Doctoral dissertation 2004-06-15
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Abstract

In this thesis various descriptions of the near surface atmospheric flow over a high latitude glacier is used in an effort to increase our understanding of the basic flow dynamics there.

Mountain glaciers play a significant role in the Earth's climate system, perhaps most so for their contribution to sea-level change. The sensitivity of glacier mass balance to climate change is determined by changes in long wave radiation and turbulent heat transfer. For this reason a good comprehension of near surface atmospheric flow over glaciers is essential for the understanding of glacier response to climate change.

In the present thesis the near surface atmospheric flow is studied from several perspectives. This approach is needed since there is no overall theory for mesoscale dynamics - in contrast to the extra-tropical synoptic scale dynamics. The different perspectives used include the effects of both rotation and slope. Rotation is an important aspect of most atmospheric flows and its significance for mesoscale flows have gained recognition over the last years, it is however often neglected in parameterizations of the near surface flow for large scale models. Similarly, the very stable boundary layer (VSBL) has lately gained interest. It differs in many ways from the weakly stable boundary layer, that has been the focus of most studies of the stable boundary layer. Within a VSBL over sloping terrain katabatic flow is known to be usual and persistent. For the present thesis a combination of numerical and simple analytical models as well as observations from the Vatnajökull glacier on Iceland have been used. The models have continuously been compared to available observations. Three different approaches have been used: linear wave modeling, analytic modeling of katabatic flow and of the Ekman layer, and numerical simulations of the katabatic flow using a state of the art mesoscale model. The analytic models for the katabatic flow and the Ekman layer used in this thesis both utilizes the WKB method to allow the eddy diffusivity to vary with height. This considerably improves the results of the models. Among other findings it is concluded that: a large part of the flow can be explained by linear theory, that good results can be obtained for surface energy flux using simple models, and that the very simple analytic models for the katabatic flow and the Ekman layer can perform adequately if the restraint of constant eddy diffusivity is relieved.

ISBN 91-7265-913-0, pp 1-22.
Printed by PrintCenter, Stockholm University, Stockholm, Sweden, 2004.

Contents

List of Papers

This thesis consists of a summary and the following four papers, which will be referred to by their Roman numerals:

I **Linear modeling of the air flow over the Vatnajökull glacier.**

Parmhed, O.

Report DM-91, 2004.

II **Describing surface fluxes in katabatic flow on Breidamerkurjökull, Iceland.**

Parmhed, O., Oerlemans, J. and Grisogono, B.

Q. J. R. Meteorol. Soc. **130**, 1137–1151, 2004.

III **An approximate Ekman layer solution for smooth eddy diffusivities.**

Parmhed, O., Kos, I., and Grisogono. B.

Submitted to Boundary-Layer Meteorology. 2003.

IV **Numerical simulation and analytical estimates of katabatic flow over a melting outflow glacier.**

Söderberg, S. and **Parmhed, O.**

Submitted to Boundary-Layer Meteorology. 2004.

In paper 2 I have done all work and writing. The observational data is supplied by the second author. The second and third authors originally developed the theory used in the paper. Paper 4 stems from the same data and is partly a continuation of paper 2. In this paper the analysis and writing was shared between the authors while the numerical simulations were performed by the first author. In paper 3 I have done all work and the vast majority of the writing. The second and third authors have done previous work on the theory, and the third author also contributed part of the introduction.

Chapter 1

Introduction

Mountain glaciers play a significant role in the climate system. Despite the fact that only a minor part of the Earth's land ice is contained in mountain glaciers (the majority is contained in the Antarctic ice-sheet), their impact on human activity is significant. Glacier meltwater is used for hydropower and irrigation. Glaciers can also threaten the safety of living in mountainous regions. In a global sense, the most important influence by glaciers is probably their significant contribution to sea-level change on a century time scale (IPCC, 2001). Although most of the melt energy comes from solar radiation (Gruell et al. 1997), the sensitivity of the melt rate to temperature change is determined by changes in long-wave radiation balance and turbulent heat flux (Oerlemans, 2001). For this reason the atmospheric near-surface flow over glaciers need to be accurately parameterized for use in larger scale models of the Earth's climate. Most current boundary layer parameterization schemes ignores the slope of the surface. This is a serious limitation when considering glaciers or ice-caps. Another import component that is often neglected is Earth's rotation. This thesis looks into various descriptions of the near surface flow over the Vatnajökull glacier on Iceland. For this purpose both analytic and numerical methods are used as well as existing observations.

1.1 Background

The global mean sea-level has risen by between 10 cm and 20 cm over the 20th century. A substantial part (on the order of 20%) of this increase can be attributed to mountain glaciers and small ice-caps. Many estimates predict a sea-level rise of some (50 ± 40) cm over the next century (IPCC, 2001). Over the next 50 years it is also believed that sea-level rise would come from thermal expansion and glacier melting in about the same proportion as that of the past century (Hartmann, 1994).

The present day ice cover on Earth is about 10%. The ice cover is however not constant. Minor advances in mountain glaciers culminated about 5300, 2800 and 150-600 years ago. The latter time coinciding with the 'little ice age' of 1250-1850 A.D. (Hartmann, 1994). During the last century there has been a strong retreat of glaciers, coinciding in time with the rise in sea-level. Oerlemans (1994) estimated a linear global warming trend from these glacier retreats. Using the data from 48 glaciers, he estimated the linear global mean warming to 0.66^oC per 100 years, i.e. about the same as the latest IPCC estimate of the observed global warming rate. The assumptions in that study of the glaciers being in equilibrium, the inhomogeneity of the data set and more leaves a large uncertainty with the given estimate. Oerlemans and Fortuin (1992) showed that glaciers are very sensitive to climate change, in particular this is true for glaciers in warm regions. The reason for the high sensitivity of glaciers in warm regions is in part the fact that during melt the glacier surface remains at the melting point of ice, about 0^oC. Any increase in temperature will therefore go directly to melting, rather than to heating the ground. However, the efficiency in transporting additional heat to the surface is controlled by the near-surface atmospheric flow. For this reason efforts have been made recently to parameterize the surface energy flux around and over glaciers (e.g. Denby and Smeets, 2000).

