FATIGUE LIFE PREDICTION OF WELDED JOINTS

Prof.dr. Damir Jelaska,
University of Split, FESB, R.Boškovića b.b., 21000 Split, Croatia
Tel.: ++385 21 305 874, Fax: ++385 21 566 877, e-mail: damir.jelaska@fesb.hr
Igor Jelaska, student
University of Zagreb, PMF, Bijenička 30, 10 000 Zagreb, Croatia
Tel. & Fax : ++385 21 514 531, e-mail: jelaska@student.math.hr

Keywords: fatigue life; probabilistic method; random loading; stress spectrum; pre-stress

Ključne riječi: trajnost do zamora; vjerojatnosna metoda; slučajno opterećenje; spektar naprezanja; prednaprezanje.

Abstract: The probabilistic approach to welded joints design is presented and methods for their fatigue life prediction are suggested. The explicit formula for estimation the mean value of fatigue life for welded joints subjected to random loading causing in the critical point of weld toe stresses with certain stress history distributed after Weibull law, is obtained. For the first time, the unique and explicit formula for estimation the fatigue life of pre-stressed welded joints subjected to acting of stress spectrum corresponding to stationary gaussian random process, is derived in this paper, by means of results of constant amplitude test. Some directions to fatigue design approach of marine welded structures are given.

Sažetak: Prikazan je vjerojatnosni pristup projektiranju zavarenih spojeva i predložene metode za predviđanje njihovog vijeka trajanja do zamora. . Izvedena je eksplicitna formula za procjenu srednje vrijednosti trajnosti do zamora zavarenih spojeva izloženih djelovanju slučajnih opterećenja, koja kritičnu točku korijena vara napreže s naprezanjima, čija povijest, prikazana u obliku spektra, odgovara Weibullovoj raspodjeli. Po prvi put je izvedena i formula za računanje vijeka trajanja zavarenih spojeva sa zaostalim naprezanjima izloženih djelovanju spektra slučajnih opterećenja distribuiranih po Gaussovom (normalnom) zakonu. Date su smjernice za pristup proračunu zamora zavarenih konstrukcija u pomorstvu.

1. Introduction

Metal fatigue is a principal mode of failure in structures and machine parts. Many of design parameters associated with fatigue are subjected to considerable uncertainty and probabilistic approach to design seems necessary and appropriate. It understands knowing of entire stress history, distribution of fatigue strength and of all other data. Prediction of fatigue life of the machine parts and structural elements subjected to random load spectrum was a subject of great number of investigations, e.g. [1,2,3,4], which are expensive, unavailable to most of designers, not completed and not always possible to be done. That is why the need for a method exists, which should make possible fatigue life estimation on the basis of only the results of constant amplitude testings, which are widely available. A number of methods was suggested, e.g. [2,4,5], but explicit formula for fatigue life estimation of the parts subjected to random stress spectrum of arbitrarily shape has never been derived. However, the parts are mostly stressed by spectra having asymmetry factor $r \neq -1$. They are often pre-stressed, too. The question arises about fatigue life prediction in such conditions.

2. Limit state design

In structural engineering, safety is generally taken into account according to two possible approaches:

- the semi-probabilistic approach which consists of verifying a criterion involving characteristic values of strengths (resistances) R_i and stresses S_j, and partial safety factors γ_{Ri} and γ_{Sj}, and which may be represented in the following form:

$$\gamma_{Sj}\, S_j \leq R_i/\gamma_{Ri} \tag{1}$$

- the probabilistic method which randomly describes the resistances R_i and stress S_j .

The probability

$$P_f=P(R_i \leq S_j) \tag{2}$$

is called the probability of failure or risk. It characterizes the reliability level of the welded (or any other) joint (or component) with regard to the considered limit state. If both resistances and stresses are distributed after normal low, the risk may be carried out from the Normal law tables:

$$P_f=P(z\leq 0)= P(u\leq -\beta) \tag{3}$$

where

$$Z=R-S \tag{4}$$

is called reliability function, also following normal distribution low, and

$$\beta = \frac{m_R - m_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{m_Z}{\sigma_Z} \tag{5}$$

is called safety index.

