NUMERICAL MODELLING OF THE CRACK PROPAGATION PATH AT GEAR TOOTH ROOT

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ABSTRACT

A computational model for determination of service life of gears in regard to bending fatigue in a gear tooth root is presented. The fatigue process leading to tooth breakage is divided into crack initiation and crack propagation period. The Coffin-Manson relationship is used to determine the number of stress cycles N_i required for the fatigue crack initiation, where it is assumed that the initial crack is located at the point of the largest stresses in a gear tooth root. The simply Paris equation is then used for the further simulation of the fatigue crack growth, where required material parameters have been determined previously by the appropriate test specimens. The functional relationship between the stress intensity factor and crack length K=f(a), which is needed for determination of the required number of loading cycles N_p for a crack propagation from the initial to the critical length, is obtained numerically in the framework of the Finite Element Method. The total number of stress cycles N for the final failure to occur is then a sum $N = N_i + N_p$. Although some influences (non-homogeneous material, travelling of dislocations, etc.) were not taken into account in the computational simulations, the presented model seems to be very suitable for determination of service life of gears because numerical procedures used here are much faster and cheaper if compared with the experimental testing.

INTRODUCTION

Two kinds of teeth damage can occur on gears under repeated loading due to fatigue; namely the pitting of gear teeth flanks and tooth breakage in the tooth root (ISO 6336, 1993). In this paper only the tooth breakage is addressed and the developed computational model is used for calculation of tooth bending strength., *i.e.* the service life of gear tooth root.

Several classical standardised procedures (DIN, AGMA, ISO, *etc.*) can be used for the approximate determination of load capacity of gear tooth root. They are commonly based on the comparison of the maximum tooth-root stress with the permissible bending stress (ISO 6336, 1993). Their determination depends on a number of different coefficients that allow for proper consideration of real working conditions (additional internal and external dynamic forces, contact area of engaging gears, gear's material, surface roughness, *etc.*). The classical procedures are exclusively based on the experimental testing of the reference gears and they consider only the final stage of the fatigue process in the gear tooth root, *i.e.* the occurrence of final failure.

However, the complete process of fatigue failure of mechanical elements may be divided into the following stages (Shang et al., 1998, Glodez et al., 1997, Glodez et al., 1997, Cheng et al., 1994): (1) microcrack nucleation; (2) short crack growth; (3) long crack growth; and (4) occurrence of final failure. In engineering applications the first two stages are usually termed as "crack initiation period", while long crack growth is termed as "crack propagation period". An exact definition of the transition from initiation to propagation period is usually not possible. However, the crack initiation period generally account for most of the service life, especially in high-cycle fatigue, see Fig. 1. The total number of stress cycles N can than be determined from the number of stress cycles N_i required for the fatigue crack initiation and the number of stress cycles N_p required for a crack to propagate from the initial to the critical crack length, when the final failure can be expected to occur:

$$N = N_i + N_p \tag{1}$$

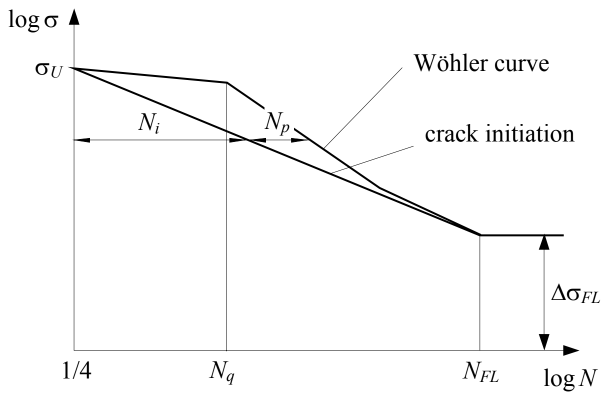

Figure 1: Schematic representation of the service life of mechanical elements

CRACK INITIATION ASSESSMENT

Presented model for the fatigue crack initiation is based on the continuum mechanics approach, were it is assumed that the material is homogeneous and isotropic, i.e. without imperfections or damages. Methods for the fatigue analyses are in that case usually based on the Coffin-Manson relation between deformations (ε), stresses (σ) and number of cycles (N_i), which can be described as follows (Manson, 1953, Tavernelli and Coffin, 1959):

$$\Delta\varepsilon = \Delta\varepsilon_{el} + \Delta\varepsilon_{pl} = \frac{\sigma'_f}{E} N_i^b + \varepsilon'_f N_i^c \qquad (2)$$

where $\Delta\varepsilon$ is the strain range, $\Delta\varepsilon_{el}$ and $\Delta\varepsilon_{pl}$ are the elastic and plastic strain range, E is the Young's modulus of the material and σ'_f, ε'_f, b and c are the strength coefficient, ductility coefficient, strength exponent and ductility exponent for crack initiation, respectively. The strain range can be obtained numerically (usually by FEM), or by strain gauges measurings in the area of tooth root, where the crack initiation is expected. The material constants σ'_f, ε'_f, b and c are obtained for each material and stress/strain ratio, from strain controlled tests.

