Robustness of PM Brushless DC Motor Drive Adaptive Controller with Reference Model and Signal Adaptation Algorithm

P. Crnosija*, T. Bjazic*, R. Krishnan, *Fellow IEEE*†, H. Y. Yang†
*University of Zagreb, Faculty of El. Eng. and Computing, Zagreb, Croatia
†The Bradley Department of El. & Comp. Eng., Virginia Tech, Blacksburg, USA
petar.crnosija@fer.hr, kramu@vt.edu

***Abstract* – This paper covers an application of MRAC with modified signal adaptation algorithm with optimal error weighting coefficients in a permanent magnet brushless DC motor drive with significant parameter changes in moment of inertia, armature resistance and flux linkage coefficient. Optimal values of the error weighting coefficients are determined for third order reference model, using Matlab and simplex optimization method. Presented results show that one set of optimal error weighting coefficients can compensate for any parameter change reducing the error from 41-57% to 4-8% in relation to a reference model thus showing that MRAC with modified signal adaptation algorithm is robust.**

I. INTRODUCTION

The influence of minor parameter changes on a drive dynamic behaviour may be satisfactorily compensated using standard control algorithms, to ensure the required quality and accuracy of the system response. However, the effects of substantial parameter variations can no longer be effectively compensated by standard control algorithms. Therefore, some kind of adaptive control should be used, i.e. a gain scheduling system [1], [6], self-tuning controller [1] or model reference adaptive control (MRAC) [1], [2], [6]. Adaptive systems with reference model and parameter adaptation algorithm are described in [1], [6] and with signal adaptation algorithm in [2], [3].

The main advantage of model reference adaptive control with signal adaptation is that it does not contain integral parts and hence it does not need online tuning of controller parameters for changed plant parameters [2], [3].

In author's previous paper [8] a model reference adaptive control with signal adaptation in modified form is investigated. Optimal error weighting coefficients are determined by *Matlab – Simulink* [9] and *simplex* optimization method [10]. They are then applied in permanent magnet brushless DC motor drive with moment of inertia varying from half to twice the nominal value.

This paper covers drive behaviour when moment of inertia changes even more drastically: 3 times lower ($J = 0.33J_n$) and 3 times higher ($J = 3J_n$) than nominal. The error weighting coefficients applied are those determined for change of moment of inertia: $J = 0.5J_n$ and $J = 2J_n$. Further optimization is conducted for change of moment of inertia: $J = 0.33J_n$ and $J = 3J_n$ and applied for lower range of change: $J = 0.5J_n$ and $J = 2J_n$. Influence of armature resistance and flux linkages coefficient changes is also considered. Results are given in fifth section.

II. MODEL REFERENCE SIGNAL ADAPTATION ALGORITHM

The signal adaptation algorithm generates additional control signal u_A which minimizes the difference between reference model y_M and system output y (Fig. 1). The adaptation signal u_A acts on the system input so that the adaptation mechanism (reference model and signal adaptation algorithm) forms an outer control loop, while the adjustable system with main PI controller forms an inner control loop.

Model reference adaptive controller with modified signal adaptation algorithm is described in [8].Due to space constraint, this paper contains only necessary final form of algorithm with saturation function:

$$u_A(t) = \text{sat}(\nu(t), h) = \begin{cases} h, & \text{for } \nu(t) > \nu_s \\ K_\nu \nu(t), & \text{for } |\nu(t)| \le \nu_s \\ -h, & \text{for } \nu(t) < -\nu_s \end{cases}, \quad (1)$$

$$\nu(t) = \mathbf{d}^T \mathbf{e}(t), \quad (2)$$

where: ν(t) is a generalized error, $\mathbf{d}^T$ is an error weighting coefficient vector, ν_s is a region where the saturation function is linear, K_ν is the gain coefficient of generalized error ν(t), h is the value of saturation.

