A New Objective Criterion and Rounding Techniques for Determining Virtual Topologies in Optical Networks

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***Abstract*— This letter addresses the problem of determining virtual topologies in wavelength-routed WDM optical networks. To solve this problem, it is necessary to determine the set of lightpaths which comprise the virtual topology. Packet-switched traffic is then routed over this set of lightpaths, completely independent of the physical topology. Common objectives include minimizing congestion or packet hop distance. In this letter, we introduce an additional objective criterion, discuss its importance, and derive an effective lower bound. Several solution approaches use MILP formulations solved using LP-relaxations and rounding techniques. Proposed are alternative rounding schemes for determining more effective virtual topologies.**

***Index Terms*— Virtual topology design, WDM optical networks, LP-relaxation**

I. INTRODUCTION

WAVELENGTH division multiplexing (WDM) is a technology that can exploit the large bandwidth of optical fibers by dividing it among different wavelengths. In *wavelength-routed* WDM networks, a virtual topology is created over the physical optical network by establishing all-optical connections, called *lightpaths*, between pairs of nodes. Lightpaths transmit data entirely in the optical domain even though they can traverse multiple physical links. In order to design a virtual topology, it is necessary to determine the set of lightpaths which comprise it. Packet-switched traffic can then be routed over the established virtual topology independent of the underlying physical optical network. In order to establish a given set of lightpaths, it is necessary to find for them corresponding paths in the physical topology and assign wavelengths to them, i.e. solve the Routing and Wavelength Assignment (RWA) problem. This letter focuses on determining virtual topologies, while RWA can be solved subsequently using various algorithms available in literature such as [1].

Determining a good virtual topology with respect to various optimization criteria is a complex problem. Several variations of the problem have been studied [2] which include designing regular [3] and arbitrary ([4], [5], [6]) virtual topologies. In [4], an exact mixed integer linear formulation (MILP) for virtual topology design in WDM networks with full wavelength conversion with the objective to minimize the average packet hop distance is given. In [6], the authors formulate a MILP for virtual topology design with the objective to minimize congestion. Most algorithms suggested for virtual topology design are evaluated by considering a single optimization criterion to be the measure of quality of their obtained solutions. In this letter, we discuss common objectives for virtual topology design and introduce a new objective criterion which we call *virtual hop distance* which is independent of the traffic matrix. We discuss its importance and derive an effective lower bound. Furthermore, efficient rounding techniques to obtain virtual topologies from LP-relaxations are proposed and tested.

II. THE VIRTUAL TOPOLOGY DESIGN PROBLEM

The physical optical network is modelled as a graph $G = (V, E)$, where V is the set of nodes ($|V| = N$) and E is the set of bidirectional edges. Given is a long-term traffic matrix $\Lambda = (\lambda^{sd}), s, d \in V$, where each element represents the average traffic flow from a source node s to a destination node d. The number of available transmitters and receivers Δ_l at each node, i.e. the maximum logical degree, is also given. The virtual topology design problem searches for a set of lightpaths which creates a virtual topology on top of the physical topology and can be modelled as a directed graph where each edge represents one lightpath. After establishing a virtual topology, packet switched traffic given in Λ must be routed over it. Various objectives can be considered and are discussed in the next section.

III. A NEW OBJECTIVE CRITERION: THE AVERAGE VIRTUAL HOP DISTANCE

The most common objective criterion used in virtual topology design is the minimization of *congestion*. Congestion is defined as the maximum traffic load on any virtual link. If delay is an important issue, it is desirable to minimize the average number of lightpaths traversed by a unit of traffic (packet) on its path from source to destination in the virtual network. This is called the *average packet hop distance* and is a function of the virtual topology and the long-term traffic matrix.

An optimization criterion that has not been considered in research dealing with virtual topology design is a measure which we refer to as the ***average virtual hop distance***. The average virtual hop distance is the average hop distance in the virtual topology between all source - destination pairs, i.e. the average path length of the virtual topology. This is a function of the virtual topology alone and is independent of the current traffic in the network. We feel that this criterion, in combination with the average *packet* hop distance, is relevant due to the following. If the average *packet* hop distance is low but the average *virtual* hop distance is very high, this means that low-traffic source - destination pairs are poorly connected. Since traffic can be prone to change, and reconfiguration of the

