LTO Heuristics for Capacitated Lot Sizing Problem with Sequence Dependent Setups and Overtimes

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Abstract

The wellknown CLSP problem is generalized including sequence dependent setup times and overtimes and modeling it as a quadratic mixed 0-1 integer programming problem called Capacitated Lot Sizing Problem with Sequence Dependent Setups and Overtimes. We develop a heuristics based on Lagrangean relaxation and tabu search for solving the problem. At the end, some computational results are presented.

Key words: capacitated lot sizing problem, two machines, capacity limitations, overtimes, sequence dependent setup times, quadratic mixed 0-1 integer programming problem, heuristics, tabu search, Lagrangean relaxation

1 Introduction

Production planning is an activity that considers the best use of production resources in order to satisfy production goals over a certain period named the planning horizon. Lot sizing decisions give rise to the problem of identifying when and how much of a product to produce such that setup, production and storage costs are minimized. Making the right decision in lot sizing will affect directly the system performance and its productivity, which are important for a manufacturing firm's ability to compete in the market.

In this paper we are generalizing the wellknown deterministic, single-level dynamic lot sizing - the capacitated lot sizing problem (CLSP) in the sense that the setup times are sequence dependent and the overtimes are introduced. Also, the availability of two machines is presented. The objects is to minimize the sum of the costs of production, storage, setup and overtime. A setup may imply two kinds of machine consumption. One is setup cost, expressed in monetary terms; the other is setup time, consuming a certain amount of machine-hours. Having the machine capacities expressed in time units, it is important to have the setup time as small as possible because in this way there is more time available for the production. Having more time for the production, we are more able to satisfy the demands in each of T periods. This problem can be formulated as a quadratic mixed 0-1 integer programming problem called Capacitated Lot Sizing Problem with Sequence Dependent Setups and Overtimes.

Capacitated lot sizing problem has been shown to be NP-hard (Bitran and Yanasse, 1982). Also, when setup times are included in the model, finding a feasible solution to the capacitated lot sizing problem also becomes an NP-complete problem (Garey and Johnson, 1979). Based on these results, it is unlikely that we can develop any effective optimal algorithm for this problem. Therefore, research on developing effective heuristics has been a profitable research area for a long time.

In solving the problem considered in this paper we decided to develop our LTO heuristics based on some relaxations and tabu search method. We are using three kinds of relaxations. The first one is socalled "decomposition" based on the structure of the sequence dependent setup times. Depending on them we are partitioning the items for the production on two available machines. Since the problem is modelled as a quadratic mixed 0-1 integer programming problem where the quadratic term appears in the capacity limitation constraints, the second relaxation is to fix these quadratic terms and overtimes in an appropriate way making the problem simpler. The third relaxation is Lagrangean relaxation where we are dualizing the capacity constraints (including the quadratic terms). The Lagrangean multipliers are choosen in an appropriate way as a result of solving a linear programmning problem defined using the second relaxation mentioned above.

The paper is organized as follows. In the second section we are presenting the mathematical model describing the considered problem. In the third section the relaxations mentioned above are describing. Combining these relaxations we are constructing the LTO heuristics in the fourth section and the computational results are presented. We performed three kinds of numerical simulations. First, we linearized the problem for the small dimensions in order to get the optimal solution. The same problems were solved using only Lagrangean relaxation and after that, applying the LTO heuristics. For the problems of larger dimensions we applied the Lagrangean relaxation and after that, the LTO heuristics. The comparison results are presented in the table considering the quality of the solutions and computational times.

2 Formulation of the problem

In order to formulate the above problem, let us introduce the following notations:

1. Index: i - item type, $i = 1, 2, ..., n$, j - machine type, $j = 1, 2$ and t - planning period, $t = 1, 2, ..., T$.

2. Parameters: d_{it} - the demand for product i in period t, p_{itj} - the unit production cost of product i in period t on machine j, h_{it} - the unit storage cost for product i in period t, k_{itj} - the fixed setup cost for item i in period t on machine j, u_{iltj} - the setup time from item i to item l in period t on machine j, q_{tj} - the overtime cost of machine j in period t, c_{tj} - the capacity of machine j in period t, a_{ij} - the consumption of machine j per unit of item i and O_{tj} - the maximum overtime of machine j in period t.