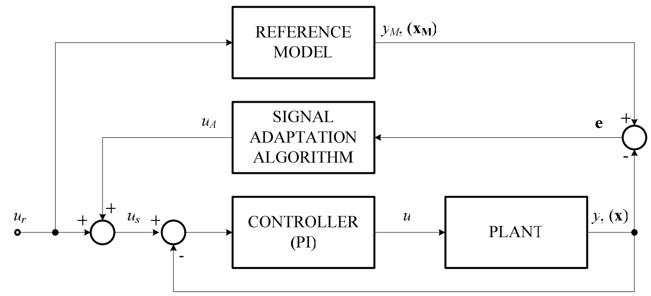

Fig. 1. Adaptive control system with reference model and signal adaptation controller.

To illustrate the method, modified signal adaptation algorithm (1) was applied for parameter changes and load torque compensation of a PMBDCM drive and results are discussed in the fifth section.

III. MODEL OF PM BRUSHLESS DC MOTOR DRIVE

This model is based on the PM brushless DC motor drive discussed and given in [4], [5]. For the sake of easy reference, the model is derived in brief and given in the following. During two phase conduction, the entire dc voltage is applied to the two phases and the transfer function for the stator current is given by (Fig. 2):

$$\frac{I_{as}(s)}{V_{is}(s)-E(s)}=\frac{K_a}{1+T_a s}, \quad (3)$$

where: $K_a = 1/R_a$, $T_a = L_a/R_a$, $R_a = 2R_s$, $L_a = 2(L - M)$, R_s is the stator resistance per phase, L is the self inductance per phase, M is the mutual inductance per phase, E is the induced emf and s is the Laplace operator.

The induced electromagnetic force (emf) E is proportional to rotor speed Ω_m:

$$E = K_b \Omega_m, \quad (4)$$

where:

$$K_b = 2\lambda_p, \quad (5)$$

and λ_p is the flux linkages per phase (volt/rad/sec).

Note that the electromagnetic torque for two phases combined is given by:

$$T_e = 2\lambda_p I_{as} = K_b I_{as}. \quad (6)$$

The load is assumed to be proportional to speed:

$$T_l = B_t \Omega_m. \quad (7)$$

With that included in the feedback path, the speed to air gap torque transfer function can be evaluated as (Fig. 2):

$$\frac{\Omega_m(s)}{T_e(s)}=\frac{K_t}{1+T_t s}, \quad (8)$$

where: K_t=1/B_t, T_t= J/B_t, B_t=B_1+B_2, B_1 is the friction coefficient of the motor, B_2 is proportional coefficient between load torque and speed and J is the inertia of the machine.

Inverter transfer function is given by:

$$\frac{V_{is}(s)}{V_c(s)}=\frac{K_r}{1+T_r s}, \quad T_r=\frac{T_{ch}}{2}=\frac{1}{2f_{ch}}, \quad (9)$$

where f_{ch} is switching (PWM) frequency.

The current and speed feedbacks have low pass filters with transfer functions (Fig. 2):

$$\frac{I_{am}(s)}{I_{as}(s)}=\frac{K_c}{1+T_c s}, \quad (10)$$

$$\frac{\Omega_{mr}(s)}{\Omega_m(s)}=\frac{K_\omega}{1+T_\omega s}. \quad (11)$$

Numerical value of the drive parameters are:

Base speed, n_b = 4000 rev/min, Base power, P_b = 373 W, Base current, I_b = 17.35 A, Base voltage, V_b = 40 V, Base torque, T_b = 0.89 Nm, Supply voltage, V_s = 160 V, Maximum phase current, I_{max} = $2I_b$ = 34.7 A, Maximum torque, T_{max} = $2T_b$ = 1.78 Nm, Gain of the inverter, K_r = 16 V/V, Time constant of the converter, T_r = 50 µs, Phase resistance, R_a = 1.4 Ω, Phase inductance L_a = 2.44 mH, Phase time constant, T_a = L_a/R_a = 1.743 ms, K_a = $1/R_a$ = 0.71428 A/V, Emf constant, K_b = 0.051297 Vs, Total friction coefficient, B_t = 0.002125 Nm/rad/sec, Inertia, J = 0.0002 kgm^2, K_t = $1/B_t$ = 41.89, Motor and load time constant, T_t = J/B_t = 94.1 ms, Current feedback gain K_c = 0.288 V/A, Current feedback time constant, T_c = 0.159 ms, Speed feedback gain, K_ω = 0.02387 Vs/rad, Speed feedback time constant, T_ω = 1 ms.