Modern climate research is to a large extent carried out by numerical methods using global climate models (GCM). These models are an invaluable tool for understanding and eventually predicting Earth's climate. They are however extremely demanding in terms of computer time. For this reason most GCMs use a rather coarse resolution, both vertically and horizontally. The low resolution lead to the use of parameterizations of physical processes on scales smaller than those resolved by the model. Considering the already mentioned sensitivity of glaciers to the climate, we need a good understanding of the glacier near surface flows. This understanding is needed to construct parameterizations that take into account the special environment of a glacier. Important factors are such a the cold sloping surface of a melting glacier or the flow around complex topography. The problem is further complicated by the fact that the size of most glaciers is such that the flow over and around them must be categorized as mesoscale, i.e. with a spatial scale of about 10-1000 km, a scale not resolved by current climate models.

1.2 Atmospheric motion

Atmospheric motions exist on a wide variety of scales, both temporal and spatial, and can be associated with many different physical phenomena and processes. At the heart of our knowledge of geophysical fluid dynamics, as well as fluid dynamics in general, lies the so called Navier-Stokes equations. The Navier-Stokes equations stems from the application of Newton's second law, on conservation of momentum, to a continuous fluid. The full Navier-Stokes equations are however not easily applied to atmospheric motions. The reason is the wide range of scales that need to be represented and that lead to a requirement on resolution, mainly spatial, which is hard to meet with todays computational resources. This problem leads to a separation in scales and the subsequent analysis of what processes are important for the flow at the scale of interest. Depending on the scale and the important processes it is easier or more difficult to uniformly and adequately describe the flow. On the synoptic scale, or planetary scale as it is also known, the atmospheric flow is today well described by the quasi-geostrophic theory. This is part of the reason why we today can enjoy fairly accurate and reliable weather predictions. However, on the smaller scales there exists today no general, self-contained, three-dimensional theory that adequately describes the flow. Rather, different phenomena are approached in different ways, using different kinds of modeling. Perhaps the most intriguing range of scales is that known in geophysical fluid dynamics as the mesoscale, reaching from approximately 10 km to 1000 km in spatial scale. The mesoscale is important because many of the phenomena occurring in this region of scales is important for the every day life of humans. For example, fronts, clouds, thunderstorms, and sea-breeze circulations are all mesoscale phenomena. Another area that falls within the mesoscale region is flow controlled by topography. In its simplest form, such flows consists of just a large scale (in a mesoscale frame of reference) flow around the topography. To increase accuracy, one can increase the focus on details and zoom into specific areas.

In this thesis I will by and large stay within the mesoscale of geophysical flow, focusing on the flow over mid-latitudes glaciers. This is done by a combination of different model approaches, analytical as well as numerical and analysis of observations. The modeling is continuously supplemented and compared to observations from the Icelandic glacier Vatnajökull, and its outlet tongue Breidamerkurjökull.

1.3 The Vatnajökull glacier

In recent years the need for observations of glacier surface energy balance have led to a number of observation campaigns. In an effort to investigate a large size glacier the Vatnajökull campaign was designed. Vatnajökull is Europe's largest glacier. It is more than 8000 km^2 and situated in a marine climate on the south-east coast of Iceland, in the North-Atlantic. In 1996 four universities joined to erect 16 weather stations on the glacier ice, as well as just off the ice. The weather stations sampled some or all of wind speed and direction, temperature, humidity, radiation, pressure and more. Some of the weather stations were also equipped to measure variables at several heights, giving the opportunity of profiling the lowest part of the boundary layer. In addition to the weather stations a tethered helium balloon was used to sample the lower couple of hundred meters of the atmosphere. This gives the opportunity of studying the vertical structure in more detail.

The Vatnajökull observation campaign is described in detail in Oerlemans et al. (1999). Data from this campaign is used in three of the four papers comprising this thesis.

Chapter 2

Linear wave theory

Perhaps the only truly three-dimensional theory known today for the mesoscale, disregarding the application of the full Navier-Stokes equations, is the linear wave theory. Although disputed when it was announced, the importance of gravity wave theory is today recognized (Nappo, 2002). Today, linear wave theory forms the basis of our understanding of the complex flow around topography. Already in 1948 Queney described the flow over an isolated two-dimensional ridge (Queney, 1948). Smith (1980) developed the theory to account for three-dimensional, isolated mountains. The simplicity and elegance of the linear wave theory together with the wide area of atmospheric studies in which wave interactions are important, make the linear wave theory a cornerstone in the understanding of flow over complex topography. The theory does however have some significant limitations. The most obvious one is of course the linearity. Atmospheric flows often experience non-linear behavior. However, in many cases non-linear effects are either small or limited in spatial/temporal extent. It may also happen that non-linear effects cancel each other or are counter-acted by other neglected processes. In such cases, the linear wave theory may be a valuable tool for describing mesoscale flow over complex terrain (see e.g. Smith, 1980; Smith, 1982; Thompson et al., 1991; Koffi et al., 1998; Nappo, 2002)