In the HCF region commonly applicated for gears, where the plastic strain can be neglected, the Coffin-Manson relation reduces only to elastic part and so transforms to an equation of the Basquin type (Nicholas and Zuiker, 1996, Jelaska, 2000):

$$(\Delta\sigma)^{k_i} \cdot N_i = C_i \qquad (3)$$

where $\Delta\sigma$ is the applied stress range and k_i and C_i are the material constants. It is easy to obtain the crack initiation life N_i using this relation, if we assume that the crack initiation curve passes the same point (N_{FL}; $\Delta\sigma_{FL}$) as the Wöhler (S-N) curve, it means at the fatigue limit level the whole fatigue life consists of the crack initiation period:

$$N_i = N_{FL} \cdot \left(\frac{\Delta\sigma_{FL}}{\Delta\sigma}\right)^{k_i} \qquad (4)$$

where N_{FL} is the number of cycles at the knee of the Wöhler curve, see Fig. 1. On the basis of the same assumption, the exponent k_i can be obtained as:

$$k_i = \frac{\log(4N_{FL})}{\log(\sigma_U / \Delta\sigma_{FL})} \qquad (5)$$

where σ_U is the ultimate strength, see Fig. 1. This relation was found to be in a good correlation with available experimental results (Jelaska, 2000).

The most important parameter when determining the crack initiation life N_i according to equation (4) is the fatigue limit $\Delta\sigma_{FL}$, which is a typical material parameter and is determined using appropriate test specimen. When determining the fatigue limit for gears, the reference test gears are usually used as the test specimens. According to ISO 6335 standard (1993), they are spur gears with normal module m_n=3 to 5 mm, tooth width B= 10 to 50 mm, surface roughness $R_z\approx10$ μm, etc, which are loaded with repeated pulsating tooth loading. If geometry, surface roughness, gear size and loading conditions of real gears in the praxis deviate from the reference testing, the previously determined fatigue limit $\Delta\sigma_{FL}$ must be modified through the appropriate correlation factors.

FATIGUE CRACK PROPAGATION

The application of LEFM to fatigue is based upon the assumption that the fatigue crack growth rate, da/dN, is a function of the stress intensity range $\Delta K = K_{max} - K_{min}$, where a is a crack length and N is a number of load cycles. In this study the simply Paris equation is used to describe the crack growth rate (Ewalds, 1989).

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C[\Delta K(a)]^m \qquad (6)$$

where C and m are the material parameters. In respect to the crack propagation period N_p according to Eq. (1), and with integration of Eq. (6) one can obtain:

$$\int_0^{N_p} \mathrm{d}N = N_p = \frac{1}{C} \cdot \int_{a_o}^{a_c} \frac{\mathrm{d}a}{[\Delta K(a)]^m} \qquad (7)$$

This equation indicates that the required number of loading cycles N_p for a crack to propagate from the initial length a_o to the critical crack length a_c can be explicitly determined, if C, m and $\Delta K(a)$ are known. C and m are material parameters and can be obtained experimentally, usually by means of a three point bending test as to the standard procedure ASTM E 399-80. For simple cases the dependence between the stress intensity factor and the crack length K=f(a) can be determined using the methodology given in (Ewalds and Wanhill, 1989, ASTM E 399-80). For more complicated geometry and loading cases it is necessary to use alternative methods. In this work the Finite Element Method in the framework of the programme package FRANC2D has been used for simulation of the fatigue crack growth. The determination of the stress intensity factor is based on the displacement correlation method using singular quarter-point elements, Fig. 2. The stress intensity factor in mixed mode plane strain condition can then be determined as:

$$
\begin{aligned}
K_I &= \frac{2G}{(3-4\nu)+1} \cdot \sqrt{\frac{\pi}{2L}} \cdot \left[4v_d - v_e - 4v_b + v_c\right] \\
K_{II} &= \frac{2G}{(3-4\nu)+1} \cdot \sqrt{\frac{\pi}{2L}} \cdot \left[4u_d - u_e - 4u_b + u_c\right]
\end{aligned}
\tag{8}
$$