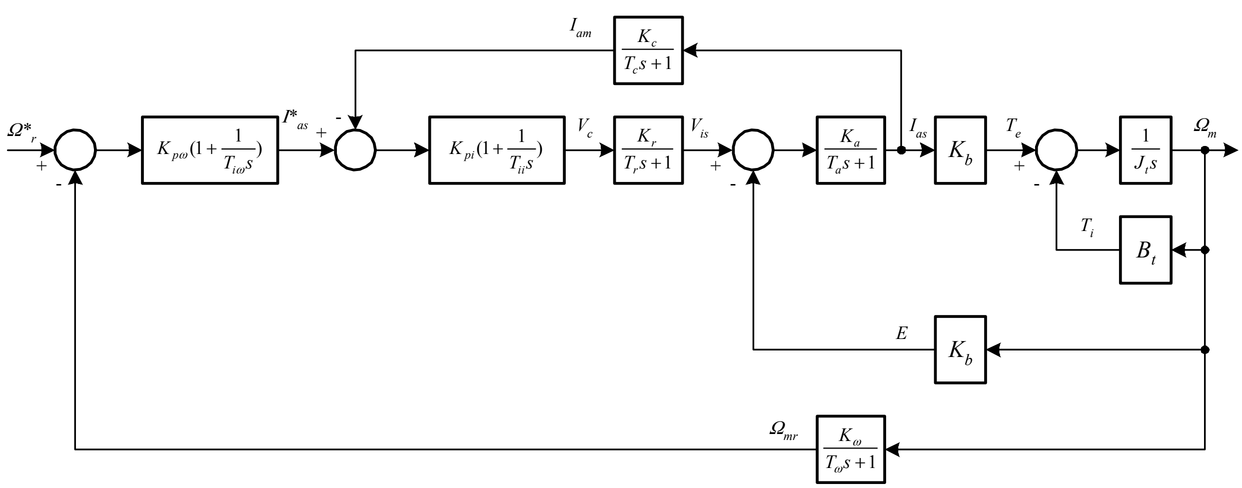

Fig. 2. Block schematic of cascade speed control system of PM brushless DC motor drive.

IV. CURRENT AND SPEED PI CONTROLLER DESIGN

Integral time constant of the current controller is usually chosen to be equal to the armature time constant (compensates maximum time constant in the current loop): $T_{ii} = T_a = 1.743$ ms. For the overshoot $M_{pi} = 5\%$, current controller gain coefficient determined from the Bode plot and simulation is $K_{pi} = 1.267$.

Main PI speed controller parameters are determined using transient performance based design optimization of PMBDCM [7]. The controller designed in this way is able to achieve faster and better load torque compensation than in the case of standard controller design [7]. Speed controller integral time constant is picked as: $T_{i\omega} = 0.125T_t = 11.76$ ms and its gain coefficient is chosen for the speed feedback signal overshoot $M_{p\omega mr} = 40\%$: $K_{p\omega} = 44.9$. To achieve system overshoot $M_{p\omega mr} = 10\%$, first order filter with time constant $T_f = 1.96$ ms has been added to the drive input.

V. IMPLEMENTATION OF MRAC WITH SIGNAL ADAPTATION ALGORITHM

This paper covers the application of modified signal adaptation algorithm (1) with three state space variables and third order reference model. First state space variable is speed feedback signal, while other two are its first and second derivative. Derivations are calculated approximately as follows:

$$G_1(z) = \frac{\dot{\Omega}_{mr}(z)}{\Omega_{mr}(z)} = \frac{z-1}{T_s z}, \qquad (12)$$

$$G_2(z) = \frac{\ddot{\Omega}_{mr}(z)}{\Omega_{mr}(z)} = \frac{z^2-2z+1}{T_s^2 z^2}, \qquad (13)$$

where $T_s = 50$ μs is sampling time of algorithm.