The theory developed by Smith (1980) is a simple description of the flow modification induced by an obstacle such as a mountain. The model considers the steady flow of a vertically unbounded, stratified Boussinesq fluid over a small-amplitude topography. Because the amplitude of the topography is small, the equations describing the flow can be linearized. The equations can be combined to yield a single governing equation for the vertical displacement, η:

$$\frac{\partial^2 \left(\nabla^2 \eta\right)}{\partial x^2} + \frac{N^2}{U^2}\nabla_H^2 \eta = 0; N^2 = -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz}, \tag{2.1}$$

in which N is the buoyancy frequency, U is the unperturbed background wind speed, g is gravity, x and z are the axial and vertical coordinates respectively, ρ_0 is the background mean density, $d\bar{\rho}/dz$ is the background density gradient, ∇ is the 3D Laplacian and the index H refers to the horizontal derivatives. If η is represented as a double Fourier integral and N is considered constant, together with U, the resulting vertical displacement can be shown to be:

$$\eta(x, y, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{h}(k, l) e^{im(k,l)z} e^{i(kx+ly)} dk dl, \tag{2.2}$$

where k and l are the wave numbers corresponding to x and y, $\hat{h}(k,l)$ is the Fourier transformed height of the topography and m is the vertical wave number given by:

$$m^2 = \frac{k^2 + l^2}{k^2}\left(\frac{N^2}{U^2} - k^2\right). \qquad (2.3)$$

From the vertical displacement field, velocity perturbations can be computed to give an overall view of the flow-field created by the topography.

In **paper I** the linear model given by (2.1)-(2.3) is modified for non-hydrostatic rotating conditions. The model is applied to the Vatnajökull topography and compared to observations of the near surface flow. When considering the influence of rotation, one normally refers to the Rossby number $Ro = U/fa$, in which U is the flow speed, a is the half-width of the topography and f is the Coriolis parameter. Rotation is normally considered important when $Ro < 1$. Over the last couple of years the importance of rotation in mesoscale flows have gained focus (see e.g. Hunt et al, 2001). Smith (1982) showed that in an expansion based on inverse Rossby number, the horizontal wind is affected to a lower order in the expansion than pressure or vertical velocity. Enger and Grisogono (1998) used a non-linear model to simulate bora-type flows and found rotation to be locally important despite a high large scale Ro.

Linear theory is generally assumed to be valid at Froude numbers $Fr = \frac{U}{Nh} \gg 1$, where N is the buoyancy frequency and h is the height of the topography. Considering the previous studies, the question arises whether rotation may include sufficient dispersion that the flow may be accurately modeled using the linear assumption even at $Fr = O(1)$.

The model is applied to Vatnajökull. Such a simple model as the linear wave model is difficult to perceive as giving a faithful representation of the actual flow. In particular this is so in areas where non linear processes are known to be important. One such area is the wake behind the topography. For a meaningful statistical estimate, weather stations located in the wake are removed from consideration. A number of statistical parameters are then computed in an effort to categorize the predicted flow. As expected, the statistics show that the linear model is less than satisfactory in quantifying the direction perturbations around the glacier although including the effect of rotation improved the predictions somewhat. Remarkable though, is the large values of correlation of the flow speed. Both correlation of wind magnitude and spatial correlation is also shown to increase with time, a feature that can only be explained by the actual flow around the glacier becoming more linear in its nature as time progresses.

In summary, **paper I** shows that despite the simplicity of the linear wave model it is capable of quantifying a large part of the flow around the Vatnajökull glacier. The predictions are however also severely limited.

Chapter 3

Stable boundary layers

The atmospheric boundary layer is often divided into the convective (potential temperature decreasing with increasing height above ground), neutral (potential temperature constant with height) and stable (potential temperature increasing with height) boundary layer. Since long the Monin-Obukhov (MO) similarity theory has been used to describe the turbulence and mean gradients of the atmospheric boundary layer. It has been shown that the MO theory is in excellent agreement with observations in conditions when the density variations are not too far from neutral stratification (e.g. Stull, 1988). However, in strong convection and strong static stability, the MO theory is not sufficiently advanced to describe the boundary layer (e.g. Zilitinkevich and Calanca, 2000; Zilitinkevich et al., 2002). Mahrt (1998) divided the stable boundary layer into the *very stable boundary layer* (VSBL) and *weakly stable boundary layer*. The weakly stable boundary layer is that most often studied, perhaps most so in the shape of the nocturnal boundary layer. The very stable boundary layer, however, is still not well known. The VSBL has been the focus for several studies over recent years (see e.g. Zilitinkevich and Calanca, 2000; Mahrt, 1998; van den Broeke, 1997; Smeets et al., 2000). Although less common than the weakly stable boundary layer the VSBL may well be common during high latitude winter.

In the following two approaches to model the atmospheric boundary layer will be presented, the Prandtl model for katabatic flow and the Ekman model for boundary layers in a rotating fluid. The studies utilize the WKB method for solving differential equations with variable coefficients and the mesoscale numerical model COAMPS. These will first be briefly introduced, followed by the analytic models.

3.1 The WKB method

The WKB method is a powerful and elegant single perturbation method for obtaining a global approximation to the solution of a linear differential equation of any order. The method is named after Wentzel, Kramers and Brillouin, who popularized the method in theoretical physics. The WKB approximation to a solution has a simple structure. Order by order in a small parameter, it consists of exponentials of elementary integrals of algebraic functions. This is the case even if the exact solution is a function of great complexity. The main limitation of the conventional WKB method is that it is only useful for linear differential equations. Another limitation is a restriction on the variability of the coefficients in the approximate solution. For the problems treated in this thesis, this implies a restriction on the variability of the vertical diffusivity with height, in relation to the variability of flow velocity and/or temperature. As will be shown later, this restriction - and the way to treat it - has significant effects on the results.