3. Variables: x_{itj} - the amount of item i produced in period t on machine j, s_{it} - the inventory (stock) of item i in period t, o_{tj} - the overtime of machine j in period t

and

$$y_{itj} = \begin{cases} 1, & \text{if machine } j \text{ is set up for item } i \text{ in period } t \\ 0, & \text{otherwise} \end{cases}$$

The quadratic mixed 0-1 integer programming formulation called (P) is

$$minf(x, y, s, o) = \sum_{i=1}^{n} \sum_{t=1}^{T} (\sum_{j=1}^{2} p_{itj} x_{itj} + h_{it} s_{it} + \sum_{j=1}^{2} k_{itj} y_{itj}) + \sum_{t=1}^{T} \sum_{j=1}^{2} q_{tj} o_{tj}$$

$$s_{i,t-1} + \sum_{j=1}^{2} x_{itj} = d_{it} + s_{it} \quad \forall i, t \tag{1}$$

$$\sum_{i=1}^{n} \sum_{l=1}^{n} u_{iltj} y_{i(t-1)j} y_{ltj} + \sum_{i=1}^{n} a_{ij} x_{itj} \leq c_{tj} + o_{tj} \quad \forall t, j \tag{2}$$

$$x_{itj} \leq M_{itj} y_{itj} \quad \forall i, t, j \tag{3}$$

$$\sum_{i=1}^{n} y_{itj} \leq 1, \quad \forall t, j \tag{4}$$

$$o_{tj} \leq O_{tj}, \quad \forall t, j \tag{5}$$

$$x_{itj}, s_{it}, o_{tj} \geq 0, \;\; y_{itj}, \in \{0, 1\} \tag{6}$$

where M_{itj} is the upper bound on the production capacities, $M_{itj} = \frac{c_{tj}+O_{tj}}{a_{ij}}$. The constraints (1) represent the flow conservation constraints for each item in each period. Also, the constraints (2) describe the capacity limitations and overtime decision for each machine in each period. The constraints (4) refer to a single mode of production.

For small dimensions we linearized the model and applied the branch and bound method offered by Cplex. But in this way the number of variables and constraints increased a lot. In order to solve the problem for larger dimensions we introduce the heuristics based on tabu search and the Lagrangean relaxation.

3 Relaxations

In order to solve the NP-hard problem defined in the previous section, we propose three relaxations.

The first relaxation: decomposition $(RP1)$

The motivation for the generation of the heuristics starting points is as follows. We introduced the 0-1 variables

$$z_{ij} = \begin{cases} 1, & \text{if item } i \text{ is produced on machine } j \\ 0, & \text{otherwise} \end{cases}$$

and constraints $y_{itj} \leq z_{ij}, \quad \forall i, t, j$. If $z_{ij} = 1$, $\forall i, j$ (every item can be produced on every machine), the dimension of the problem is very large. The feasibility set is as large as possible. But, by fixing some z_{ij} to 0, we decrease the number of variables and constraints. Let $\mathcal{N} = \{1, ..., n\}$. We assign the items to the machines in order to allow their productions on them. Let $\mathcal{N}_1$ be the set of all items that are going to be produced on machine 1, let $\mathcal{N}_2$ be the set of all items that are going to be produced on machine 2, and let $\mathcal{N}_0$ be auxiliary set. We allow an item to be produced on both machines as well. Because of machine deterioration, without a loss of generality we can assume that setup times u_{iltj} are proportional by some coefficient of proportionality $b > 1$, i.e. $u_{il,t+1,j} = bu_{iltj}$. Therefore we fix time to be $t = 1$. Then we fix the machine. If for the sum of the setup times on the first machine for a pair of items i and l the inequality $u_{il11} + u_{li11} \leq k$ holds for a certain value of k, then we allow both items i and l to be produced on the first machine, i.e. $i, l \in \mathcal{N}_1$. We do the same for the second machine, i.e. if $u_{il12} + u_{li12} \leq k$, then we allow items i and l to be produced on the second machine, $i, l \in \mathcal{N}_2$. If there exists an item i not assigned to any machine, we put it to set $\mathcal{N}_0 = \{i \in \mathcal{N} : i \notin \mathcal{N}_1 \bigcup \mathcal{N}_2\}$. After we've done this, the items from the set $\mathcal{N}_0$ are allowed to be produced on the machines, randomly. Now the dimension of the problem (P) is reduced significantly.