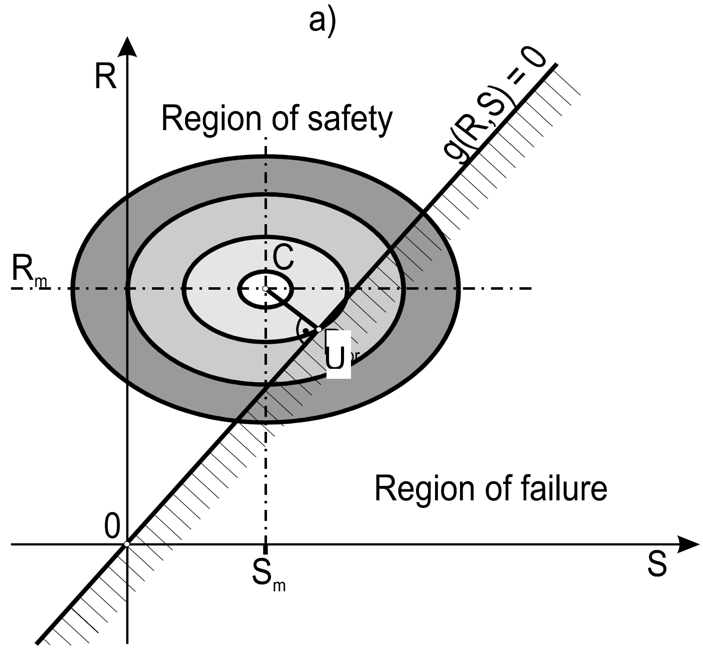

Fig.1-Diagram of limit state field

This probability must be smaller then a reference probability fixed by design code. The condition of reliability (3) may be also expressed in the form

$$\xi \le m_R / m_S \tag{6}$$

where

$$\xi = 1 + \sqrt{\beta^2 (v_R^2 + v_S^2) - \beta^4 v_R^2 v_S^2} \,/ (1 - \beta^2 v_R^2) \tag{7}$$

is the central coefficient of reliability, and

$$v_R = \sigma_R / m_R \text{ and } v_S = \sigma_S / m_S \tag{8}$$

are the coefficients of variation.

Both approaches are clearly reviewed in limit state field on Fig.1, where the fields of less probability are more strongly shadowed. Every point has its own probability of state, but also it represents appropriate reliability of welded joint or structural element, and level of possibility, too [6]. The ultimate limit state is represented by point U, and straight line connecting it and origin splits the diagram in the region of failure and region of safety. It is possible to get various probability levels by various design values of failure probability P_f (usually 10^{-7} to 10^{-3}) or by corresponding value of design safety index β whose value vary from 3,09 to 5,20. The design standard EUROCODE 3 determined safety index by a value of 3,8 for a reference time of 50 years. This is valid for all welded joints of steel structures except for those applied in marine and maritime structures, which are not yet involved by this design code.
In the general case, the limit state is defined not only by two random variables, but n of them. The limit state is then described by the relation

$$Z = g(X_1, X_2, \ldots, X_n) = 0 \quad , \tag{9}$$

and the reliability is

$$P_R=P(g>0)=\int_G f_x(x_1,x_2,...,x_n)dx_1...dx_n \ , \tag{10}$$

where f_x is joint probability density function of $X_1, X_2,..., X_n$. The integration is performed over the region G in which $g>0$. Safety is insured by requiring that P_f be acceptably small for any possible design.

3. Fatigue life prediction at random loading

Machine parts and structural elements are mostly subjected to random loading. It results with random stress history $S(t)$, which causes fatigue. In general, $S(t)$ wil be a random process, meaning it is a random variable at any time t. The central notion involved in the concept of a random stress process is that not just one time history is described, but the whole family of possible time histories, which might be result of same experiment, are described. A sample time function belonging to this family is called a stress history. Thus, it becomes clear that process, at any time, has an appropriate distribution with mean value m_S and standard deviation σ_S . If distribution is normal, such process is called gaussian. If m_S and σ_S remain constant with time, the process is stationary. It was found that all random processes might be considered as stationary, except those resulting from the sea waves loads. In such a cases the nonstationary process can be modeled as a sequence of stationary gaussian process or by distribution of long term peaks if sequence effects are ignored. One of the most appropriate is Weibull's distribution. If the random process is treated as a sequence of peaks, the question of stationarity is not relevant.