Reference model is chosen to satisfactorily describe drive behavior with nominal parameters. Its behavior can be satisfactorily described by third order transfer function:

$$G_M(s) = \frac{\Omega_{Mmr}(s)}{U_r(s)} = \frac{1}{(1+T_f s)(1+2\zeta T_n s + T_n^2 s^2)}, \qquad (14)$$

where Ω_{Mmr} is reference model output, $T_f = 1.96$ ms is a filter time constant, while ζ and T_n are determined by optimization and equal: $\zeta = 0.318$, $T_n = 1.197$ ms.

Discretization of the reference model is made with *zero order hold* (ZOH) blocks in input and output of the reference model. Sample time of ZOH blocks is set to $T_s = 50$ μs.

Error weighting coefficient vector **d** (size 3×1) is determined by optimization based on *integral square error* (ISE) criterion:

$$I = \int e^2(t)\,dt, \qquad (15)$$

where:

$$e(t) = \omega_{Mmr}(t) - \omega_{mr}(t). \qquad (16)$$

Optimization is initially carried out on reference step change $\Delta\omega^*_r(t) = 0.1\mathrm{S}(t)$, change of moment inertia to values $J = 0.5J_n$ and $J = 2J_n$, saturation value $h = 0.2$ and gain coefficient $K_v = 1$ [8]. Optimization resulted in following:

$$\mathbf{d}^T = \begin{bmatrix} 25.99 & 5.41\cdot10^{-3} & 1.97\cdot10^{-6} \end{bmatrix}. \qquad (17)$$

With these error weighting coefficients, maximum transient errors in drive on a reference step change $\Delta\omega_r^*=0.1\mathrm{S}(t)$ are reduced from 32.4% (30.4%) to 0.91% (1.88%) for $J = 0.5J_n$ ($2J_n$) (TABLE I). Maximum speed feedback signal drops $\Delta\omega_{mr}$ for nominal load torque step change are given in TABLE II.

TABLE I
MAXIMUM TRANSIENT ERROR VALUES FOR REFERENCE STEP CHANGE.

J/J_n	$\mathbf{d}^T$	K_v	h	e_m (%)	Figure
0.5	-	-	-	32.4	-
2				30.4	
0.33	-	-	-	47.9	
3				47.1	
0.5	(17)	1	0.2	0.91	-
2				1.88	
0.33	(17)	0.8	0.2	1.397	-
3				4.897	
0.33	(18)	1	0.2	1.409	Fig. 5
3				4.984	
0.5	(18)	1	0.2	1.134	-
2				2.452	
0.5	(18)	1.25	0.2	0.91	-
2				1.94	

TABLE II
MAXIMUM SPEED FEEDBACK SIGNAL DROPS FOR NOMINAL LOAD TORQUE STEP CHANGE.

J/J_n	$\mathbf{d}^T$	K_v	h	$\Delta\omega_{mr}$ [V]	$\overline{\Delta\omega_{mr}}$ [%]	Figure
1	-	-	0.2	−0.13338	−1.33	-
0.5				−0.16712	−1.67	
2				−0.108	−1.08	
1	-	-	0.2	−0.13338	−1.33	
0.33				−0.19204	−1.92	
3				−0.09601	−0.96	
1	(17)	1	0.2	−0.0088231	−0.088	-
0.5				−0.01538	−0.154	
2				−0.0069731	−0.070	
1	(17)	0.8	0.2	−0.0092588	−0.093	-
0.33				−0.022667	−0.227	
3				−0.0082865	−0.083	
1	(18)	1	0.2	−0.0094717	−0.095	Fig. 6
0.33				−0.022666	−0.227	
3				−0.0084919	−0.085	
1	(18)	1	0.2	−0.0094717	−0.095	-
0.5				−0.015876	−0.159	
2				−0.0088645	−0.089	
1	(18)	1.25	0.2	−0.0089291	−0.089	-
0.5				−0.01538	−0.154	
2				−0.0072404	−0.072	

Maximum speed feedback signal drops on the step change of nominal load torque in the drive without adaptation for $J = J_n$, $J = 0.5J_n$ and $J = 2J_n$ equal −1.33%, −1.67% and −1.08%, respectively. Adaptive controller with optimal error weighting coefficients (17) reduces these drops to −0.088%, −0.154% and −0.070% respectively (that is 10 to 15 times).

