Optimization of PM Brushless DC Motor Drive Speed Controller Using Modification of Ziegler-Nichols Methods Based on Bodé Plots

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Abstract - The most commonly used methods for design of the industrial speed controllers are: experimental, root locus, frequency and optimization methods. The most commonly used experimental methods are Ziegler-Nichols methods: stability margin and step response function.

Ziegler-Nichols methods are applied for design of speed controller of PM brushless DC motor drive. Both methods give the controller parameters, which result in relatively high overshoot in response to reference signal. Therefore, optimal speed controller parameters are determined by modification of Ziegler-Nichols methods based on Bodé's frequency diagrams. Thereby, controller integral time constant is increased and controller gain is reduced (increased) to achieve faster and better load torque compensation than traditional controller synthesis based on compensation of maximum time constant of drive. Desired overshoot in response to reference change is achieved by adding a first order filter at the drive input. Synthesis results and responses are given in this paper.

I. INTRODUCTION

The most commonly used methods for design of the industrial speed controllers are [1], [2], [10]: experimental, root locus, frequency and optimization methods.

The most frequently used experimental methods are Ziegler-Nichols methods. Very often applied procedures of controller parameters determination in practice, for systems with expressive dead time, are based on empirical researches by Ziegler and Nichols. There are two methods available [1], [2], [10]: stability margin and step response function. Applying these methods for determining controller parameters of electric motor drives with insignificant delay time, result in relatively high drive overshoot. Therefore, in this paper optimal speed controller parameters are determined by modification of Ziegler-Nichols methods based on Bodé's frequency diagrams.

The most suitable frequency domain method for design of controller parameters is Bodé's line approximation of magnitude and phase-frequency characteristics.

In [3] a drive control system is approximated as a second order system (T_1, T_s) , where the elements with small time constants are replaced by first order element with time constant T_s equal to sum of the small time constants. The design according to the symmetrical optimum has been described for the case of integral time constant $T_i = 4T_s$.

Better load torque compensation is achieved by controller design based on symmetrical frequency characteristics for desired overshoot in a change of the reference value $M_{pd}=10-20\%$ and increased gain coefficient (2-3 times) for response overshoot $M_p=40\%$ [4]. Desired response overshoot is achieved by adding a filter at the drive input, which time constant can be determined by optimization. This synthesis procedure results in 3-4 times higher integral time constant T_i and 2-3 times higher gain coefficient than in the case of symmetrical optimum synthesis [3], [4] for response overshoot M_p =40%. In that way, the optimum values of controller parameters are obtained, resulting in the best load torque compensation and faster than technical optimum synthesis (maximum time constant compensation), but slower than symmetrical optimum synthesis.

There are different optimization methods which can be applied for electric drive controller design: gradient, simplex and Hooke-Jeves. Program package *Matlab* [7] uses gradient and simplex methods. Thereby, different optimization criteria can be used, i.e. integral error criteria and response quality indices. When standard integral error criteria are used for optimization of controller parameters of electric motor drive (ISE, ITSE, IAE, ITAE) in relation to ideal system response, they result in approximately 20% system overshoot. Very small values up to zero overshoot response can be achieved using integral square error criterion and weighted square derivative error. However, in such a case controller integral time constant is much greater than maximum time constant of system, which is not favorable for load torque compensation.

To achieve controller integral time constant smaller than maximum time constant of system when using optimization of electric motor drive according to integral criteria, it is necessary to apply reference model for generation of system behavior [5] or derive dependence between cascade control system overshoot and controller gain coefficient for different values of controller integral time constants (smaller than maximum time constant of a drive) [6]. A design procedure of electric motor drive controller parameters using reference model [5] and dynamic performance based design optimization [6] results in better and faster load torque compensation than standard (traditional) controller design and desired system overshoot in a change of reference value.

The paper is organized on the following lines. Second section of this paper describes a cascade speed control system of PM brushless DC motor drive. Third section describes speed controller parameter optimization using modification of Ziegler-Nichols methods based on Bode plot of frequency characteristics. The conclusion and references are given in fourth and fifth sections, respectively.