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virtual topology can be very costly due to service disruption, it seems that such a virtual topology could perform poorly on the long run. On the other hand, if the virtual topology has not only a low average *packet* hop distance, but a low *virtual* hop distance as well, we know that *all* the source - destination pairs are fairly well connected. Therefore, in addition to performing well for current traffic trends, the virtual topology would perform well for changing traffic and thus postpone reconfiguration for a longer period of time. Furthermore, ensuring a finite average virtual hop distance would eliminate unconnected virtual topologies. Suppose that there is zero traffic between a pair of nodes in the current traffic matrix. If such is the case, the hop distance between these nodes would not enter into the calculation of the average *packet* hop distance since there are no packets delivered between these two nodes. Therefore, without considering the average virtual hop distance, the distance between these nodes could be arbitrarily long or the two could even be unconnected. In the case of the latter, not even a single packet could be sent between these nodes without reconfiguration of the virtual topology. Thus, we suggest considering the virtual hop distance in conjunction with congestion and packet hop distance when solving the virtual topology design problem.

A. A lower bound on the average virtual hop distance

We now derive a lower bound for the average virtual hop distance which we will refer to as $\overline{H_v^{LB}}$. Since the average virtual hop distance is independent of the traffic matrix, the lower bound on the average virtual hop distance from any node $s \in V$ to all the other nodes in the network is the same for each node s. Therefore, the lower bound on the overall average virtual hop distance in the network is the same as the lower bound for any one node.

As noted in [6], if a network has a maximum logical degree of Δ_l, for some node $s \in V$ there can be at most Δ_l nodes one hop away from s, at most Δ_l^2 nodes two hops away, at most Δ_l^3 nodes three hops away, etc. An ideal virtual topology with respect to virtual hop distance from some node s to the remaining nodes in the network would be such a topology in which node s had Δ_l neighbors, each of which had Δ_l neighbors of their own without creating a cycle, and so on, until all the nodes were connected. This would create a tree structure of degree Δ_l, where only the last non-leaf node could have a degree less than Δ_l, depending on the total number of nodes in the network.

Let m be the largest integer such that $N \geq 1 + \Delta_l + \ldots + \Delta_l^{m-1} = \frac{\Delta_l^m - 1}{\Delta_l - 1}$ holds. In the ideal virtual topology with respect to virtual hop distance from node s, Δ_l nodes would be one hop away from s, Δ_l^2 nodes would be two hops away, etc., up until Δ_l^{m-1} nodes that would be $(m-1)$ hops away. The remaining $(N-1) - (\Delta_l + \ldots + \Delta_l^{m-1})$ nodes would be m hops away. It follows that a lower bound on the average virtual hop distance $\overline{H_v^{LB}}$ is

$$\begin{aligned} \overline{H_v^{LB}} &= \frac{\sum_{k=1}^{m-1} k\Delta_l^k + m[(N-1) - \sum_{k=1}^{m-1} \Delta_l^k]}{N-1} \\ &= \frac{\Delta_l[\frac{(m-1)\Delta_l^m - m\Delta_l^{m-1} + 1}{(1-\Delta_l)^2}] + m(N - \frac{\Delta_l^m - 1}{\Delta_l - 1})}{N-1}. \qquad (1) \end{aligned}$$

IV. ALTERNATIVE ROUNDING ALGORITHMS TO DETERMINE VIRTUAL TOPOLOGIES FROM LP-RELAXATIONS

Solution approaches using MILP formulations for virtual topology design most often include solving the corresponding LP-relaxations and then apply various rounding techniques. Two such approaches with the objective to minimize congestion, $LPLDA$ [6] and what we refer to as $MILP + WA$ [5], run as follows. After solving the corresponding LP-relaxations, the fractional values of the variables representing the virtual topology, b_{ij}, are sorted in decreasing order and sequentially rounded to 1 if the degree constraints are not violated. If variable b_{ij} is set to 1, that means that a lightpath will be established between nodes i and j.

Such a rounding scheme may not be very effective when there is a limited number of transmitters and receivers (commonly referred to as *transceivers*) available in the network. In such a case, several potential lightpaths with lower fractional values in the LP-relaxation may be rejected due to the lack of resources. Suppose some lightpath variables representing source-destination pairs with very high traffic have slightly smaller fractional values than certain ‘low-traffic’ variables. By using the rounding techniques from [6] and [5], it is possible that these ‘high-traffic’ lightpaths are rejected in favor of ‘low-traffic’ lightpaths. Since establishing lightpaths between nodes with higher traffic (i.e. maximizing single-hop traffic) has been shown to significantly lower congestion, as well as the average packet hop distance, we think that taking traffic trends into consideration when rounding variables obtained by solving LP-relaxations may yield better results.

