Passive Internal Model Based Repetitive Control of Robot Manipulators

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***Abstract*— In this paper a new class of globally stable finite dimensional repetitive controller for robot manipulator is proposed. The passivity based design of the proposed repetitive controller avoid the problem of tight stability conditions and slow convergence of the conventional, internal model based, repetitive controllers. The passive interconnection of the controller with nonlinear mechanical systems provide stability margin which is the same as stability margin of the controller with exact feed-forward compensation of robot dynamics. The simulation results illustrate the convergence properties of the proposed controller.**

I. INTRODUCTION

An important subject in control of mechanical systems is tracking periodic reference signals and attenuating periodic disturbances. Many tracking systems, such as computer disk drives, rotation machine tools, or robots, have to deal with periodic reference and/or disturbance signals. A promising control approach for achieving tracking of periodic references signal is learning control or repetitive control.

In most of the conventional approaches to robot trajectory control, including parametric adaptive control, it is necessary to compute in real time the so called inverse dynamics equations of the robot or regression matrix. However, due to the model uncertainties, it is difficult to derive the exact description of the system. In other side, using neural networks for learning feed-forward control has some drawback: slow converges and relatively large tracking error.

There have been many studies in the topic of repetitive control for controlling of mechanical systems in an iterative manner. In contrast with conventional approaches to robot trajectory control, repetitive control schemes are easy to implement and do not require exact knowledge of the dynamic model.

Repetitive controllers can be classified as being either internal model based or external model based [1]. Controllers using the internal model are linear and have periodic signal generators [2], [3]. In external model controllers the disturbance model is placed outside the basic feedback loop [4], [5].

The internal model controllers are based on a delayed integral action of the form $(1-\exp(-sT))^{-1}$ which produce an infinite number of poles on imaginary axes. However, the asymptotic convergence can only be guaranteed under restrictive conditions in the plant dynamics - zero relative degree or direct transmission term. These conditions are generally not satisfied in robot control applications because they are imply acceleration measurement. Further, the positive feedback loop used to generate the periodic signal decreases the stability margin. In the end, the repetitive controller is likely to make the system unstable. To enhance the robustness of these repetitive control schemes, the repetitive update rule is modified to include the so-called Q-filter [2], [3].Unfortunately, the use of the Q-filter eliminates the ability of the tracking errors to converge to zero. Therefore, the trade-off between stability and tracking performance has been considered to be an important factor in the repetitive control system.

The another problem is that, due to infinite dimensional dynamics of delayed line, a large memory space is required for digital implementation of the control law. To overcome this problem, in [6] a finite dimensional approximation of delayed line is proposed in the form of cascade connection of N harmonic oscillators and one integrator.

The advantages of internal model controllers are that it is linear, making analysis and implementation easier. The disadvantages are that the stability is almost entirely governed by the feedback loop of the repetitive compensator. The frequency response of the system is altered and robustness to noise and unmodelled dynamics is reduced.

The external model controllers are based on feedforward compensation of inverse dynamics. The disturbance model is adjusted adaptively to match the actual disturbance. The central idea in [4] is that the disturbance can be represented as a linear combination of the basis functions like Fourier series expansion. On this way, an adaptive control law with regressor matrix containing basis functions is obtained. In [5] unknown disturbance functions are represented by integral equations of the first kind involving a known kernel and unknown influence functions. The learning rule indirectly estimates the unknown disturbance function by updating the influence function.

The main advantage of the external model approach is that there are no significant influence on the stability margin of the control system. The map between the feedforward function error and the tracking errors is strictly passive. Thus, the control system is robust to the imprecise estimation of the robot inverse dynamics. The disadvantage are that the analysis and implementation are more complex than for the internal model based algorithms.

In this paper a new class of internal model based repetitive controllers for robot manipulators is proposed. The proposed finite dimensional repetitive controller is founded on passivity based design and has structure in the form of parallel connection of N linear oscillators and one integrator. The passive interconnection of the controller with nonlinear mechanical systems has no influence on the stability margin which is the same as stability margin of controller with exact feed-forward compensation of robot dynamics [7], [8].

This paper is organized as follows. The robot dynamics and its main properties are presented in Section II. In Section III a class of finite dimensional internal model based repetitive controllers is introduced and conditions for global asymptotic stability are established in Section IV. The passivity properties of proposed controllers are considered in Section V. The simulation results are presented in Section VI. Finally, the concluding remarks are emphasized in Section VII.