II. MODEL OF A PM BRUSHLESS DC MOTOR DRIVE

This model is based on the PM brushless DC motor drive discussed and given in [8], [9]. For the sake of easy reference, the model is derived in brief and given in the following. During two phase conduction, the entire dc voltage is applied to the two phases and the transfer function for the stator current is given by (Fig. 1),

$$\frac{I_{as}\left(s\right)}{V_{is}\left(s\right) - E\left(s\right)} = \frac{K_{a}}{1 + T_{a}s},\tag{1}$$

where $K_a = 1/R_a$, $T_a = L_a/R_a$, $R_a = 2R_s$, $L_a = 2(L - M)$, R_s is the stator resistance per phase, *L* is the self inductance per phase, *M* is the mutual inductance per phase, *E* is the induced emf and *s* is the Laplace operator.

The induced emf *E* is proportional to rotor speed Ω_m ,

$$E = K_{b}\Omega_{m}, \qquad (2)$$

where

$$K_{b} = 2\lambda_{p}, \qquad (3)$$

 λ_p is the flux linkages per phase (volt/rad/sec).

Note that the electromagnetic torque for two phases combined is given by,

$$T_e = 2\lambda_p I_{as} = K_b I_{as}.$$
 (4)

The load is assumed to be proportional to speed,

$$T_{l} = B_{l} \Omega_{m}. \tag{5}$$

With that included in the feedback path, the speed to air gap torque transfer function can be evaluated as (Fig. 1),

$$\frac{\Omega_m(s)}{T(s)} = \frac{K_t}{1+T_s},\tag{6}$$

where: $K_t=1/B_t$, $T_t=J/B_t$, $B_t=B_1+B_2$, B_1 is the friction coefficient of the motor and *J* is the inertia of the machine. Transistor chopper transfer function is given by,

$$\frac{V_{is}(s)}{V_{c}(s)} = \frac{K_{r}}{1 + T_{r}s},$$
(7)

where

$$T_{r} = \frac{T_{ch}}{2} = \frac{1}{2f_{ch}},$$
(8)

 f_{ch} is chopper frequency.

The current and speed feedbacks have a low pass filters with a transfer functions (Fig. 1),

$$\frac{I_{am}(s)}{I_{ax}(s)} = \frac{K_c}{1 + T_c s},\tag{9}$$

$$\frac{\Omega_{mr}\left(s\right)}{\Omega_{m}\left(s\right)} = \frac{K_{\omega}}{1+T_{\omega}s}.$$
(10)

Numerical value of the drive parameters are:

Base speed, $n_b = 4000$ rev/min, Base power, $P_b = 373$ W, Base current, $I_b = 17.35$ A, Base voltage, $V_b = 40$ V, Base torque, $T_b = 0.89$ Nm, Supply voltage, $V_s = 160$ V, Maximum phase current, $I_{max} = 2I_b = 34.7$ A, Maximum torque, $T_{max} = 2T_b = 1.78$ Nm, Gain of the inverter, $K_r =$ 16V/V, Time constant of the converter, $T_r = 50\mu$ s, Phase resistance, $R_a = 1.4\Omega$, Phase inductance, $L_a = 2.44$ mH, Phase time constant, $T_a = L_a/R_a = 1.743$ ms, $K_a = 1/R_a =$ 0.71428 A/V, Emf constant, $K_b = 0.051297$ Vs, Total friction coefficient, $B_t = 0.002125$ Nm/rad/sec, Inertia, J =0.0002 kgm², $K_t = 1/B_t = 41.89$, Motor and load time constant, $T_t = J/B_t = 94.1$ ms, Current feedback gain $K_c =$ 0.288 V/A, Current feedback time constant, $T_c = 0.159$ ms, Speed feedback gain, $K_{\omega} = 0.02387$ Vs/rad, Speed feedback time constant, $T_{\omega} = 1$ ms.