Maximum transient errors in drive without adaptation on a reference step change $\Delta\omega_r^*=0.1\mathrm{S}(t)$ for $J = 0.33J_n$ ($3J_n$) equal 47.9% (47.1%) (Fig. 3). Responses of drive without adaptation on a step change of nominal load torque are shown in Fig. 4. Adaptive controller with optimal error weighting coefficients (17) and lowered gain coefficient $K_v = 0.8$ reduces the errors to 1.4% (4.9%) (TABLE I). Gain coefficient K_v had to be lowered because adaptation algorithm produces constant high frequency oscillations in drive when moment of inertia equals $J = 0.33J_n$.

Therefore, error weighting coefficients are optimized for $J = 0.33J_n$ and $J = 3J_n$ with gain coefficient $K_v = 1$. Optimization resulted in following:

$$\mathbf{d}^T = \left[20.81 \quad 4.098\cdot10^{-3} \quad 1.449\cdot10^{-6}\right]. \tag{18}$$

Maximum transient errors are slightly increased (TABLE I, Fig. 5) because optimization resulted in lower algorithm gain as seen from coefficient d_1. Responses of drive with adaptation on a step change of nominal load torque are shown on Fig. 6, while maximum speed feedback signal drops are given in TABLE II.

Applying error weighting coefficients (18) on drive with $J = 0.5J_n$ ($2J_n$) results in slightly increased transient errors in relation to coefficients given in (17), from 0.91% (1.88%) to 1.08% (2.33%) (TABLE I). This increase is insignificant, but can be compensated by increasing the gain coefficient K_v to value $K_v = 1.25$.

It is important to consider the influence of change of armature resistance R_a and flux linkage coefficient K_b on drive behaviour. Maximum error values and speed feedback signal drops for different combinations of change of armature resistance, flux linkages coefficient and moment of inertia are given in TABLE III and TABLE IV. MRAC with optimal error weighting coefficients (18) result in significant error reduction (TABLE III), but the responses on reference step change and step change of nominal load torque are a little oscillatory. Therefore, another optimization is executed for drive with worst case change of parameters, i.e. $J = 0.33J_n$, $J = 3J_n$, $R_a = 1.25R_{an}$ and $K_b = 0.8K_{bn}$. Optimization resulted in following:

$$\mathbf{d}^T = \left[18.018 \quad 4.429\cdot10^{-3} \quad 1.438\cdot10^{-6}\right]. \tag{19}$$

Maximum transient errors with optimal error weighting coefficients (19) are reduced from 37.9% (56.5%) to 1.41% (7.97%) for the worst case scenario (TABLE III, Fig. 7).

Optimal error weighting coefficients (19) applied in drive with smaller change of parameters result in slightly increased errors (TABLE III), but that increase is insignificant.

Maximum speed feedback signal drops for step change of nominal load torque are given in TABLE IV. For the worst case scenario maximum drops are reduced 5 to 10 times with optimal error weighting coefficients (19) (Fig. 8).

TABLE III
MAXIMUM TRANSIENT ERROR VALUES FOR REFERENCE STEP CHANGE.

J/J_n	R_a/R_{an}	K_b/K_{bn}	$\mathbf{d}^T$	e_m (%)
1	1.25	0.8	-	11.86
0.5	1.25	0.8	-	21.3
2				41.1
0.33	1.25	0.8	-	37.9
3				56.5
1	1.25	0.8	(18)	0.99
0.5	1.25	0.8	(18)	0.93
2				3.92
0.33	1.25	0.8	(18)	1.25
3				7.31
0.33	1.25	0.8	(19)	1.41
3				7.97
0.5	1.25	0.8	(19)	1.00
2				4.25
1	1.25	0.8	(19)	1.02
0.33	1	1	(19)	1.58
3	1	1	(19)	5.41
0.5	1	1	(19)	1.28
2	1	1	(19)	2.62
1	1	1	(19)	0.59

TABLE IV
MAXIMUM SPEED FEEDBACK SIGNAL DROPS FOR NOMINAL LOAD TORQUE STEP CHANGE.