A thorough description of the WKB method can be found in Bender and Orszag (1999).

3.2 COAMPS^{TM}

The Coupled Ocean/Atmosphere Mesoscale Prediction System (COAMPS) model, developed at the U.S. Naval Research Laboratory, is a nonhydrostatic compressible model, with terrain-following sigma-z vertical coordinate system. The model supports nested grids and can be used with idealized or real-case background. For the work presented here, ECMWF analyses were used for initial and lateral boundary data in the real-case simulations. This is essential to achieve realistic atmospheric conditions within the model when comparing to the observations from Vatnajökull. COAMPS physical parameterization schemes include radiation (Harshvardhan et al. 1987) and ”level-2.5” turbulence closure (Mellor and Yamada, 1982), among others. The model is used operationally by the U.S. Navy. A more detailed description of the model can be found in Hodur (1997).

3.3 Katabatic flow

Katabatic flows are formed when the air close to a sloping surface is cooled relative to the surrounding atmosphere. This temperature difference is the cause of buoyancy forces that, because of the sloping surface, also act along the slope. The buoyancy force is usually small compared to other components of the momentum budget, and katabatic flows are therefore normally only seen when nighttime radiative cooling of the surface peaks. On melting glaciers the situation is rather different. Because of the surface melt, the difference in temperature between the surface and adjacent air at the same altitude can be large enough to continuously sustain a katabatic flow. In fact, over sloping surfaces katabatic flow is a ubiquitous feature of the VSBL (e.g. Stull, 1988; Egger, 1990). Field experiments have shown that over land ice, katabatic flows are both usual and persistent (e.g. Oerlemans et al., 1999).

The simplest description of the katabatic flow, or glacier wind, is the Prandtl model for gravity-driven flow down a cooled inclined surface (Prandtl, 1942; Defant,1949; Mahrt, 1982). The simplicity of the Prandtl model and the fact that it is analytical makes it attractive for an understanding of the pure katabatic flow. The model equates the divergence of the turbulent fluxes of momentum and heat to the advected background temperature lapse rate and the buoyancy acceleration (Prandtl, 1942; Mahrt, 1982; Egger, 1990). In a coordinate system where the x-axis is aligned with the slope, the equations are:

$$\frac{\partial \theta}{\partial t} = -\gamma u \sin(\alpha) - \frac{\partial(\overline{w'\theta'})}{\partial z} \tag{3.1}$$

$$\frac{\partial u}{\partial t} = g\frac{\theta}{\theta_0}\sin(\alpha) - \frac{\partial(\overline{w'u'})}{\partial z}. \tag{3.2}$$

In eqs. (3.1) and (3.2) θ is the potential temperature perturbation (i.e., actual potential temperature minus background potential temperature), θ_0 a reference temperature (typically the melting point of ice), γ is the background potential temperature lapse rate, g is gravity, z is the vertical coordinate (perpendicular to the slope), α the slope of the surface and $\overline{w'\theta'}$ and $\overline{w'u'}$ are turbulent heat and momentum fluxes parameterized with K-theory according to:

$$\overline{w'\theta'} = -K_h \frac{\partial \theta}{\partial z}, \quad \overline{w'u'} = -K_h Pr \frac{\partial u}{\partial z}. \tag{3.3}$$

Here K_h is the eddy diffusivity for heat and Pr is the turbulent Prandtl number ($Pr = K_m/K_h$, where K_m is the eddy diffusivity for momentum). Under steady-state conditions and assuming that K_h and Pr are constants, the single governing equation resulting from (3.1)-(3.3) becomes (Prandtl, 1942)

$$\frac{d^4\theta}{dz^4} + N^2 \frac{\sin^2(\alpha)}{Pr K_h^2}\theta = 0; N = \sqrt{\frac{g\gamma}{\theta_0}}. \tag{3.4}$$

The solution to (3.4) is

$$u = -C\mu \cdot e^{\lambda z} \cdot \sin(\lambda z) \tag{3.5}$$

$$\theta = C \cdot e^{\lambda z} \cdot \cos(\lambda z) \tag{3.6}$$

with

$$\mu = \sqrt{\frac{g}{\theta_0 Pr \gamma}}; \lambda = \sqrt[4]{\frac{g\gamma \sin^2(\alpha)}{4 Pr \theta_0 K_h^2}}. \tag{3.7}$$

The typical shape of the Prandtl model katabatic boundary profiles are shown as an example in fig. 3.1.

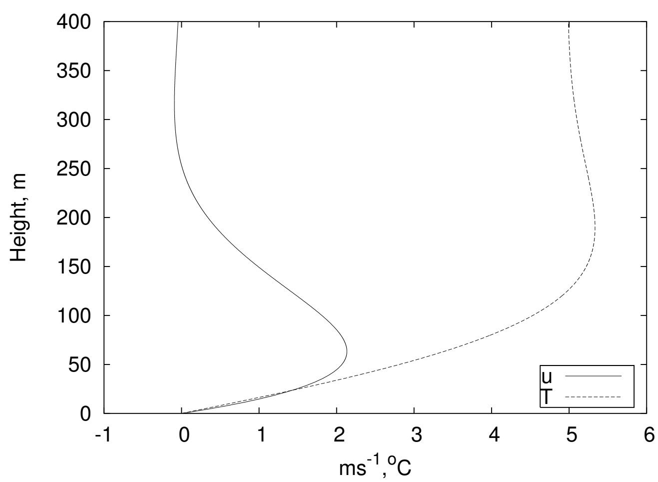

Figure 3.1: An example of the Prandtl model profiles for a katabatic boundary layer.