Integral time constant of the current controller is usually chosen to be equal to the armature time constant (compensates maximum time constant in the current loop): $T_{ii} = T_a = 1.743$ ms. For the overshoot $M_{pi} = 5\%$ current controller gain coefficient determined from the Bodé plot and simulation is $K_{pi} = 1.267$. With these armature current PI controller parameters, magnitude of the closed loop frequency response and closed loop transfer function follows from magnitude of the open loop frequency response [4]:



Fig. 1. Block schematic of cascade speed control system of PM brushless DC motor drive.

$$\frac{I_{as}(s)}{I_{as}^{*}(s)} = \frac{K_{ci}}{T_{ci}s+1}$$
(11)

where: $K_{ci} = 1/K_c = 3.472$, $T_{ci} = 1/\omega_{ci} = 0.4524$ ms.

With transfer function (11), open loop speed transfer function without controller follows (Fig. 1Fig.):

$$G_{p}(s) = \frac{\Omega_{mr}(s)}{I_{as}^{*}(s)} = \frac{K_{p}}{(T_{ci}s+1)(T_{i}s+1)(T_{\omega}s+1)} = \frac{2}{(0.4524 \cdot 10^{-3}s+1)(0.0941s+1)(0.001s+1)},$$

(12) where: $K_p = K_{ci}K_bK_{\omega}K_t$; $K_t = 1/B_t$; $T_t = J/B_t$.

Speed PI controller parameters are determined according to Ziegler-Nichols methods of stability margin and step response. To achieve desired value of overshoot, a filter with following transfer function has been added to the drive input:

$$G_{f}(s) = \frac{1 + T_{f^{2}}s}{1 + T_{f^{1}}s}.$$
 (13)

Numerator time constant T_{f2} has been added to achieve faster response for desired response overshoot M_{pownr} , i.e. lower maximum response time t_p than in the case without that time constant.

III. MODIFICATION OF ZIEGLER-NICHOLS METHODS FOR CONTROLLER PARAMETERS DETERMINATION

Very often applied procedures of controller parameters determination in practice, for systems with expressive dead time, are based on empirical researches by Ziegler and Nichols. There are two methods available [1], [2], [10]: stability margin and step response.

A. Determination of drive's controller parameters by modification of Ziegler-Nichols method of stability margin

The stability margin method can be found in different simulations of control systems, as well as in plants where putting the control systems at the margin of stability is not dangerous. The procedure of control parameters determination using this method is as follows [1], [2], [10]:

- 1. only proportional (P) action is chosen for standard controller (integral (I) component is shut off),
- controller gain K_C is increasing to ultimate value K_{Cu}, which causes constant oscillations in closed loop control system,
- 3. ultimate period of oscillations T_{up} at the margin of stability is determined,
- PI controller parameters are determined from ultimate gain K_{Cu} and ultimate period T_{up}, according to relations:

$$K_{c} = 0.45 K_{cu}, \quad T_{i} = 0.8 T_{up}.$$
 (14)

Speed controller ultimate gain and period of oscillations are determined by drive simulation (Fig. 1), according to Ziegler-Nichols method of stability margin:

$$K_{pou} = 168.802, \quad T_{up} = 0.00353 \text{ s.}$$
 (15)

From (14) i (15) for PI speed controller parameters (Fig. 1Fig.), follows:

$$K_{p\omega} = 75.9609, \quad T_{i\omega} = 0.0030005 \text{ s.}$$
 (16)

With speed controller parameters (16) overshoot in speed feedback signal equals M_{pomr} =97.15%. To achieve overshoot in response M_{pomr} =50% it is necessary to reduce the speed controller gain coefficient to value $K_{p\omega}$ =0.1074. That value of speed controller gain coefficient is significantly lower than (16), and because of that speed of response to reference change will be considerably lower and load torque compensation significantly worse than with controller gain coefficient (16). Therefore, speed controller parameters correction is carried through, so that integral time constant and controller gain are corrected using Bodé frequency diagrams.