where G is the shear modulus of the material, ν is the Poisson ratio, L is the finite element length on crack face, u and v are displacements of the crack tip elements. The combined stress intensity factor is then:

$$K = \sqrt{\left(K_I^{\,2} + K_{II}^{\,2}\right) \cdot (1-\nu^2)} \tag{9}$$

The computational procedure is based on incremental crack extensions, where the size of the crack increment is prescribed in advance. In order to predict the crack extension angle the maximum tensile stress criterion (MTS) is used. In this criterion it is proposed that crack propagates from the crack tip in a radial direction in the plane perpendicular to the direction of greatest tension (maximum tangential tensile stress). The predicted crack propagation angle can be calculated by, see Fig. 2:

$$\theta_0 = 2\tan^{-1}\left[\frac{1}{4} \cdot \frac{K_I}{K_{II}} \pm \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right] \tag{10}$$

A new local remeshing around the new crack tip is then required. The procedure is repeated until the stress intensity factor reaches the critical value K_c, when the complete tooth fracture is expected. Following the above procedure, one can numerically determine the functional relationship K=f(a).

PRACTICAL EXAMPLE

The presented model has been used for the computational determination of the service life of real spur gear with complete data set given in Table 1. The gear is made of high strength

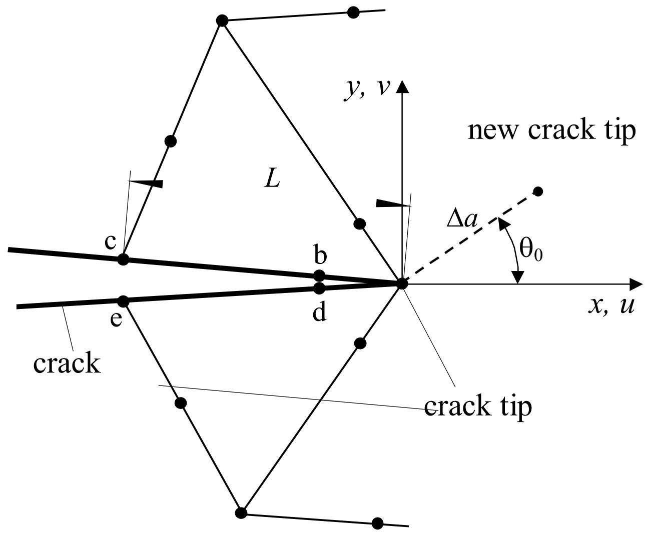

Figure 2: Triangular quarter-point elements around crack tip

alloy steel 42CrMo4 (0.43 %C, 0.22 %Si, 0.59 %Mn, 1.04 %Cr, 0.17 %Mo) with Young's modulus E=2.1·10^5 MPa and Poison's ratio ν=0.3. The gear material is thermally treated as follows: flame heated at 810 °C; 2 min, hardened in oil; 3 min and tempered at 180 °C; 2 h.

Table 1. Basic data of a treated spur gear

module	m_n = 4.5 mm
number of teeth	z = 39
pressure angle on pitch circle	α_n = 24°
coefficient of profile displacement	x = 0.06
tooth width	B = 28 mm
gear material	42CrMo4
surface roughness	R_z = 10 μm

Fatigue crack initiation

The procedure as described before has been used to determine the number of stress cycles N_i required for the fatigue crack initiation. The ultimate tensile strength σ_u=1100 MPa, fatigue limit $\Delta\sigma_{FL}$=550 MPa and number of cycles at the knee of the Wöhler curve N_{FL}=3·10^6 have been taken from ISO 6335, Niemann and Winter (1983) and Abersek (1993) for the same material as used in this study. The computational analysis have been done for different values of normal pulsating force F, which is acting at the outer point of single tooth contact, see Fig. 3. As a consequence of F the maximum principal stress $\Delta\sigma$ in a gear tooth root has been determined numerically with the Finite Element Method, where the FE-model shown in Fig. 3 has been used. The results are summarised in Table 2.

