinite programming [13], which is oriented towards the optimization of matrices, in contrary to the classical optimization techniques working on the optimisation of the scalar type variables. As the spectrum is by the definition a set of all eigenvalues of the certain matrix, the eigenvalue optimisation techniques [14] based on the semidefinite programming should be applied here. As the semidefinite programming and eigenvalue optimisation algorithms imposse very high demand on the computational hardware, it will be a real challenge to see if the movement optimisation can fit in the standard and cheap computational environment, or much more expensive hardware, e. g..supercomputers, should be used instead.

The tensor of inertia evolution operator contains the data about rotating inertial properties of the virtual model. It is a relatively simple task to include both linear and rotational inertial properties of the model by generalizing this evolution operator to the evolution operator of the so-called mass matrix [15]. By computing the evolution operator matrix of the mass matrix for each segment, it is possible to store the dynamic behaviour of the virtual model in a concise and space efficient way, for the later reconstruction and analysis.

CONCLUSIONS AND FUTURE WORK

We have presented a virtual 3D human biomechanical character of the real human walking model based on the inertial tensor properties and the evolution operator theory. A program that can compute the rotational inertial characteristics of the walking model by computing the tensor of inertia evolution operator matrix has been developed. From this matrix one can compute its spectrum and use it in various application fields; for example in the recognition process and the movement optimization.

We are continuing to develop our virtual model and the next step will be to build a proposed self-learning expert system that with high and acceptable accuracy will be able to decide if the spectral curve computed from the captured frames correlates to the already existing spectral data set in the knowledge database or not.

We will also continue to investigate and implement optimization techniques that hopefully will lead us to our ultimate goal, i.e. optimizing movements in various sport disciplines and rehabilitation medicine, both for humans and animals.

Moreover, we hope this technique will enable the optimization of human body movements via semidefinite optimization of the underlaying tensor of inertia matrices, coupled with expert system using knowledge base.

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