The Prandtl model does have some significant drawbacks. Perhaps the most serious limitation is the models inability to correctly describe the sharp near surface gradients in temperature and wind that are often observed (Defant, 1949; Munro, 1989; Egger, 1990). Conversely, above the jet the simulated gradient is often too sharp, as compared to observations (Oerlemans, 1998; Denby, 1999). These inabilities of the model are due to the use of a constant value for the eddy diffusivity, that makes it possible for the model to be analytically solvable. Another limitation is the fact that the height of the jet in the Prandtl model does not increase with the strength of the jet maximum, a correlation that is found in observations (e.g. Oerlemans and Grisogono, 2002). The latter limitation is the focus of **paper IV**, and will be further introduced later in this text.

The first of the limitations, a constant value of the eddy diffusivity, can be overcome by the use of the WKB method as shown in Grisogono and Oerlemans (2001a). In **Paper II** this approach is used and the solutions are compared to observations from the Breidamerkurjökull.

Assuming that $K = K(z)$, i.e. varying with height, the equivalent of (3.4) becomes significantly more complex. However, the requirement of the WKB method that the eddy diffusivities vary slowly with height leads to a dominant balance analysis where the higher order derivatives of the diffusivity can be neglected for sufficiently smooth diffusivities and only the 0^{th} order terms retained. A proper justification of this simplification can be found in Grisogono and Oerlemans (2002). Then the governing equation is simplified to the equivalent of (3.4) with the difference that $K = K(z)$ which is a form suited for the WKB method. However, an expression for the eddy diffusivity is also needed. The polynomial eddy diffusivity profile by O'Brien (1970) have been used in many modeling studies, e.g. Pielke (1984) and Stull (1988). The O'Brien profile was simplified in Grisogono and Oerlemans (2001a) to the linear-Gaussian expression

$$K(z) = \frac{K_{max} e^{1/2}}{H_K} \cdot z \cdot e^{-\frac{1}{2}(z/H_K)^2}, \tag{3.8}$$

where K_{max} is the maximum value of the eddy diffusivity that is reached at the height H_K. The coefficients K_{max} and H_K are calculated from the jet height, z_j, the height of the inversion, z_i, and the slope. This profile meets the important requirement that at the surface and when approaching the free atmosphere the eddy diffusivity should approach zero. To solve the equation using the WKB method the solution can be split into an inner region where K is strictly increasing from 0 and an outer region approaching the free atmosphere. The two solutions are then patched together at the height of the maximum in K, H_K. The resulting profiles for u and θ can now be computed from:

$$u_{inner} = -C\mu \cdot e^{-I(z)} \cdot \sin\left(I(z)\right) \tag{3.9}$$

$$\theta_{inner} = C \cdot e^{-I(z)} \cdot \cos\left(I(z)\right) \tag{3.10}$$

$$(u, \theta)_{outer} = (u, \theta)_{inner} \cdot \left(\frac{K(z)}{K_{max}}\right)^{-\frac{1}{4}} \tag{3.11}$$

$$I(z) = \sqrt{\frac{\sigma_0}{2}} \int_0^z \frac{1}{\sqrt{K}} dz \tag{3.12}$$

$$\sigma_0 = \sqrt{\frac{g\gamma \sin^2(\alpha)}{Pr\theta_0}}. \tag{3.13}$$

In **paper II** the above equations are used to calculate the katabatic flow on the Breidamerkurjökull, an outlet glacier of Vatnajökull. Using the patching described above and profile observations the model is evaluated. Although the model was presented in Grisogono and Oerlemans (2001a) and justified in Grisogono and Oerlemans (2002) this is the first time the theory is tested on a second data set, in a more complex environment and using vertical information to a larger height. To further prove the value of the model jet height, maximum jet speed and the surface fluxes are compared to observations.

Simple and conservative criteria for katabatic flow are used for both the weather station and the balloon sounding data. These criteria confirm that katabatic flow is highly persistent on Breidamerkurjökull (cf. Oerlemans et al. 1999). Despite the conservative nature of the criteria about 40% of the data was categorized as katabatic. A fair agreement was found between model and observations. In particular, the flux calculations from the model are in good agreement with independently observed turbulent flux. The values of the katabatic jet speed are not in such good agreement. In fact, the model is consistently overestimating the jet speed. In **paper II** this error was attributed to the uncertainty in determining the surface temperature deficit. However, a large part of this error can be removed by using an additional improvement to the WKB approach. This improvement is further discussed in **paper III** and its impact on the katabatic flow estimates will be discussed in section 3.4.1.

An additional issue in determining the eddy diffusivity is that one has to determine a value for z_j. Oerlemans (1998) suggested a relation for z_j as

$$z_j = B \cdot \frac{-C}{\gamma\sqrt{\sin(\alpha)}}, \tag{3.14}$$

which would tie in z_j with the spatial locality. Despite the fact that relations such as this are important for a proper way of treating the katabatic flow and that the relation is not new it has not previously been verified against observations. In **paper II** a first effort at such a validation is made and the coefficient $B = 9.7 \cdot 10^{-4}$ is determined. However, the value of this coefficient is associated with a very large uncertainty. This uncertainty prompts for further investigation. Support for this approximate value is given in **paper IV**.