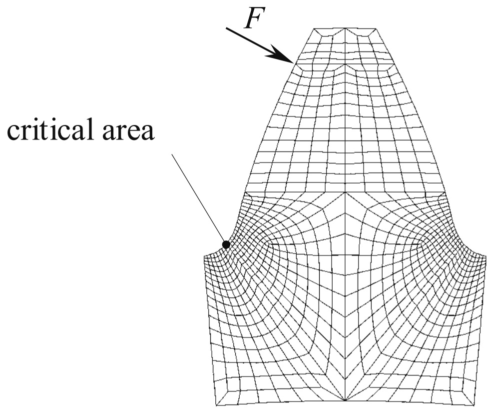

Figure 3: Finite element model

Fatigue crack propagation

The FEM-programme package FRANC2D as described before, has been used for the numerical simulation of the fatigue crack growth. The initial crack has been located perpendicularly to the surface at the point of the maximum principal stress on the tensile side of gear tooth, see Fig. 4.

Table 2. Computational results for the fatigue crack initiation

Loading F [N/mm]	Maximum principal stress in a gear tooth root σ [MPa]	Number of cycles N_i
800	527	$8{,}192 \cdot 10^{6}$
900	593	$5{,}109 \cdot 10^{5}$
1000	659	$4{,}271 \cdot 10^{4}$
1100	725	$4{,}526 \cdot 10^{3}$
1200	790	$6{,}010 \cdot 10^{2}$
1300	857	$8{,}861 \cdot 10^{1}$
1400	922	$1{,}588 \cdot 10^{1}$
1500	988	$3{,}087 \cdot 10^{0}$

In numerical computations it has been assumed that the initial crack a_o corresponds to the threshold crack length a_{th}, below which LEFM is not valid. The threshold crack length may be estimated approximately (Bhattacharya and Ellingwood, 1998) as

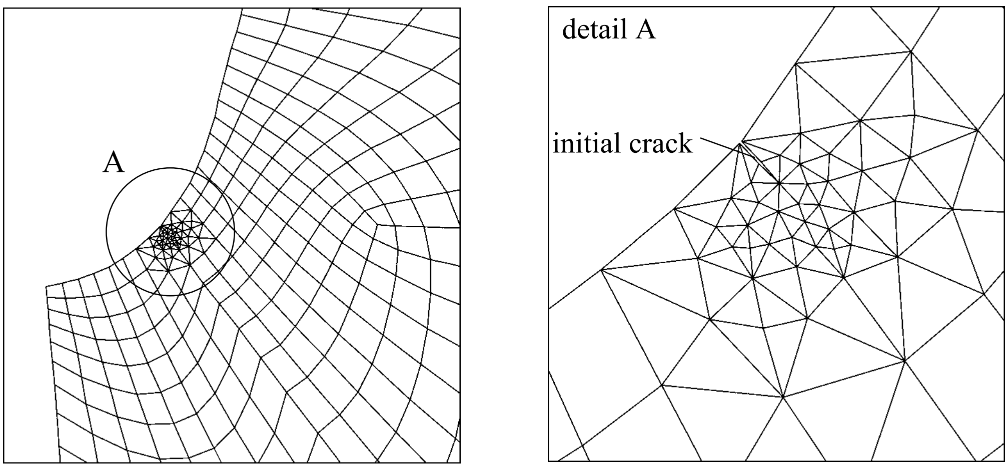

Figure. 4: Finite element mesh around initial crack in a gear tooth root

$$a_{th} \approx \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_{FL}}\right)^2 \quad (11)$$

where $\Delta\sigma_{FL}$ is the fatigue limit ΔK_{th} is the threshold stress in0tensity range. However, a wider range of values have been selected for a_{th} in the literature, usually between 0.05 and 1 mm for steels, where high strength steels take the smallest values.

Considering the material parameters $\Delta\sigma_{FL} \approx 550$ MPa and $K_{th} \approx 269$ MPa√mm available in (Niemann and Winter, 1983, Abersek, 1993), the threshold crack length is equal to $a_{th} \approx 0.1$ mm. The fracture toughness $K_{Ic} \approx 2620$ MPa√mm, and the material parameters $C=3.31\cdot10^{-17}$ mm/cycl/(MPa√mm)^m and $m=4.16$ have been determined previously by the three-point bending samples according to ASTM E 399-80 standard and for the same material as used in this study (Abersek, 1993).