II. Robot Dynamics

The model of n-link rigid-body robotic manipulator, in the absence of friction and disturbances, is represented by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u, \tag{1}$$

where q is the $n \times 1$ vector of robot joint coordinates, $\dot{q}$ is the $n \times 1$ vector of joint velocities, u is the $n \times 1$ vector of applied joint torques and forces, $M(q)$ is $n \times n$ inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n\times 1$ vector of centrifugal and Coriolis torques and $g(q)$ is the $n \times 1$ vector of gravitational torques and forces, obtained as the gradient of the robot potential energy $U(q)$

$$g(q) = \frac{\partial U(q)}{\partial q}. \tag{2}$$

We assume that the matrix $C(q,\dot{q})$ is defined using the Christoffel symbols.

The following well known properties of the robot dynamics, [9], [10], [11], [12], are important for stability analysis.

Property 1. The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e.,

$$z^T(\dot{M}(q) - 2C(q,\dot{q}))z = 0, \quad \forall z \in \mathbb{R}^n. \tag{3}$$

This implies

$$\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T. \tag{4}$$

Property 2. The inertia matrix $M(q)$ is a positive definite symmetric matrix which satisfies

$$\lambda_m\{M\}\|\dot{q}\|^2 \le \dot{q}^T M(q)\dot{q} \le \lambda_M\{M\}\|\dot{q}\|^2, \tag{5}$$

where $\lambda_m\{M\}$ and $\lambda_M\{M\}$ denotes strictly positive minimum and maximum eigenvalues of $M(q)$, respectively.

Property 3. (see e.g. [7], [8]) There exist positive constants k_M, k_{C1}, k_{C2} and k_g such that for all $x, y, z, v, w \in \mathbb{R}^n$, we have

$$\|M(x)z - M(y)z\| \le k_M\|x-y\|\,\|z\|, \tag{6}$$

$$\begin{aligned}\|C(x,z)w - C(y,v)w\| \le\ & k_{C1}\|z-v\|\,\|w\| + \\ &+ k_{C2}\|z\|\,\|x-y\|\,\|w\|,\end{aligned} \tag{7}$$

$$\|g(x) - g(y)\| \le k_g\|x-y\|, \tag{8}$$

$$\|C(x,y)z\| \le k_{C1}\|y\|\,\|z\|. \tag{9}$$

III. Control Problem Formulation

A. Finite-Dimensional Repetitive Controller

The periodic reference trajectory $q_d(t)$ with the period T can be represented in the form of Fourier series

$$q_d(t) = a_0 + \sum_{k=1}^{\bar{N}}[a_k\cos(k\omega t) + b_k\sin(k\omega t)], \tag{10}$$

where $\omega = \frac{2\pi}{T}$ is the fundamental frequency, and a_0, a_k and b_k are some known constant vectors.

We consider the control law given by

$$u = -K_P\tilde{q} - K_D\dot{\tilde{q}} - k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}} - K_I z_0 - \sum_{k=1}^{N} Q_k\dot{z}_k, \tag{11}$$

$$\ddot{z}_k + k^2\omega^2 z_k = Q_k(\dot{\tilde{q}} + \alpha\tilde{q}), \qquad k = 1,\dots,N, \tag{12}$$

$$\dot{z}_0 = \dot{\tilde{q}} + \alpha\tilde{q}, \tag{13}$$

where $\tilde{q} = q - q_d$, $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$ are the joint position error and velocity, respectively, q_d is the time periodic desired joint position represented by (10), K_P, K_D, K_I, Q_k ($k = 1,\dots,N$) are $n\times n$ constant positive-definite diagonal matrix, $k_D^{(1)}$ and α are some positive constants.

The desired joint position q_d is assumed to be twice continuously differentiable. In other words, we assume that exist a finite upper bounds on the norm of the desired velocity and acceleration, denoted by $\|\dot{q}_d\|_M$ and $\|\ddot{q}_d\|_M$. The nonlinear derivative term $k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}}$ in control law (11) is introduced to ensure global asymptotic stability of closed-loop system [13].

The parallel interconnection of N harmonic oscillators (12) and integrator (13) represents the internal model of the periodic reference signal $q_d(t)$ including higher order harmonics which are induced by nonlinear robot dynamics, so that condition $N \ge \bar{N}$ must be satisfied.

Remark 1. Note that repetitive controller (11)-(13) is not a finite dimensional approximation of delayed line as the repetitive controller proposed in [6]

$$G_{rc}(s) = \frac{1}{s}\prod_{k=1}^{N}\frac{1}{s^2 + k^2\omega^2}, \tag{14}$$

which can be interpreted as a cascade connection of N linear oscillators and one integrator.