Using the notations $\mathcal{N} = \{1, ..., n\}$, $\mathcal{N}_j = \{i \in \mathcal{N} : z_{ij} = 1\}, j = 1, 2$, the indexing $i = 1, ..., n; j = 1, 2$ in every constraint is substituted by $i \in \mathcal{N}_j, j = 1, 2$ resulting in the new problem $(RP1)$.

The second relaxation: fixing F_{tj} and o_{tj} $(RP2)$

Let us consider the quadratic constraints (2). In order to eliminate the quadratic term $F_{tj}(y) = \sum_i \sum_l u_{iltj} y_{i(t-1)j} y_{ltj}$ in the capacity constraints $g_{tj}(x, y, o) = F_{tj}(y) + \sum_i a_{ij} x_{itj} - c_{tj} - o_{tj} \leq 0$, we will fix $F_{tj}(y)$ randomly (using the uniform distribution offered by Cplex) to some value F^r_{tj} from the interval $[0, maxF_{tj}]$, where $maxF_{tj} = max\{u_{iltj} : i, l \in \mathcal{N}_j\}, j = 1, 2; t = 1, ..., T$. Also, we will fix the term o_{tj} to some value o^r_{tj} from the interval $[0, O_{tj}]$, where O_{tj} is the maximum overtime of machine j in period t. In this way we construct the problem $(RP2)$ which has the same constraints as the problem (P), but the constraints (2) are substituted with the constraints obtained in the following way:

$$\sum_i a_{ij} x_{itj} \leq c_{tj} - F^r_{tj} + O^r_{tj}, \quad j = 1, 2; \quad t = 1, ..., T \tag{7}$$

Let $c_{tj} - F^r_{tj} + o^r_{tj} = b^r_{tj}$. Now we can substitute the constraints (2) by $a_{ij} x_{itj} \leq b^r_{tj} y_{itj}$ $\forall i, t, j$ and reduce the constraints (3). Fixing F_{tj} and o_{tj} and solving the resulting capacitated lot sizing problem without setup times and overtimes, $(RP2)$, R times will give us the sequence of optimal solutions (x^r, y^r, s^r, o^r), $r = 1, ..., R$. These optimal solutions may satisfy all the constraints of (P), except maybe the constraints (2). If some of the solutions (x^r, y^r, s^r, o^r) is feasible for (P) we have an upper bound on the optimal objective function value f_{min}. Having more than one upper bound, we take the minimal one.

The third relaxation: Lagrangean relaxation (LR)

In order to obtain the Lagrangean relaxation, we do as follows. Let us dualize the constraints (2) of the problem (P) obtaining the problem (LR):

$$v(\lambda) = \{min f(x,y,s,o) + \sum_{t,j} \lambda_{tj} g_{tj}(x,y,o)\}$$

subject to (1), (3), (4), (5) and (6) of the problem (P). The problem $max\{v(\lambda) : \lambda \geq 0\}$ is the Lagrangean dual of (P) relative to the complicating constraints $g_{tj}(x,y,o) \leq 0$, $\forall t,j$.

How to select an appropriate value for the λ? Having the sequence of optimal solutions for $(RP2)$, (x^r, y^r, s^r, o^r), $r = 1,...,R$, let us solve the problem $(\overline{D})$

$$max \;\; w$$

$$w \leq f(x^r, y^r, s^r, o^r) + \sum_{t,j} \lambda_{tj} g_{tj}(x^r, y^r, o^r), r = 1,...,R \tag{8}$$

$$\lambda_{tj} \geq 0, \quad t = 1,...,T, \; j = 1,2$$

The optimal solution $\lambda = (\lambda_{tj}),\ t = 1,...,T,\ j = 1,2$ of the $(\overline{D})$ represents the initial Lagrangean multipliers for (LR). Let us note the optimal solution of (LR) by $(x(\lambda), y(\lambda), s(\lambda), o(\lambda))$. Now, we have the lower bound for the optimal objective function value of the problem (P) as follows:

$$f(x(\lambda), y(\lambda), s(\lambda), o(\lambda)) + \sum_{t,j} \lambda_{tj} g_{tj}(x(\lambda), y(\lambda), o(\lambda)) \leq f_{min}$$

Let $(x(\lambda), y(\lambda), s(\lambda), o(\lambda)) = (x^{R+1}, y^{R+1}, s^{R+1}, o^{R+1})$ and solve the problem $(\overline{D})$ for $r = 1,...,R+1$. We applied the constraint generation method also called cutting plane method (CP). One substantial advantage of (CP) over subgradient algorithms for (LR) is the existence of a true termination criterion incorporated in the algorithm.