The tooth loading was equal as by the computational analysis of the fatigue crack initiation, see the previous section During numerical simulations the crack increment size Δa was 0.2 mm up to the crack length $a = 4$ mm, and after this 0.4 mm up to the critical crack length a_c, see Fig. 2. To be able to determine the number of loading cycles N_p required for the crack to propagate from the initial crack length a_{th} to the critical crack length a_c according to Eq. (7), it is necessary to determine the dependence ΔK=f(a) first. Figure 5 shows the functional relationship between the equivalent stress intensity factor K and crack length a, where K is obtained by Eq. (9) using numerically determined values of K_I and K_{II}. Numerical analysis have shown that the K_I stress intensity factor is much higher if compared with K_{II} (K_{II} was less than 5 % of K_I for all load cases and crack lengths). Therefore, the fracture toughness K_{Ic} can be considered as the critical value of K and the appropriate crack length can be taken as the critical crack length a_c. The loading cycles N_p for the crack propagation to the critical crack length can than be estimated using Eq. (7), see Table 3. Figure 6 shows the numerically determined crack propagation path in a gear tooth root.

On the basis of the computational results for crack initiation (N_i) and crack propagation (N_p) period in Tables 2 and 3, the complete service life of gear tooth root can be obtained according to Eq. (1), see Fig. 7. It is clear from Fig. 7 that the ratio among the periods of initiation and of end of propagation (i.e. final breakage) depends on the stress level. At low stress level almost all service life is spent in crack initiation, but at high stress levels the significant part of the life is spent in the crack propagation. The computational results for total service life are in a good agreement with the available experimental results, which are taken from (Niemann and Winter, 1983).

Table 3. Computational results for the fatigue crack propagation

Loading F [N/mm]	Critical crack length a_c [mm]	Number of cycles N_p
800	8.6	$9{,}473\cdot10^5$
900	8.4	$5{,}845\cdot10^5$
1000	8.2	$3{,}768\cdot10^5$
1100	7.9	$2{,}534\cdot10^5$
1200	7.7	$1{,}773\cdot10^5$
1300	7.5	$1{,}264\cdot10^5$
1400	7.3	$9{,}322\cdot10^4$
1500	7.1	$6{,}993\cdot10^4$

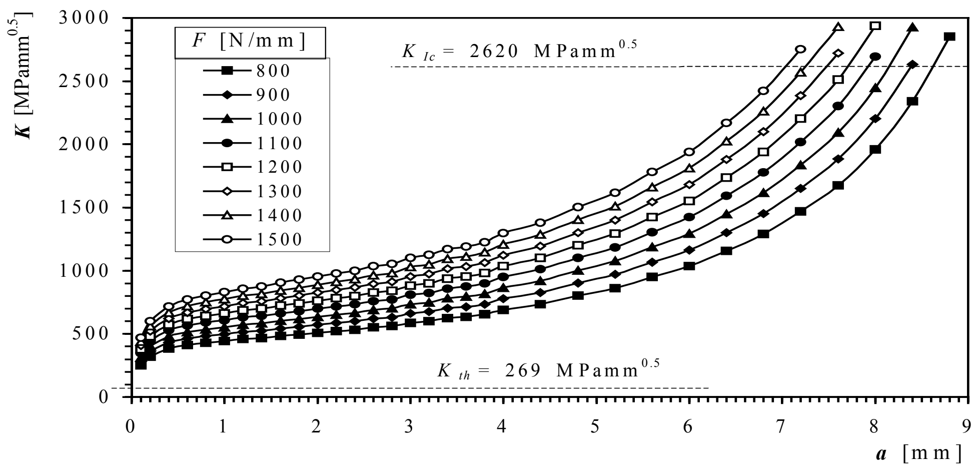

Fig. 5: Functional relationship between the stress intensity factor and crack length