Since controller integral time constant (16) is much lower than maximum time constant of drive $(T_t = J/B_t =$ 94.1 ms) and only 3 times higher than next time constant of drive (T_{ω} =1 ms), controller parameters are corrected at first so that integral time constant $T_{i\omega}$ is increased (TABLE I). In that way amplitude frequency characteristic is changing at lower frequencies and phase frequency characteristic is changing at middle frequencies, thus increasing the phase margin Φ_s (Fig. 2 and TABLE I) and reducing the response overshoot M_{pomr} (TABLE I). Since crossing frequency ω_c is thereby not changing significantly (Fig. 2 and TABLE I), maximum response time t_p is not changing (TABLE I.). Besides that, increase of controller integral time constant doesn't significantly effect on maximum speed feedback signal drop $\Delta \omega_{mr}$ on change of nominal load torque $T_b = 0.89$ Nm (TABLE I.).

Optimal value of controller integral time constant is chosen to be $T_{i\omega}$ =12ms, since it's higher values doesn't result in significantly higher phase margin Φ_s and reduced overshoot in response M_{pomr} (TABLE I.).

Lowering controller gain coefficient to value: $K_{p\omega} = 0.8 \cdot K_{p\omega0} = 60.77$ results in decrease of response overshoot to value $M_{p\omega nnr} = 51.42\%$.

TABLE IDEPENDENCE OF OVERSHOOT M_{pountr} , MAXIMUM TIME t_p , MAXIMUM SPEEDFEEDBACK SIGNAL DROP $\Delta \omega_{mr}$, CROSSING FREQUENCY ω_c and PhaseMARGIN Φ_s in relation to $T_{i\omega}$ with $K_{p\omega} = 75.961$.

$\begin{bmatrix} T_{i\omega} \\ [ms] \end{bmatrix}$	3	6	9	12	15	18
$M_{p\omega mr}$ [%]	97.15	73.04	65.24	61.25	58.79	57.13
t_p [ms]	2.95	2.95	2.9	2.9	2.9	2.9
$\Delta \omega_{mr}$ [s ⁻¹]	-0.087	-0.092	-0.093	-0.094	-0.095	-0.095
ω_c [s ⁻¹]	1094	1071.3	1067	1065.4	1064.7	1064.3
$\Phi_{\rm s}[^{\circ}]$	9.88	18.9	22	23.5	24.4	25.0



Fig. 2. Bodé frequency characteristics for $K_{p\omega} = 75.961$ and: 1. $T_{i\omega} = 3$ ms, 2. $T_{i\omega} = 6$ ms, 3. $T_{i\omega} = 9$ ms, 4. $T_{i\omega} = 12$ ms.

Further decrease of response overshoot to value $M_{pomr}=10\%$ is achieved by adding a first order filter (13) at the drive input. Values of filter time constants T_{f1} i T_{f2} , by which desired response overshoot $M_{pomr}=10\%$ is achieved, are determined by optimization and given in TABLE II.

Lowering controller gain coefficient from K_{po} =75.961 to K_{po} =60.77, results in increased time of first maximum from t_p =2.9 ms to t_p =3.25 ms, i.e. approximately 10% (TABLE II).

Responses of drive on a reference step change $\Delta \omega_r^*(t) = 0.1S(t)$ and nominal load torque $T_i = 0.89S(t)$ with different values of controller gain coefficient $K_{p\omega}$ and controller integral time constant $T_{i\omega}$ are shown on Fig. 3 and Fig. 4.

By adding a first order filter at the drive input, response overshoot is lowered to $M_{pcomr}=10\%$. Responses to reference change $\Delta \omega_r^*(t) = 0.1S(t)$ with different values of filter time constants are shown on Fig. 5. When time constant in filter numerator (13) equals zero ($T_{f2}=0$) time of response maximum increases from $t_p=3.25$ ms to $t_p=4.8$ ms (TABLE II. and Fig. 5). Adding time constant in filter numerator ($T_{f2}=1.45$ ms) results in faster response, i.e. reduces time of response maximum from $t_p=4.8$ ms to $t_p=3.75$ ms (Fig. 5. and TABLE II.).