We propose two rounding algorithms to determine effective virtual topologies in the following subsections. Traffic routing over the chosen set of lightpaths is solved using an LP which minimizes *congestion* with only the traffic constraints from [5].

A. The TW_LPLDA algorithm

The fractional virtual topology variables obtained by solving the LP-relaxation of a MILP formulation for the problem (such as that in [6]) are multiplied by the values of their respective traffic, $\lambda_{i,j}$. The variables are then sorted in decreasing order and sequentially rounded to 1 if the degree constraints are not violated (i.e. there are available transceivers). We will refer to this algorithm as TW_LPLDA, for *T*raffic *W*eighed $LPLDA$. Alternatively, the variables could be multiplied by some factor representing the relative values of traffic normalized to a lower value, and in this manner, vary the influence of traffic on the rounding procedure.

B. The $FRHT$ algorithm

The *F*lexible *R*ounding of *H*igh *T*raffic algorithm is as follows. The virtual topology variables obtained by solving the LP-relaxation are sorted in decreasing order of their corresponding single hop traffic, i.e. $\lambda_{i,j}$. Each variable is then rounded to 1 in sequential order if its value is greater than some parameter a, where $0 \leq a \leq 1$, and if the degree constraints are not violated. The algorithm could be run multiple times with various values for a, and the best virtual topology could be selected.

TABLE I

COMPARISON OF THE CONGESTION OBTAINED USING VARIOUS ROUNDING TECHNIQUES IN THE NSF NETWORK FOR TRAFFIC MATRIXES P1 AND P2.

Traffic Matix	*Δ_l*	*LB*	*LPLDA [6]*	*MILP+WA [5]*	*FRHT*				*TW_LPLDA*
					a=0.2	*a=0.3*	*a=0.4*	*a=0.5*	
p_1	2	126.18	243.43	**145.74**	181.98	218.29	244.00	231.93	191.91
	3	84.53	102.82	**84.58**	91.76	93.93	93.93	102.89	91.61
	4	63.43	82.03	70.03	67.38	**65.85**	72.52	66.12	65.86
	5	50.75	53.49	50.94	51.56	51.59	**50.75***	**50.75***	**50.75***
	6	42.29	44.45	44.39	42.78	**42.29***	43.11	43.01	45.34
	7	36.25	36.55	36.43	37.68	**36.25***	**36.25***	**36.25***	**36.25***
	8	31.72	32.27	31.77	33.28	31.92	31.98	**31.72***	32.39
p_2	2	282.51	345.42	345.42	467.24	496.31	**329.67**	360.16	530.11
	3	189.62	195.71	195.71	**189.78**	**189.78**	**189.78**	**189.78**	**189.78**
	4	142.32	**142.33**	**142.33**	**142.33**	**142.33**	**142.33**	**142.33**	**142.33**
	5	113.87	**113.87***	**113.87***	**113.87***	**113.87***	**113.87***	**113.87***	**113.87***
	6	94.89	**94.89***	**94.89***	**94.89***	**94.89***	**94.89***	**94.89***	**94.89***
	7	81.33	**81.33***	**81.33***	**81.33***	**81.33***	**81.33***	**81.33***	**81.33***
	8	71.17	**71.17***	**71.17***	**71.17***	**71.17***	**71.17***	**71.17***	**71.17***

TABLE II

COMPARISON OF THE AVERAGE PACKET HOP DISTANCES OBTAINED USING VARIOUS ROUNDING TECHNIQUES IN THE NSF NETWORK FOR TRAFFIC MATRIXES P1 AND P2.

Traffic Matix	*Δ_l*	*LB*	*LPLDA [6]*	*FRHT*				*TW_LPLDA*
				a=0.2	*a=0.3*	*a=0.4*	*a=0.5*	
p_1	2	1.22	2.52	**1.86**	2.38	2.58	2.52	2.30
	3	1.10	1.88	1.67	1.69	**1.65**	1.88	1.67
	4	1.06	1.67	**1.56**	1.60	1.79	1.67	**1.56**
	5	1.04	1.92	1.52	**1.50**	1.53	1.54	**1.50**
	6	1.02	1.63	**1.57**	**1.57**	1.60	1.58	1.59
	7	1.01	1.57	**1.56**	**1.56**	1.57	1.60	1.57
	8	1.01	1.82	1.65	**1.59**	1.57	**1.59**	1.61
p_2	2	1.66	2.97	3.36	3.54	**2.72**	2.97	3.01
	3	1.37	2.45	2.50	2.45	2.54	2.45	**2.37**
	4	1.28	2.58	2.57	2.42	2.57	2.59	**2.37**
	5	1.21	2.36	2.37	**2.35**	2.51	2.43	2.38
	6	1.15	2.51	2.54	2.46	2.45	2.51	**2.44**
	7	1.10	**2.47**	2.50	2.50	2.48	2.55	2.51
	8	1.06	2.60	2.56	2.62	2.66	2.61	**2.42**
Average		1.16	2.21	2.13	2.16	2.16	2.18	**2.09**

TABLE III

COMPARISON OF THE AVERAGE VIRTUAL HOP DISTANCES OBTAINED USING VARIOUS ROUNDING TECHNIQUES IN THE NSF NETWORK FOR TRAFFIC MATRIXES P1 AND P2.