4 LTO Heuristics and Computational Results

In order to define finaly the tabu search heuristics, we have to construct the neighborhood. For each point of the tabu search heuristics we construct exactly four neighborhood points. First we find the pair of items i and l

allowed to be produced on the first machine for which the setup time u_{ilt1} is maximum. Then we construct the first neighborhood point by leaving item l to be produced on the first machine and sending item i to be produced on the second machine, and second neighborhood point by leaving item i to be produced on the first machine and sending item l to be produced on the second machine. The other two neighborhood points are obtained by finding the pair of items allowed to be produced on the second machine for which the setup time u_{ilt2} is maximum, and by sending either item i or l to be produced on the first machine.

The heuristics is implemented in the AMPL programming language and uses CPLEX. All computations were performed on a PC having Pentium IV 2.4 GHz processor and 1Gb RAM.

Each problem was solved in three ways: by solving the linearized model using standard CPLEX branch-and-bound procedure, by using Lagrange relaxation where the variables z_{ij} are fixed only in order to get the initial solution of the subproblem $(RP2)$ and, at the end, by using the tabu search heuristics (where the Lagrangean relaxation is used to solve the subproblems appearing in the heuristics. Because of this, while performing the tabu search heuristics we put the upper bound on the Lagrangean relaxation execution time).

The next table shows the comparison of the results in terms of computational time, where Time-Imp LL denotes the average improvement (in %) of the computational time of Lagrange relaxations compared to the linearized problem, and Time-Imp TL denotes the average improvement of the computational time (in %) of the tabu search heuristics compared to the linearized problem

	Average time (in seconds)				
Problem Class (T,n)	Linearized model	Lagrange relaxation	Tabu search heuristics	Time-Imp LL (in %)	Time-Imp TL (in %)
$(10, 5)$	2.8	2.4	11.9	14.29	–
$(15, 5)$	4.8	2.8	16.8	41.67	–
$(15, 8)$	4301.9	265.7	354.3	93.82	–
$(20, 5)$	8.4	4.2	21.1	50	–
$(20, 10)$	110425.1	> 130000	483.5	–	99.56
$(20, 15)$	> 130000	> 130000	1523.4	–	> 98

Table 1. Comparison of results in terms of computational time

Table 2 shows the comparison of the results in the terms of the quality of solutions, where GAP-LL represents the average difference (expressed in percents) between the objective function value obtained by Lagrange relaxation and obtained by solving the linearized model, and GAP-TL represents the average difference (expressed in percents) between the objective function value obtained by tabu search heuristics and obtained by solving the linearized model.

Problem Class (T,n)	**GAP-LL (in %)**	**GAP-TL (in %)**
$(10, 5)$	-0.80	0.85
$(15, 5)$	-0.22	0.80
$(15, 8)$	-0.04	$1, 05$
$(20, 5)$	-0.62	0.84
$(20, 10)$	$-$	0.91
$(20, 15)$	$-$	$-$

Table 2. Comparison of results in terms of quality of solution

In terms of computational time, for smaller dimensions of the problem tabu search heuristics is not as effective as Lagrange relaxation method, or even linearized model. However, for higher dimensions of the problem it is by far the most effective method in terms of computational time, producing solutions within 1% gap and significantly reducing computational time.

References

[1] G.R. Bitran, H.H. Yanasse, Computational complexity of the capacitated lot size problem, *Management Science* 28 (1982) 1174-1186

[2] M. Garey, D. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, *Freeman, San Francisco, CA* (1979)

[3] B. Karimi, S.M.T. Fatemi Ghomi, J.M. Wilson, The capacitated lot sizing problem: a review of models and algorithms, *Omega, The International Journal of Management Science* 31 (2003) 365-378

[4] J. Maes, J.O. McClain, L.N. Van Wassenhove, Multilevel capacitated lot-sizing complexity and LP-based heuristics, *European Journal of Operational Research* 53 (1991) 131-148