For an acceptable probability P_f of fatigue failure, determined by design point, the corresponding design life is

$$N_D = u_N (\ln \frac{1}{1-P_f})^{1/\alpha_N} \tag{12}$$

in which the mean value of N and its coefficient of variation are related to u_N and α_N through

$$E(N) = u_N \Gamma(1+1/\alpha_N) \tag{13}$$

$$V_N = \sqrt{\frac{\Gamma(1+2/\alpha_N)}{\Gamma^2(1+1/\alpha_N)} - 1} \tag{14}$$

where Γ denotes Gamma function. Required mean fatigue life enabling safe operating design life N_D is now obtained from Eq. (12) and (13):

$$E(N) = N_D \Gamma(1+1/\alpha_N) \left(\ln \frac{1}{1-P_f} \right)^{-1/\alpha_N} \tag{15}$$

This method requires that *E(N)* and V_N be estimated for various random loading conditions.

The mean life (expectation of *N*) can be obtained from constant amplitude *S-N* curve

$$NS^k=C \tag{16}$$

or Palmgren-Miner's rule

$$\int_S \frac{dn}{N} = \int_S \frac{N_S f_S(s)ds}{N} = D$$

as

$$N_S = E(N) = \frac{CD}{E(S^k)} , \tag{17}$$

where

$$E(S^k) = \int_{-\infty}^{+\infty} s^k f_S(s)ds \tag{18}$$

and $f_S(s)$ is the probability density function (PDF) for the random variable *S* denoting the magnitude of fatigue stress cycles. The relations (17) and (18) are valid for any distribution of random variable *S*.

For the Pearson's distribution, which represents a generalisation of all exponential distributions having PDF

$$f_S(s) = \frac{s^{n-1} e^{-\frac{s^2}{2\sigma_1^2}}}{\sigma_1^n \psi(n)} ,$$

where σ_1 and *n* are the parameters of distribution, and

$$\psi(n) = 2^{\frac{n-2}{2}} \Gamma(n/2) ,$$

the result of integration is

$$E(S^k) = \frac{\psi(k+n)\sigma_1^k}{\psi(n)} .$$

For *n*=1 the Pearson's distribution transforms to normal one, and expectation is

$$E(S^k) = \frac{2^{k/2}}{\sqrt{\pi}} \Gamma(\frac{k+1}{2})\sigma_1^k ,$$

where σ_1 equals the standard deviation σ. Finally, the explicit formula for predicting the mean value of the fatigue life of the parts or joints subjected to random loading cousing stresses whose stress history corresponds to stationary gaussian random process, is obtained:

$$N_S = \frac{\sqrt{\pi}CD}{2^{\frac{k}{2}}\Gamma\left(\frac{k+1}{2}\right)\sigma^k}$$

For n=2 the Pearson's distribution transforms to Regleigh's, which gives significantly less fatigue lives for the same stress histories.

For the Weibull's distribution with PDF

$$f_S(s)=1-e^{-(s/u_S)^{\alpha_S}},$$

the result of integration of (18) is:

$$E(S^k) = u_S^k\Gamma(1+k/\alpha_S).$$

where u_S and α_S are the parameters of distribution.

When random stress history is described by the gaussian random process, and if the stress amplitude always exceeds a certain level p (called spectrum density factor and equals share of minimum spectrum amplitude), corresponding spectrum is then defined by maximum stress amplitude, spectrum density factor and cumulative frequency. A great number of investigations, e.g. [1,3], were performed in order to obtain the fatigue strength and fatigue life of parts subjected to acting of such spectra. The results were presented by corresponding *S-N* curves. It was evident that the rate of fatigue strength amplitudes at $p<1$ spectrum density factor and $p=1$ spectrum density factor (constant amplitude spectrum) and corresponding fatigue lives ratio, depend only on the spectrum density factor p, but appropriate analytical expression was not obtained. It was later done by some authors, e.g. [10,11,12]. Different analitical expressions for fatigue lives ratio at spectrum and constant amplitude loading was obtained, but every one of exponential type. Thus, it can be written in general case:

$$N_S / N_C = Ae^{ap+b}$$

where a constants A, a and b have to be determined from boundary conditions:

$$N_S/N_C=1 \text{ for } p=1,$$
$$N_S/N_C=k_N \text{ for } p=0.$$

It was obtained:

$$N_S/N_C= k_N^{1-p}.$$

By means of regression analyses of mentioned experimental investigations, it was experienced for welded joints

$$k_N=k^k$$
$$N_S/N_C=k^{(1-p)k}.$$

Otherwise

$$k_N=k^{(0,68k-1,34)(1-p)k}.$$

When pre-stressed parts are subjected to the acting of variable amplitude stresses whose history is presented by stress spectrum having *r* asymmetry factor, the equation of fatigue limit line in Smith's diagram (Fig.3), after author [13], is

$$R_{DNS} = S_m + R_{-1,NS}\left[1-(S_m / R_s)^2\right], \tag{23}$$

where R_S is an ultimate strength.

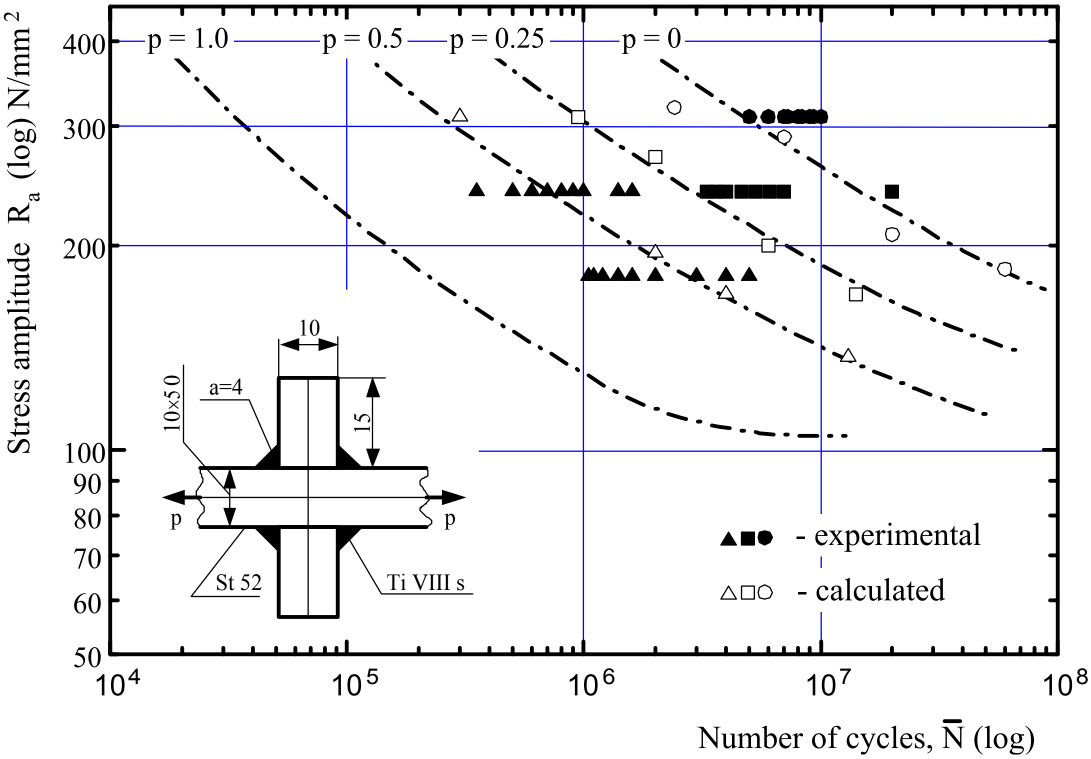

Fig.2- Comparison of calculated and experimental fatigue lives after Haibach [1]

The equation of the *r*=const line, after which maximum stresses of the stress spectrum change versus the mean stress, was obtained to be

$$S_{\max} = S_{pr} + \frac{2}{1+r}(S_m - S_{pr}), \tag{24}$$

where S_m is a mean stress of stress spectrum, and S_{pr} is pre-stress. From this equation, it is easy to obtain

$$S_m = \frac{1+r}{2}S_{\max} + \frac{1-r}{2}S_{pr}. \tag{25}$$

Involving in Eq. (23) $R_{DNS} = S_{\max}$, and S_m after Eq. (25), it is obtained a fatigue limit of welded joint for stress spectrum having asymmetry number *r=-1* ,at fatigue life *N* :