To relieve the second limitation of the Prandtl model - the lack of dependence of the jet height on the jet speed - Oerlemans and Grisogono (2002) defined scales that when put into the Prandtl model characterize a katabatic state. The katabatic scales are then chosen proportional to the maximum wind speed, temperature deficit and height of the wind maximum via constants k_i. Their analytical model consists of equations predicting the steady state jet speed (u_m), jet height (z_m) and surface sensible heat flux (F_h) from the background parameters:

$$u_m = -\frac{k_2}{k_1} C \left(\frac{g}{\theta_0 \gamma Pr}\right)^{1/2}, \tag{3.15}$$

$$z_m = -\frac{k_2 k}{k_3} \frac{C}{\gamma\sqrt{\sin\alpha}}, \tag{3.16}$$

$$F_h = -k k_2^2 C^2 \left(\frac{g}{\theta_0 \gamma Pr}\right)^{1/2}. \tag{3.17}$$

The constants k, k_1, k_2, k_3 are empirical and yet unknown. Oerlemans and Grisogono (2002) point out that there are currently not sufficient observational data to allow a reliable determination of these constants. Using the first estimates of k_i from Oerlemans and Grisogono (2002), Munro (personal communication) has shown that the flux parameterization given by (3.17) constitutes an effective approach to extensive modeling of turbulent heat exchange over glaciers. This was done by comparison of estimates from (3.17) and Monin-Obukhov theory for data from the Peyto glacier.

Although observations are required for a final word on the value of empirical constants, a numerical model can be used to gain valuable insight prior to the observations. In **paper IV** the analytical predictions from (3.15)-(3.16) are compared to the simulations of a modern mesoscale numerical model, COAMPS. The study was performed in three steps:

(i) The realistic ability of COAMPS to simulate a katabatic flow was verified by comparison to available data.
(ii) A series of idealized simulations were performed. These simulations were set up to mimic as closely as possible the conditions of the analytic scaling.
(iii) The analytic model was applied to the real-case simulated background parameters, and its predictions compared to the corresponding quantities simulated by COAMPS.

In this way, the limitation of lack of data is overcome by determining the constants from simulated fields. The realistic ability of COAMPS to simulate a katabatic flow supports the representativity of the values found.

A principal agreement between the observed and simulated flows were found in **paper IV**. In addition scaling arguments provided support for classifying the simulated low-level flow as a shooting flow (Mahrt, 1982), supporting the notion that COAMPS realistically simulates katabatic flows. The scaling analysis also revealed that cross-slope advection can be of importance for the flow.

Idealized simulations were set up to emulate as far as possible the horizontal homogeneity of the analytical model. The constants were determined from several simulations using different slope angles and different potential temperatures in the lowest level of the soundings. Finally the analytical model results were compared to the simulated fields. Discrepancies were found where the analytical model underestimates the jet speed. These discrepancies were mainly found in the lower part of the glacier were the simulated jet height is above 10 m. A possible reason put forward in **paper IV** is that the proximity of the jet to the surface is important. In the paper it is hypothesized that local effects like surface inhomogeneity (the surface was very inhomogeneous during the observations) and slope geometry may lift the jet to a higher height than would have been the case under the pure katabatic conditions assumed in the analytical model. This would lead to weaker friction and higher wind speed in the jet. Similar observations were also made for the jet height and the surface sensible heat flux.

3.4 The Ekman layer

In steady homogeneous conditions the simplest boundary layer model for rotating fluids, such as the atmospheric boundary layer, is the Ekman layer. Although almost 100 years old, the Ekman layer remains an important part of our understanding of the geophysical boundary layer (see e.g. Pedlosky, 1987; Chereskin, 1995; Kundu and Cohen, 2002) In the oceans the Ekman layer is perhaps best known through the definition of the Ekman depth, or *upper frictional depth*, where the wind-driven current is in exactly the opposite direction of the wind forcing it (e.g. Defant, 1961).

In the Ekman layer the pressure gradient, Coriolis force and turbulent friction forces are balanced to yield a flow vector that spirals with height (or depth). Direct confirmation of the existence of the Ekman spiral in real geophysical boundary layers have long eluded science. The reason for this is the strong noise to signal ratio in observations. Although some evidence have appeared in the literature (e.g. Kundu, 1977) it is not until recently that we have found more detailed evidence of the existence of the Ekman layer (e.g. Chereskin, 1995).

The Ekman layer is governed by the equations (Ekman, 1905; Kundu and Cohen, 2002):

$$\frac{Du}{Dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + fv - \frac{\partial \overline{u'w'}}{\partial z}, \tag{3.18}$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} - fu - \frac{\partial \overline{v'w'}}{\partial z}. \tag{3.19}$$

Assuming steady and nonadvective state ($\frac{D}{Dt} \equiv 0$), geostrophy ($v_g = \frac{1}{f\rho_0}\frac{\partial p}{\partial x}$, $u_g = -\frac{1}{f\rho_0}\frac{\partial p}{\partial y}$), and again using the flux-gradient theory ($\overline{u'w'} = -K\frac{\partial u}{\partial z}$, $\overline{v'w'} = -K\frac{\partial v}{\partial z}$), and after (3.19) is multiplied by $i = \sqrt{-1}$, these equations can be summed to yield:

$$K\frac{\partial^2 \Phi}{\partial z^2} + \frac{\partial K}{\partial z}\frac{\partial \Phi}{\partial z} - if\Phi = 0, \tag{3.20}$$

which is the single, complex, governing equation for the Ekman layer. Here, u, v and w are the components of the wind in x, y and z direction respectively, p is the atmospheric pressure, ρ_0 is the reference density, f is the Coriolis parameter and K is the eddy diffusivity for momentum. Φ is a complex variable, $\Phi = (u - u_g) + i(v - v_g)$. The boundary conditions are: $\Phi(0) = -u_g - iv_g$, $\Phi(\infty) = 0$.