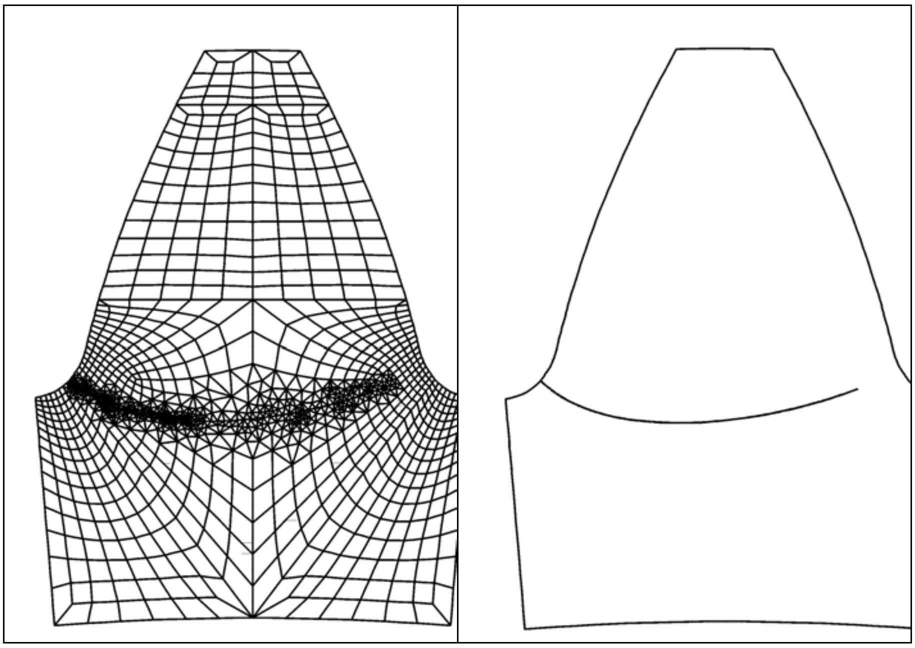

Figure 6: Crack propagation path in a gear tooth root

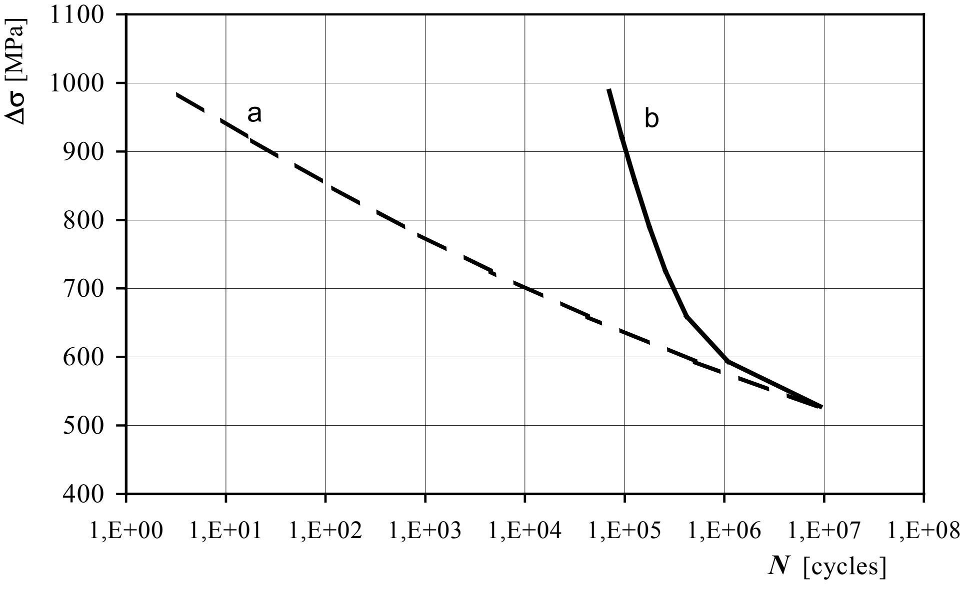

Figure 7: The computed service life of treated gear
a) crack initiation, and b) final fracture

CONCLUSIONS

The paper presents a computational model for determination of service life of gears in regard to bending fatigue in a gear tooth root. The fatigue process leading to tooth breakage in a tooth root is divided into crack initiation (N_i) and crack propagation (N_p) period, which enables the determination of total service life as $N = N_i + N_p$. The simple Basquin equation is used to determine the number of stress cycles N_i. In the model it is assumed that the crack is initiated at the point of the maximum principal stress in a gear tooth root, which is calculated numerically using FEM. The displacement correlation method is then used for the numerical determination of the functional relationship between the stress intensity factor and crack length K=f(a), which is necessary for consequent analysis of fatigue crack growth, i.e. determination of stress cycles N_p.

The model is used to determine the complete service life of spur gear made from high strength alloy steel 42CrMo4. The final results of the computational analysis are shown in Fig. 7, where two curves are presented: the crack initiation curve (a) and the curve of tooth breakage (b), which at the same time represents the total service life. The results show that at low stress levels near fatigue limit almost all service life is spent in crack initiation. It is very important cognition by determination the service life of real gear drives in the praxis, because majority of them really operate with loading conditions close to the fatigue limit.

The computational results for total service life are in a good agreement with the available experimental results. However, the model can be further improved with additional theoretical and numerical research, although additional experimental results will be required to provide the required material parameters.

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