J/J_n	R_a/R_{an}	K_b/K_{bn}	$\mathbf{d}^T$	$\Delta\omega_{mr}$ [V]	$\overline{\Delta\omega_{mr}}$ [%]
1	1.25	0.8	-	−0.15862	−1.586
0.5				−0.19762	−1.976
2				−0.12858	−1.286
1	1.25	0.8	-	−0.15862	−1.586
0.33				−0.22596	−2.260
3				−0.11416	−1.142
1	1.25	0.8	(18)	−0.012969	−0.130
0.5				−0.022305	−0.223
2				−0.011171	−0.112
1	1.25	0.8	(18)	−0.012969	−0.130
0.33				−0.031758	−0.318
3				−0.010585	−0.106
1	1.25	0.8	(19)	−0.012981	−0.130
0.33				−0.031758	−0.318
3				−0.011182	−0.112
1	1.25	0.8	(19)	−0.012981	−0.130
0.5				−0.02231	−0.223
2				−0.011656	−0.117
1	1	1	(19)	−0.0096214	−0.096
0.33				−0.022667	−0.227
3				−0.0089044	−0.089
1	1	1	(19)	−0.0096214	−0.096
0.5				−0.015887	−0.159
2				−0.0092015	−0.092

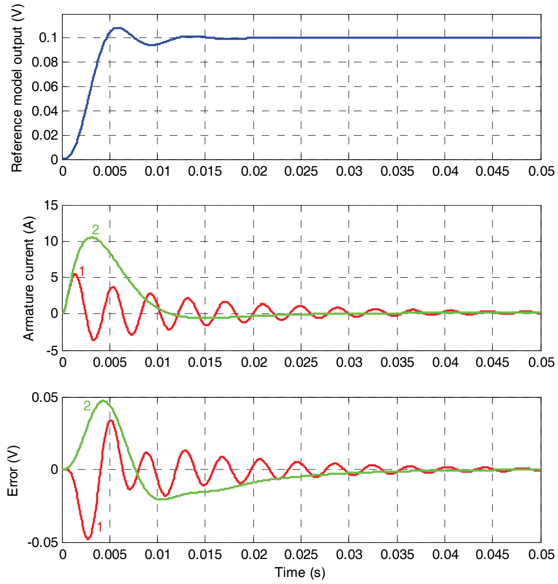

Fig. 3. Reference model output signal $\Delta\omega_{Mmr}$, armature current Δi_{as} and error e responses for a step input $\Delta\omega_r^*$=0.1S(t) and moment of inertia change in the drive without adaptation: 1. $J = 0.33J_n$, 2. $J = 3J_n$.

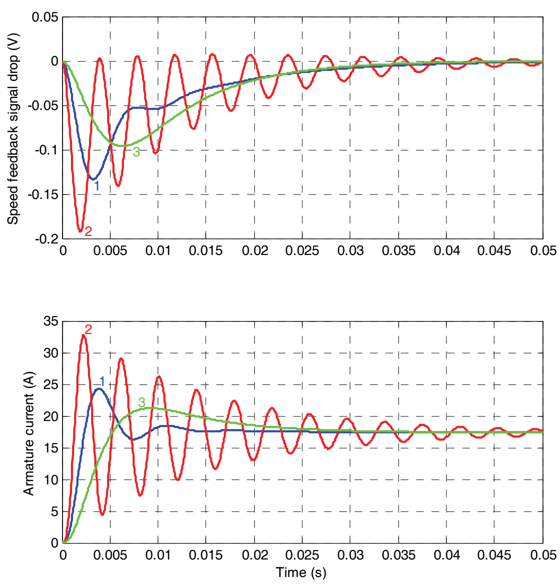

Fig. 4. Speed feedback signal $\Delta\omega_{mr}$ and armature current Δi_{as} responses for a step change of the nominal load torque T_l= 0.89S(t) and moment of inertia change in the drive without adaptation: 1. $J = J_n$, 2. $J = 0.33J_n$, 3. $J = 3J_n$.