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DEPENDANCE OF OVERSHOOT $M_{p \omega m r}$, TIME OF MAXIMUM t_p , MAXIMUM SPEED FEEDBACK SIGNAL DROP $\Delta \omega_{m r}$, CROSSING FREQUENCY ω_c and Phase Margin Φ_s in relation to T_{f1} and T_{f2} with T_i =12 ms and $K_{p \omega}$ =75.961; $K_{p \omega}$ =60.77.

$K_{p\omega}$	75.961		60.77	
$T_{i\omega}$ [ms]	12	12	12	12
$M_{p\omega mr}$ [%]	61.25	51.42	10.14	9.97
$t_p [\mathrm{ms}]$	2.9	3.25	4.8	3.75
$\Delta \omega_{mr} [s^{-1}]$	-0.094	-0.109	-0.109	-0.109
$\omega_c [s^{-1}]$	1065.4	924.8		-
$\Phi_{\rm s}$ [°]	23.5	28.98		-
$T_{\rm fl}$ [ms]	0	0	1.89	2.93
T_{f2} [ms]	0	0	0	1.45



Fig. 3. Responses to reference change $\Delta \omega_{i}^{\circ}(t) = 0.1S(t)$ for controller parameters: 1. $K_{p\omega}$ =75.961, $T_{i\omega}$ =3 ms, 2. $K_{p\omega}$ =75.961, $T_{i\omega}$ =12 ms, 3. $K_{p\omega}$ =60.77, $T_{i\omega}$ =12 ms.

Speed controller parameters determined by Bodé plots of frequency characteristics, with compensation of maximum time constant of system [4], and corrected controller gain by simulation for achieving desired response overshoot M_{pom} =10% equal:

$$K_{reg} = 24.67, \quad T_{reg} = 0.0941 \text{ s.}$$
 (17)

With controller parameters (17) time of first maximum equals $t_p = 5.66$ ms and maximum speed feedback signal drop equals $\Delta \omega_{mr} = -0.215$.

Speed controller gain coefficient determined by modification of Ziegler-Nichols stability margin method ($K_{p\omega}$ =60.77) is approximately 2.5 times higher, while controller integral time constant ($T_{i\omega}$ =12 ms) is approximately 8 times smaller than value determined by maximum time constant compensation (17).

Therefore, response on reference change is approximately 30% faster, and load torque compensation about 8 times faster and about 2 times better in the case of controller parameters determined by modification of Ziegler-Nichols stability margin method (TABLE II) than in the case of controller parameters determined by compensation of maximum time constant of system (17).



Fig. 4. Responses to nominal load torque change $T_{i} = 0.89S(t)$ with controller parameters:

1. $K_{p\omega}$ =75.961, $T_{i\omega}$ =3 ms, 2. $K_{p\omega}$ =75.961, $T_{i\omega}$ =12 ms, 3. $K_{p\omega}$ =60.77, $T_{i\omega}$ =12 ms.



Fig. 5. Responses to reference change $\Delta \omega_{r}^{*}(t) = 0.1S(t)$ with $K_{p\omega} = 60.77$, $T_{i\omega} = 12$ ms and: 1. $T_{f1}=0$, $T_{f2}=0$, 2. $T_{f1}=2.93$ ms, $T_{f2}=1.45$ ms, 3. $T_{f1}=1.89$ ms, $T_{f2}=0$.

B. Determination of drive's controller parameters by modification of Ziegler-Nichols method of step response

In industrial plants, which are not possible to bring to stability margin, a step response method is often used, i.e. system response on a step change of input value. Recording of system step response $h_p(t)$ is often allowed and available without any major difficulties. Certain system behavior with expressive dead time can be satisfactorily described with proportional (P) behavior with one time constant (T_1) and dead time (T_{dt}) (FODT):

$$G_{p}\left(s\right) = \frac{K_{p}}{1+T_{s}s}e^{-T_{a}s}$$
(18)

where: K_p – system gain coefficient,

 T_{dt} – dead time,

 T_1 – time constant.