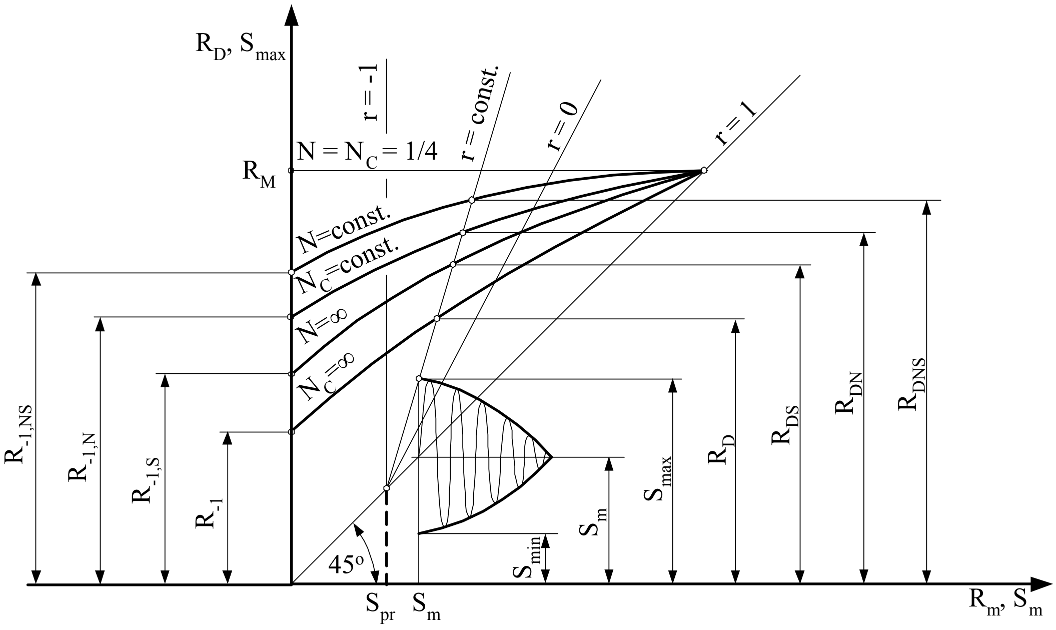

Fig.3 Fatigue limit estimation of the pre-stressed welded joint subjected to acting of stress spectrum

$$R_{-1,NS} = \frac{2(1-r)(S_{\max} - S_{pr})}{4 - \left[(1+r)k_m + (1-r)k_p\right]^2} \tag{26}$$

where k_m denotes the ratio S_m/R_S, and k_p the ratio S_{pr}/R_S. Substituting Eq. (21) in Eq. (26), it is easy to obtain the expression for the fatigue life prediction of a welded joints having in critical point the built in stress S_{pr} and subjected to acting of stress spectrum having r asymmetry factor:

$$N = N_{gr}\left[3^{1,14(1-p)}\,\frac{4 - \left[(1+r)k_m + (1-r)k_p\right]^2}{2(1-r)}\,\frac{R_{-1}}{S_{\max} - S_{pr}}\right]^k . \tag{27}$$

It is clear that N represents the mean value if all other random variables X_i (R_{-1}, S_{max} and N_{gr}) are represented with their mean values. Whereas all of three random variables are statistically independent, the standard deviation of N may be approximated by gaussian approximation formula:

$$\sigma_N = \sqrt{\sum_{i=1}^{3}\left(\sigma_i\,\frac{\partial N}{\partial X_i}\right)^2}\,, \tag{28}$$

where the derivatives has to be calculated in the mean of the distribution of X_i.

4. Conclusion

The probabilistic approach to fatigue design is presented and methods for the fatigue life prediction are suggested. The relations for estimation the fatigue life at random loading are obtained. For the first time, the unique and explicit formula for estimation the fatigue life of pre-stressed parts subjected to acting of stress spectrum corresponding to stationary gaussian random process, is derived in this paper, by means of results of constant amplitude test. Many parameters are included in these formulae, and one has to make a certain effort to get its reliable values, because fatigue life estimation is good in so far as parameters existing in it are well estimated. The first task is to define stress spectrum in the critical point of a part, as the consequence of the load spectrum, usually by means of corresponding counting method. It implies determining of minimum and maximum spectrum stress, its shape and frequency of occurrence. In case of vibrations, the stress amplitude, frequency and phase are different from corresponding values of load spectrum.