Traffic Matix	*Δ_l*	*LB*	*LPLDA [6]*	*FRHT*				*TW_LPLDA*
				a=0.2	*a=0.3*	*a=0.4*	*a=0.5*	
p_1	2	2.38	2.88	3.05	2.92	**2.85**	2.88	2.90
	3	1.85	2.10	**2.08**	2.09	2.10	2.10	2.13
	4	1.69	**1.78**	1.81	1.82	1.83	**1.78**	1.85
	5	1.62	1.65	1.66	1.65	**1.63**	1.64	1.65
	6	1.54	**1.54***	**1.54***	**1.54***	**1.54***	**1.54***	1.55
	7	1.46	1.47	1.48	1.47	1.47	**1.46***	**1.46***
	8	1.38	1.39	**1.38***	1.39	1.40	1.39	1.39
p_2	2	2.38	2.98	2.96	2.92	**2.80**	2.98	3.05
	3	1.85	2.05	**2.02**	2.03	2.07	2.05	2.14
	4	1.69	1.87	**1.82**	1.85	1.85	1.86	1.86
	5	1.62	1.67	1.68	1.69	1.67	**1.66**	1.68
	6	1.54	**1.54***	1.55	1.55	1.55	**1.54***	**1.54***
	7	1.46	1.50	**1.48**	1.49	1.50	1.49	1.49
	8	1.38	1.41	**1.40**	**1.40**	1.41	1.41	1.41
Average		1.70	1.90	1.92	1.87	**1.86**	1.93	1.93

V. NUMERICAL RESULTS

We tested the TW_LPLDA and $FRHT$ algorithms for the 14 node NSF network [6] with two traffic matrices, p1 and p2, which correspond to Tables III and IV in [6]. These traffic matrices were used to test the $LPLDA$ and $MILP + WA$ heuristics in [6] and [5], respectively. In traffic matrix p1, most of the traffic is concentrated around 42 pairs of nodes, while traffic in p2 is more evenly distributed. The number of transmitters and receivers at each node, Δ_l, ranged from 2 to 8. The LP-relaxations were solved using the CLPEXv6 solver.

$FRHT$ was run with different values for a ranging from 0 to 1 in 0.05 increments. The best results were obtained when a ranged from 0.25-0.45. We show results with $a =$ 0.2, 0.3, 0.4 and 0.5. Lower bounds (LB) and the values for congestion obtained by the proposed rounding techniques, as well as the $LPLDA$ [6] and $MILP + WA$ [5] algorithms, are shown in Table I. Since the $MILP + WA$ algorithm has a limited number of wavelengths, the best obtained solutions from [5] are shown[1]. The best obtained solution for each case is marked in bold. If the obtained solution is equal to the lower bound, i.e. the obtained solution is surely optimal, it is marked as '*'. The proposed rounding techniques outperformed the previously suggested ones in many cases.

The average packet and virtual hop distances for the proposed heuristics are shown in Tables II and III, respectively. Since we ran our rounding techniques on the LP-relaxation used by $LPLDA$, we found the average packet and virtual hop distances for the the virtual topologies obtained by $LPLDA$ as well. For all cases, at least one of the proposed approaches obtained a solution better than or equal to that obtained by $LPLDA$.

VI. CONCLUSION

In order to efficiently utilize resources in wavelength-routed optical networks, it is necessary to successfully solve the virtual topology design problem. We suggest an additional objective criterion, referred to as the virtual hop distance, and derive an effective lower bound. This criterion is aimed at postponing the need for costly reconfiguration and ensuring connected virtual topologies. Furthermore, alternative rounding techniques are suggested to improve upon solutions obtained from LP-relaxations of MILP formulations of the problem. Further avenues of research will include developing similar algorithms for establishing multiple virtual topologies on a single fiber plant.

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[1] Testing indicated that raising the number of wavelengths further does not necessarily give better results for $MILP + WA$.