Thereby, three values are determined from step response function, which are defined by tangent in inflection point:

 K_p – system gain coefficient,

 t_1 – rise time and

 t_{dt} – hold time.

Relatively good approximation of system response $h_p(t)$ is achieved if equivalent dead time T_{dt} is chosen to be equal to hold time t_{dt} , while equivalent time constant T_1 is chosen to be equal to rise time t_1 :

$$T_{dt} = t_{dt}, \quad T_1 = t_1.$$
 (19)

There is a second possibility of FODT element parameters choice. For rise time t_1 , i.e. time constant T_1 and hold time t_{dt} , i.e. dead time T_{dt} , x-line cuttings defined by straight line which intersects axis 0% and 100% of step response function value in points where step response function has 10% and 63% of final value.

Relations for PI controller parameters determination according to hold time t_{dt} , i.e. equivalent dead time T_{dt} and rise time t_1 , i.e. equivalent time constant T_1 , are given [1], [2], [10]:

$$K_{c} = 0.9 / a, \quad T_{i} = 3.3 T_{di}.$$
 (20)

Coefficient a in (20) represents cutting, which creates tangent of system response on negative y-axis, and it is determined by system response parameters as follows:

$$a = K_n T_{dt} / T_1. \tag{21}$$

Open loop speed transfer function without controller is given by relation (12). If we choose, according to Ziegler-Nichols step response method, to be:

$$T_1 = T_t = 94.1 \text{ ms}, \text{ and } T_{dt} = T_{o} + T_{ci} = 1.465 \text{ ms}, (22)$$

then according to (12), (20), (21) and (22) speed controller parameters follow:

$$K_{\mu\nu} = 30.08 \text{ and } T_{\mu\nu} = 4.836 \text{ ms.}$$
 (23)

Controller integral time constant (23) is about 20 times smaller than maximum time constant of drive $(T_t = J/B_t =$ 94.1 ms) and about 5 times higher than next time constant of drive (T_{ω} =1 ms), while controller gain coefficient (23) is only 20% higher than in a case of maximum time constant compensation (17). Therefore, controller parameters are corrected so that integral time constant $T_{i\omega}$ is increased first (TABLE III.), thus enabling increase of gain coefficient. Increase of integral time constant changes the amplitude frequency characteristic at lower frequencies and phase frequency characteristic at middle frequencies. Phase margin Φ_s is increased and response overshoot M_{pomr} is decreased (TABLE III). Since thereby crossing frequency ω_c doesn't change significantly time of maximum t_p doesn't change (TABLE III). Besides that, increased integral time constant doesn't significantly effect on maximum speed feedback signal drop $\Delta \omega_{mr}$ in change of nominal load torque $T_b = 0.89$ Nm (TABLE III).

Optimal value of controller integral time constant is chosen to be $T_{i\omega} = 9.68$ ms, since it's higher values doesn't result in significantly higher phase margin Φ_s and reduced overshoot in response M_{pomr} (TABLE III). Since in that case response overshoot equals M_{pomr} =32.18%, controller gain coefficient can be increased 2 times: $K_{p\omega} =$ 60.6, for better load torque compensation.

In that case response overshoot increases to value: $M_{pomr} = 53.62\%$, while the effect of load torque is reduced by 35% (Fig. 6 and TABLE IV).

Decrease of response overshoot to value $M_{ponn}=10\%$ is achieved by adding a first order filter (13) at the drive input. Values of filter time constants T_{f1} i T_{f2} , which give desired response overshoot $M_{ponn}=10\%$, are obtained by optimization and given in TABLE IV.