In the estimation of the dynamic stress levels at a local structure, both the global and local stress components need to be considered. The global stress components include wave-induced vertical and horizontal hull girder bending stresses. The local stress component results from the external sea pressure and the pressure loads from internal cargo. For each loading condition, the local stress components need to be combined with global stresses.

It must be also taken into account that marine structures are mostly biaxial stressed near the welded attachments. Principal stress directions may be constant or they may vary during the load cycle. The former case is normally termed proportional loading, and the latter is nonproportional loading. Eurocode 3 design code recommends that the maximum principal stress range may be used as fatigue damage parameter if the loading is proportional. For nonproportional loading, the components of damage for normal and shear stresses are assessed separately using the Palmgren-Miner rule and then combined using an interaction equation. Maximum shear stress may also be used as an equivalent stress for welded joints when the direction of the principal stresses changes during the stress cycle.

The fatigue strength data of standard recommendations are partly in agreement with lower fatigue strength data of the nominal stress approach, since the effect of the true shape of the joint is included in strength data of the resistance side. When the fatigue test results are presented as measured strains, extrapolated to the location of the site of the fatigue crack initiation, the effect of all imperfections increasing the true stress need to be taken into account on the loading side [12].

In estimating the fatigue properties of a part, the special attention has to be paid to fatigue curve exponent *k,* which depends also on its shape. The necessary attention has to be paid also to spectrum equivalence factor estimation, to dynamic impact factor calculations, particularly to fatigue stress concentration factor influence, to possible elevated temperature influence and to other environmental conditions.

References

[1] Haibach, E. *:"The Allowable Stresses Under Variable Amplitude Loading of Welded Joints*", Proc. Conf. on Fatigue of Welded Struc., Vol. 2, (1971), pp 328-346.

[2] Haibach, E. *:"Fatigue life prediction by amplitude transformation"*, proc. 4th Int. Conf. on Fracture, Univ. of Waterloo, Ontario, Vol.2, 1977, pp 1117-1124.

[3] Solin, J.:*"Methods for Comparing Fatigue Lives for Spectrum Loading",* Int. Jour. of Fatigue 12, 1/1990.
[4] Olivier, R.: *"Zur bemessung von Schweissverbindungen aus Baustählen"*, Fraunhofer-Institut fürBetriebsfestigkeit, Darmstadt 1985.
[5] Leipholz, H. H. E., Topper, T. H., El Menoufy, M. :*"Lifetime prediction for metallic components subjected to stochastic loading"*, Computer & Structures, 1983, 16 (1-4), pp 499-507.
[6] Cremona, C., Gao, Y. :*"The possibilistic reliability theory: theoretical aspects and applications"*, Structural safety 19 (2), 1997, pp 173-201.
[7] Wirsching, P. H. :*"Fatigue reliability of welded joints in offshore structures"*, Proc. of Offshore Techn. Conf., 1989, OTC paper 3380, pp 113-119.
[8] "*Fatigue reliability: An introduction"*, Journal of Struct. Divis., ASCE, Vol. 108, No. ST1, Proc. Paper 16788, 1982, pp 3-23.
[9] Haibach, E. *:"Betriebfestigkeit",* VDI Verlag, Dusseldorf 1989.
[10] Jelaska, D. *:"Estimation of Fatigue Life at Random Loading",* Proc.Danubia-Adria Symp.on Exp. Methods in Mechanics, Poreč '97, pp 142-144.
[11] Hobbacher, A. :*"Recommendations on Fatigue Design of Welded Joints and Components",* Woodhead Publishing, Abington, Cambridge, 1966.
[12] Niemi, E. :*"Stress Determination for Fatigue Analysis of Welded Components"*, Woodhead Publishing, Abington, Cambridge, 1993.
[13] Jelaska, D. *:"Fatigue Stress Concentration Factor in the Zone of Elastic-plastic Strain*", Materialove inžinierstvo No.8, 1997, pp 10-15.