Quantum Experiments without Classical Counterparts

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Summary

Algorithms for finding arbitrary sets of Kochen-Specker (KS) vectors have been devised. The algorithms are based on linear MMP diagrams which generate orthogonalities of KS vectors, on an algebraic definition of states defined on the vectors, and on interval analysis solving of nonlinear equations.

Kochen-Specker Vectors

In $\mathcal{H}^n$, $n \geq 3$, KS vectors are those vectors to which it is impossible to assign 1s and 0s in such a way that (1) no two of mutually orthogonal vectors are both assigned 1; (2) 0 is not assigned to all of them.

Kochen-Specker-Penrose-Pavičić

MMP Diagram Algorithm

1. Every vertex belongs to at least one edge;
2. Every edge contains at least 3 vertices;
3. Every edge which intersects with another edge at most twice contains at least 4 vertices.

McKay-Megill-Pavičić

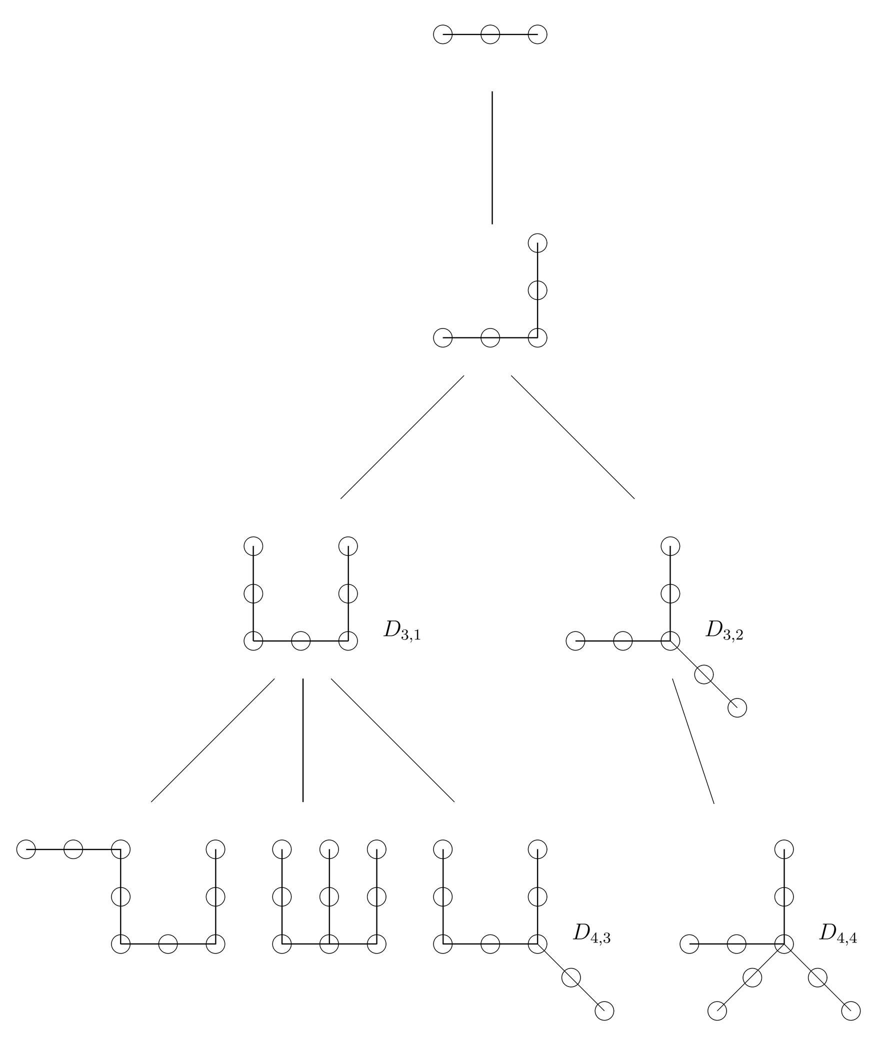

Figure 1: Generation tree for MMP diagrams with loops of size 5 for 3-dim vectors.