TABLE IIIDEPENDENCE OF OVERSHOOT M_{poumr} , TIME OF MAXIMUM t_p , MAXIMUM SPEEDFEEDBACK SIGNAL DROP $\Delta \omega_{mr}$, CROSSING FREQUENCY ω_c and PhaseMARGIN Φ_s in Relation to $T_{i\omega}$ with $K_{p\omega}$ = 30.08.

$T_{i\omega}$ [ms]	4.84	9.68	14.52	19.36
$M_{p\omega mr}$ [%]	49.28	32.18	26.00	22.82
$t_p [\mathrm{ms}]$	4.975	5.0	4.975	4.95
$\Delta \omega_{mr} [s^{-1}]$	-0.163	-0.173	-0.177	-0.179
$\omega_c [s^{-1}]$	580.19	560.83	556.8	555.34
$\Phi_{\rm s} \left[\circ \right]$	32.59	42.9	46.5	48.4

TABLE IV DEPENDENCE OF OVERSHOOT M_{pomm} , TIME OF MAXIMUM t_p , MAXIMUM SPEED FEEDBACK SIGNAL DROP $\Delta \omega_{mr}$, CROSSING FREQUENCY ω_c and PHASE MARGIN Φ_s IN RELATION TO T_{f1} AND T_{f2} WITH T_i =9.68 ms, $K_{p\omega}$ =30.08 AND

$K_{p\omega}$	<i>K</i> _{pω} 30.08		60.16		
$T_{i\omega}$ [ms]	9.68	9.68	9.68		
$M_{p\omega mr}$ [%]	32.18	53.63	10.05		
$t_p [\mathrm{ms}]$	5.0	3.275	3.775		
$\Delta \omega_{mr} [s^{-1}]$	-0.173	-0.109	-0.109		
$\omega_c [s^{-1}]$	560.83	920.04	-		
$\Phi_{\rm s}$ [°]	42.9	27.9	-		
T_{fl} [ms]	0	0	3.16		
T_{f2} [ms]	0	0	1.56		

 $K_{p\omega} = 60.16.$

Responses on reference value change, with different values of filter time constants, are shown on Fig. 7.



Fig. 6. Responses to nominal load torque change $T_{l} = 0.89S(t)$ with $K_{p\omega} = 30.08$ and:

1. $T_{i\omega} = 4.84 \text{ ms}$, 2. $T_{i\omega} = 9.68 \text{ ms}$, 3. $T_{i\omega} = 14.52 \text{ ms}$ te 4. $K_{p\omega} = 60.16$, $T_{i\omega} = 9.68 \text{ ms}$.



Fig. 7. Responses to reference change $\Delta \omega_r^{*}(t) = 0.1S(t)$ with $T_{i\omega} = 9.68$ ms and: 1. $K_{p\omega} = 30.08$, $T_{f1} = 0$, $T_{f2} = 0$, 2. $K_{p\omega} = 60.16$, $T_{f1} = 0$, $T_{f2} = 0$, 3. $K_{p\omega} = 60.16$, $T_{f1} = 3.16$ ms, $T_{f2} = 1.56$ ms.

IV. CONCLUSION

This paper describes the procedure of controller parameters determination by using Ziegler-Nichols's methods of stability margin and step response function. Both of these methods are applied for controller parameters determination of permanent magnet brushless DC motor drive. Optimal controller parameters are determined by modification of results obtained by original Ziegler-Nichols's methods by using Bodé plots of frequency characteristics.

Contributions of this paper lie in derivation of modification procedure of original Ziegler-Nichols methods for determination of optimal speed controller parameters, which in relation to standard maximum time constant compensation give:

- i. Controller integral time constant 8-10 times smaller, resulting in 8-10 times faster compensation of load torque influence on speed,
- ii. Controller gain coefficient approximately 2.5 times higher, resulting in approximately 2 times better load torque compensation, i.e. 2 times lower speed drop on change of load torque,
- iii. Desired overshoot on change of reference signal and approximately 30% faster response is achieved by optimization of time constants of first order filter with time constant in numerator added to drive input.

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