If $K = Constant$, (3.20) is analytically solvable and, assuming that the coordinate system is aligned such that $v_g \equiv 0$, the real and imaginary parts of Φ yield:

$$u = u_g \cdot \left(1 - e^{-\zeta z}\cos\zeta z\right) \tag{3.21}$$

$$v = u_g \cdot e^{-\zeta z}\sin\zeta z \tag{3.22}$$

$$\zeta = \sqrt{\frac{f}{2K}}, \tag{3.23}$$

where ζ relates to the inverse of the layer height. An example of typical Ekman layer profiles is given in fig. 3.2 together with a hodograph of the flow.

Grisogono (1995) calculated Ekman layer profiles for different eddy diffusivities. However, in that paper all eddy diffusivities are non-zero at the ground. This property is unphysical. In view of the success with applying the patched two-solution WKB method to the Prandtl model in **paper**

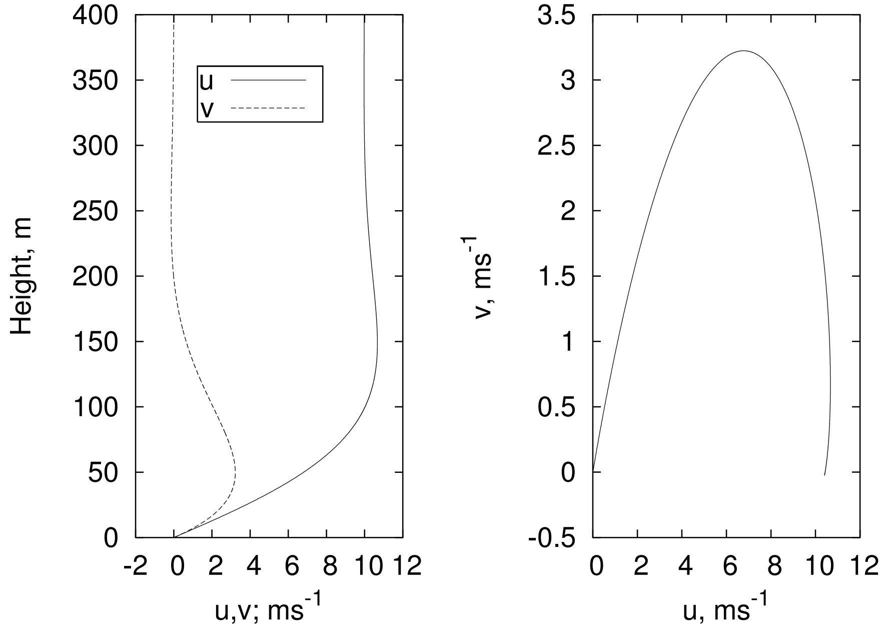

Figure 3.2: An example of the Ekman layer profiles and their hodograph.

II this approach was tried on the Ekman layer also in **paper III**. In **paper III** a profile of the eddy diffusivity was assumed like that used in **paper II**. In absence of observed data a fully numerical solution was obtained from the governing equation (3.20). It was concluded that although the WKB approach used in **paper II** - where the 0^{th} and 1^{st} order solutions were patched at the height of the maximum eddy diffusivity, H_K - lowered the height of the maxima in both velocity components, it did little to lower the maximum of the components. Since the latter is an important feature of the fully numerical solutions, this was clearly discomforting. An improvement was devised by remaining with the initial idea to patch two solutions, but performing a more detailed analysis of the validity of the WKB approximation. This led to a new requirement on the height of patching, z_p, to as low as possible while retaining the asymptotic convergence of the mathematical model:

$$z_p = \frac{1}{4}\left[W\left(\frac{2}{\sqrt{a}}\right)\right]^2. \tag{3.24}$$

In (3.24) the eddy diffusivity near the surface have been expanded for small z as $K(z) \approx az$. Also, W is Lambert's W-function[1] (e.g. Corless et al. 1997).

Using the improved patching height given by (3.24) considerably improves the structure of the Ekman layer. This is most evident when looking at hodographs of the flow components. Comparison of the hodographs in **paper III** to those in Kundu (1977) or in particular to those in Chereskin (1995) show a clear similarity. It was also shown in **paper III** that the error in the vertically integrated cross-isobaric mass-flux was lowered to about $1/10$ of that for a solution using a constant eddy diffusivity.

[1]Lambert's W(c) function is the solution to the transcendental equation $We^W = c$.

3.4.1 Improving the results from paper II

The principles followed when deriving the improved patching height given by (3.24) are equally valid for the Prandtl model of **paper II**. A comparison of the governing equations for the Prandtl model (3.4) and the Ekman layer (3.20) shows that they are very similar in their mathematical structure. The same is true for the solutions, even though (u, θ) for the Prandtl model is replaced by (u, v) for the Ekman layer.

From the similarity of the governing equations and solutions one may anticipate that the improvement to the Ekman layer from using (3.24) - lowering of the maximum wind speed - would be found in the Prandtl model also. Figure 3.3 shows scatter plots of the observed and modeled maximum jet wind speed in the Prandtl model, given by (3.10)-(3.14). The left panel is the same as that found in **paper II**, i.e. with a solution patched at H_K. The right panel shows the same variables with the difference that the patching is done at z_p. Although an increased scatter around the one to one line may be seen in the solution using z_p, the systematic overestimation is gone.