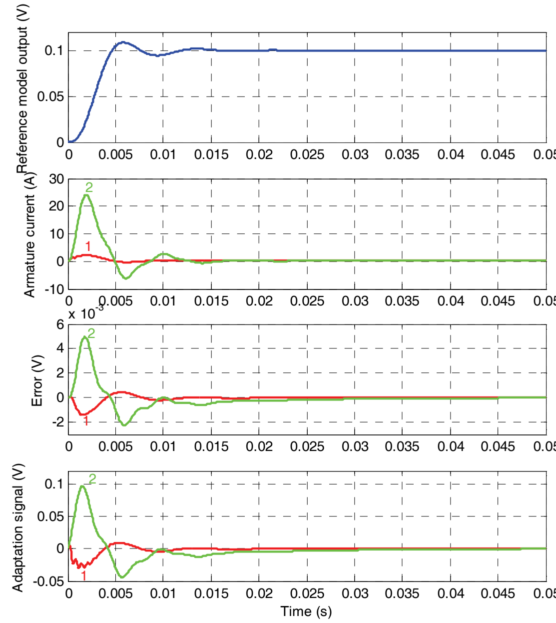

Fig. 5. Reference model output signal $\Delta\omega_{Mmr}$, armature current Δi_{as}, error e and adaptation signal u_A responses for a step input $\Delta\omega_r^*$=0.1S(t) and moment of inertia change in the drive with adaptation (18) (h = 0.2, K_v = 1): 1. $J = 0.33J_n$, 2. $J = 3J_n$.

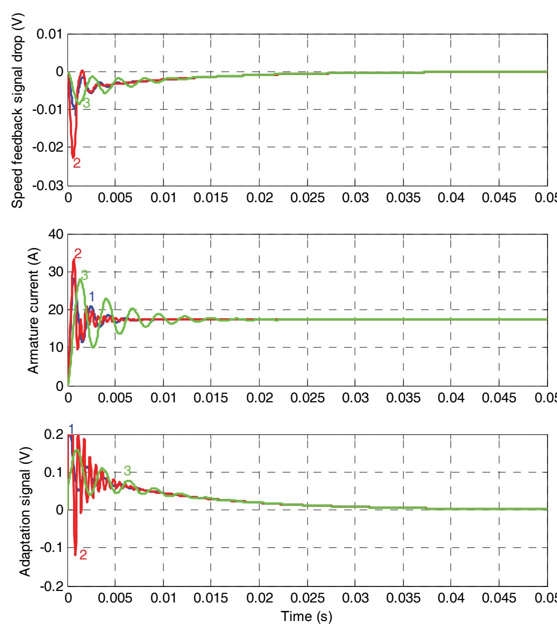

Fig. 6. Speed feedback signal $\Delta\omega_{mr}$, armature current Δi_{as} and adaptation signal u_A responses for a step change of the nominal load torque T_l= 0.89S(t) and moment of inertia change in the drive with adaptation (18) (h = 0.2, K_v = 1): 1. $J = J_n$, 2. $J = 0.33J_n$, 3. $J = 3J_n$.