No 0-1 State Algorithm

The algorithm is exhaustive search of MMP diagrams with backtracking. The criterion for assigning non-dispersive (0-1) states is that each edge must contain exactly one vertex assigned to 1, with the others assigned to 0. As soon as a vertex on an edge is assigned a 1, all other vertices on that edge become constrained to 0, and so on. The algorithm scans the vertices in some order, trying 0 then 1, skipping vertices constrained by an earlier assignment. When no assignment becomes possible, the algorithm backtracks until all possible assignments are exhausted (no solution) or a valid assignment is found. In principle the search is of exponential complexity, but because the diagrams of interest are tightly coupled, constraints build up quickly. The algorithm uses this feature to avoid the exponential behaviour of the search.

Megill-Pavičić

The Main Non-Linear Orthogonality Algorithm

The number of vertices within edges corresponds to the dimension of $\mathbb{R}^n$ and that edges correspond to $n(n-1)/2$ equations resulting from inner products of vectors being equal to zero which means the orthogonality. So, e.g., an edge of length 4, BCDE, represents equations the first of which is: $a_{B1}a_{C1} + a_{B2}a_{C2} + a_{B3}a_{C3} + a_{B4}a_{C4} = 0$. Each possible combination of edges for a chosen number of vectors (vertices) and edges makes a diagram which corresponds to a system of such nonlinear equations. A solution to such a system is a set of KS vectors we want to find.

Pavičić

Interval Analysis Self-Teaching Algorithms for Solving Nonlinear Equations

Several interval analysis algorithms are used. For avoiding the exponential growth of the number of generated MMP diagrams KS-system are generated incrementally, i.e., sequentially, starting with a given m n-tuples before modifying the mth n-tuple. Thus as soon as the preliminary pass determines that an initial set of m n-tuples has no solution and that no further systems starting with this set will be generated. E.g., for 18 vectors and 12 quadruples without such a filter one should generate $> 2.9 \cdot 10^{16}$ systems—what would require more than 30 million years on a 2 GHz CPU—while the filter reduces the generation to 100220 systems (obtainable within < 10 mins on a 2 GHz CPU). Thereafter no-0-1-state-algorithm gives us 22 systems without 0-1 states in < 5 secs. For the remaining systems two algorithms have been developed which gives 1 solution (see Fig. 2) within less then 1 sec.

Merlet-Pavičić

Chosen Results

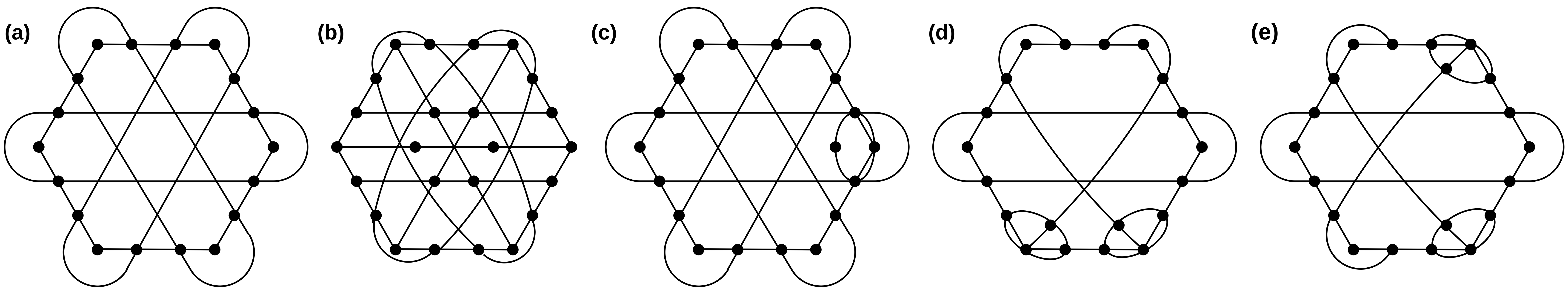

Figure 2: Smallest 4-dim MMP diagrams with: (1) loops of size 3: (a) 18-9 (isomorphic to Cabello-Estebaranz-García-Alcaine); (b) 24(22)-13 not containing (a), with values $\notin \{-1, 0, 1\}$; (2) loops of size 2: (c) 19(18)-10; (d) 20-11; (e) 20-11 (isomorphic to Kernaghan).