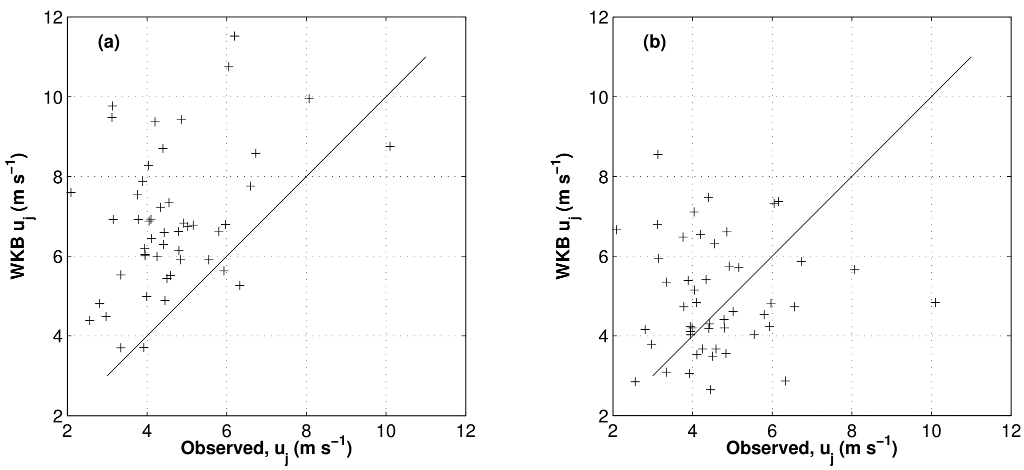

Figure 3.3: Observed and calculated maximum wind speed in the katabatic jet. a) Results from paper II, b) results using z_p from (3.24).

Chapter 4

Conclusions and outlook

The goal of the studies in this thesis has been to improve the description of air flow close to glaciers. The question of near surface flow over glaciers and the associated energy balance is an important part of our understanding of the response of Earth's climate system to an increased greenhouse gas concentration. Mountain glaciers contain only a minor part of Earth's land ice, yet their impact on human activity is significant. Mountain glaciers are also expected to contribute significantly to sea-level rise, on a century time scale. Although the majority of the melting energy on a glacier comes from solar radiation, it has been shown that the glacier sensitivity to climate change is controlled by longwave radiation and turbulent energy fluxes at the surface. Thus, the understanding of the near surface flow over mountain glaciers is important. In an effort to shed light on some aspects of this question a combination of models, analytical and numerical, and observations have been used in this thesis. The combination of models and observations is a powerful tool for understanding the basic principles governing the atmospheric behavior. Three different approaches have been used: linear wave modeling, analytic modeling of katabatic flow and analytic modeling of the Ekman layer. Among the findings are:

A large part of the flow over a large glacier like Vatnajökull can be described by linear wave theory. However, due to its simplicity, such a model can not in an adequate way describe the flow in regions that are shielded from the surroundings (like Breidamerkurjökull). It is also inherently unable to estimate the surface energy flux unless connected to another model.

Very good results can be obtained for surface energy fluxes in katabatic flow using simple analytical models.

Care must be taken when applying the very simplest models so that dominant features in the environment are not neglected and thereby degrade the model results.

The very simple Prandtl and Ekman models illustrate important features of geophysical flow. Despite their reputation as inaccurate, they can perform well if the strong restraint of constant eddy diffusivity is relieved.

Todays sometimes breathtaking development in computer technology promises to let us run even larger and more detailed numerical models in the near future. Despite this, simple analytical models will remain valuable. One reason for this is that they are simple to understand, we can see a direct coupling between forcings and results. Another reason, and perhaps the most important, is that for the near future, or as long as we can predict, our numerical models will need to parameterize small scale processes. In particular this may be the case for the stable boundary layer in which a very high vertical resolution is required for accurate simulations. With the increase in computer power, our requirements on the results, and thereby resolution and computational domain, will also increase.

Naturally, these issues become even more important when considering climate models that are run for hundreds (or thousands) of years and whose simulations guide us in our decisions for the future. It is important that we strive to include in our models as many as possible of the important physical processes interacting in the climate system.

Acknowledgments

Finally, the last chapter! Pfeeuuw!
This thesis would never have come about if it had not been for several people. Outstanding of these are of course my two supervisors Erland Källén and Branko Grisogono. Thank you Erland, for putting me on to the track of PhD studies in the first place, and for your continuous support as my supervisor during these years. And thank you, Branko, for all that knowledge you've shared and all the discussions we've had (and the food and wine). And not least for your support when my mood was down. I would also like to mention Hans (aka Johannes) Oerlemans, who has taught me a lot about science and research. I'm happy to have cooperated with you. Several people have helped me with good advice on the way. Among them are: Michael Tjärnström, Peter Lundberg, Carmen Nappo, Larry Mahrt and Donal Murtagh.

When I first entered MISU as an expectant student, I was happy to find a light, friendly and enjoyable department. That feeling has stayed with me over the years. Thank you all for making my days at MISU pleasant. In particular I would like to thank Ulla with whom I worked when I was student counselor. That was lots of fun. Thank you for your support. And of course, I would like to thank Eva for.., for.., running the show, basically. What would any of us do without you? I would also like to thank my past roommates: Jenny, Annika, Nedjeljka and Anna. Thank you all for discussions on all conceivable matters, flowers, cookies and what else. Stefan turned into not only a friend but also a colleague. Thank you for a very nice cooperation on paper IV. It really is much more fun to work together than alone. All the PhD students at MISU, from the day I started as a student until today. The good spirit among the students at MISU is really valuable, and I owe you all.

After I left MISU I ended up deep in the woods at the computational physics group at FOI. Imagine that I could find another place where the scientific atmosphere is just as good, and friendship too. Thank you guys for making my out-of-university time so nice.

Finally, I would like to thank my family and friends for having had patience with me over the last few years, as I grew more and more busy. I'm back now. Lastly, the one person who carried me through it all, and to whom I owe so much. Sara, I love you!

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