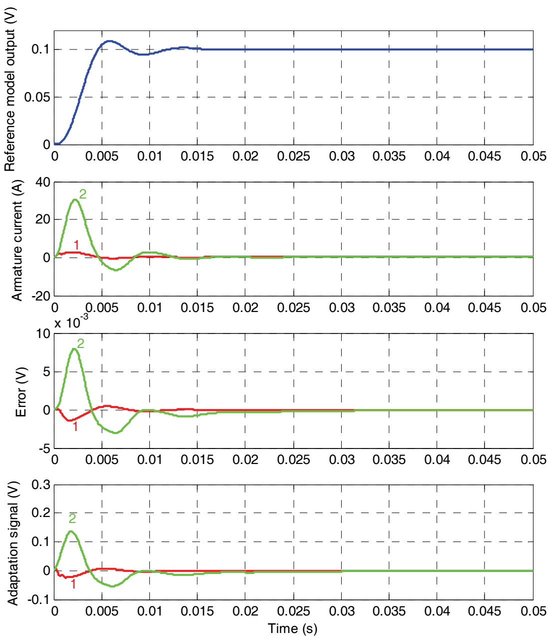

Fig. 7. Reference model output signal $\Delta\omega_{Mmr}$, armature current Δi_{as}, error e and adaptation signal u_A responses for a step input $\Delta\omega_r^*$=0.1S(t) and moment of inertia change in the drive with adaptation (19) (h = 0.2, K_v = 1): 1. $J = 0.33J_n$, 2. $J = 3J_n$, $R_a = 1.25R_{an}$, $K_b = 0.8K_{bn}$.

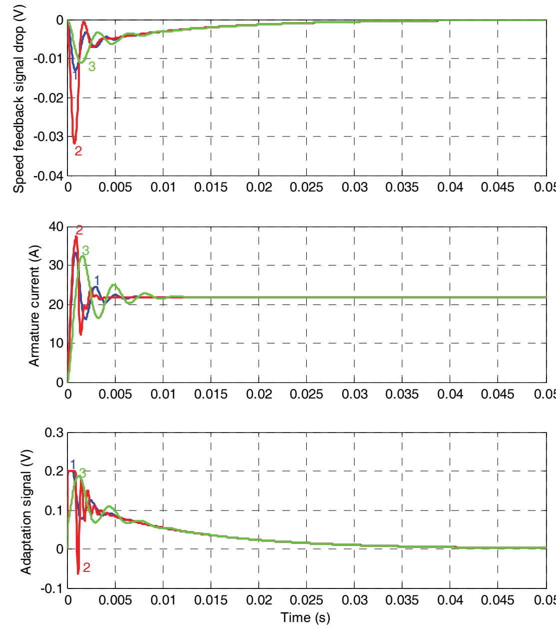

Fig. 8. Speed feedback signal $\Delta\omega_{mr}$, armature current Δi_{as} and adaptation signal u_A responses for a step change of the nominal load torque T_l = 0.89S(t) and moment of inertia change in the drive with adaptation (19) (h = 0.2, K_v = 1): 1. $J = J_n$, 2. $J = 0.33J_n$, 3. $J = 3J_n$, R_a = 1.25R_{an}, K_b = 0.8K_{bn}.

The nonlinearities in the motor drive system such as limitations on voltage supply and armature current, may have an influence on the magnitude of adaptation error. For responses on change of reference value and load torque, shown in Figures 3 to 8, those limits are inactive and, therefore, stated nonlinearities don't have an effect on maximum error values.

VI. Conclusion

Model reference adaptive control and modified signal adaptation algorithm with optimal error weighting coefficients have been applied to minimize the effects of parameters variations (moment of inertia, armature resistance and flux linkages coefficient) on the performance of PM brushless DC motor drive.

It is shown that one set of optimal error weighting coefficients can compensate those parameters variations, i.e. for the drive with $J = 0.5J_n$, $J = 2J_n$, $R_a = 1.25R_{an}$, $K_b = 0.8K_{bn}$ error is reduced from 21.3% (41.1%) to 1.00% (4.25%) and for the drive with $J = 0.33J_n$, $J = 3J_n$, $R_a = 1.25R_{an}$, $K_b = 0.8K_{bn}$ error is reduced from 37.9% (56.5%) to 1.41% (7.97%).

Maximum speed feedback signal drops for step change of nominal load torque and the worst case scenario are reduced 7 to 10 times (from 1.14 - 2.26% to 0.112 - 0.318%) with optimal error weighting coefficients of modified signal adaptation algorithm.

As the parameters variations considered are quite severe, achieved results (error 5-8%) show that MRAC and signal adaptation controller with optimal error weighting coefficients is robust. It is known that 8% error is acceptable for majority of applications in process control.

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