B. Residual Robot Dynamics

The dynamic model of robot manipulator (1) can be rewritten in the following form

$$M(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + h(\tilde{q},\dot{\tilde{q}}) = u - f(q_d,\dot{q}_d,\ddot{q}_d), \tag{15}$$

where

$$h(\tilde{q},\dot{\tilde{q}}) = [M(q)-M(q_d)]\ddot{q}_d + [C(q,\dot{q})-C(q_d,\dot{q}_d)]\dot{q}_d + \\ + g(q)-g(q_d), \qquad (16)$$

is the so-called *residual robot dynamics* introduced in [14], [15] and

$$f(q_d,\dot{q}_d,\ddot{q}_d) = M(q_d)\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + g(q_d), \qquad (17)$$

represents the unknown *desired robot inverse dynamics*.

The function $f(q_d,\dot{q}_d,\ddot{q}_d)$ is a periodic function with the same fundamental frequency as $q_d(t)$ and can be represented by infinite Fourier series expansion

$$f(q_d,\dot{q}_d,\ddot{q}_d) = \bar{a}_0 + \sum_{k=1}^{\infty}[\bar{a}_k\cos(k\omega t) + \bar{b}_k\sin(k\omega t)], \quad (18)$$

where $\bar{a}_0$, $\bar{a}_k$ and $\bar{b}_k$ are some unknown constant vectors.

The following property of the function $h(\tilde{q},\dot{\tilde{q}})$ is important in subsequent stability analysis.

Property 4. By defining

$$c_1 = k_g + k_M\|\ddot{q}_d\|_M + k_{C2}\|\dot{q}_d\|_M^2, \qquad (19)$$

$$c_2 = k_{C1}\|\dot{q}_d\|_M, \qquad (20)$$

the norm of residual dynamics (16) satisfies (see [7], [8])

$$\|h(\tilde{q},\dot{\tilde{q}})\| \le c_1\|\tilde{q}\| + c_2\|\dot{\tilde{q}}\|. \qquad (21)$$

From the inequality (21) follows

$$-(\dot{\tilde{q}}+\alpha\tilde{q})^T h(\tilde{q},\dot{\tilde{q}}) \le \alpha c_1\|\tilde{q}\|^2 + c_2\|\dot{\tilde{q}}\|^2 + \\ + (c_1+\alpha c_2)\|\tilde{q}\|\,\|\dot{\tilde{q}}\|. \qquad (22)$$

The parameters c_1 and c_2 can be estimated on the base of Fourier representation of desired periodic signal (10)

$$c_2 = k_{C1}\sum_{k=1}^{\bar{N}} k\omega(a_k+b_k), \qquad (23)$$

$$c_1 = k_g + k_M\sum_{k=1}^{\bar{N}} k^2\omega^2(a_k+b_k) + \frac{k_{C2}c_2^2}{k_{C1}^2}, \qquad (24)$$

where we used properties $|\sin(k\omega t)| \le 1$ and $|\cos(k\omega t)| \le 1$.

Remark 1. One of the simplest motion control scheme for the system (15) is PD control with feedforward compensation [16], [17]

$$u = -K_P\tilde{q} - K_D\dot{\tilde{q}} + f(q_d,\dot{q}_d,\ddot{q}_d). \qquad (25)$$

The local asymptotic stability of the control law (25) is proven in [7] and conditions for global asymptotic stability are established in [8]. Implementation of control law (25) requires the exact knowledge of matrices $M(q)$, $C(q,\dot{q})$ and vector $g(q)$. However, due to the model uncertainties, it is difficult to derive the exact dynamic model of the robot manipulator. The main idea of using controller (11)-(13) is model-free feedback compensation of periodic function $f(q_d,\dot{q}_d,\ddot{q}_d)$. This idea will be more clarified through derivation of the error equations of the closed-loop system.

C. Error Equations

Introducing the change of variables $\tilde{z}_k = z_k - z_k^*$, $k = 0,1,...,N$, with

$$z_0^* = -K_I^{-1}\bar{a}_0, \qquad (26)$$

$$z_k^* = Q_k^{-1}[\bar{b}_k\cos(k\omega t) - \bar{a}_k\sin(k\omega t)], \quad k=1,...,N, \quad (27)$$

the following error equations are obtained

$$M(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + h(\tilde{q},\dot{\tilde{q}}) = u + w(t;N+1), \qquad (28)$$

$$u = -K_P\tilde{q} - K_D\dot{\tilde{q}} - k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}} - K_I\tilde{z}_0 - \sum_{k=1}^{N} Q_k\dot{\tilde{z}}_k, \quad (29)$$

$$\ddot{\tilde{z}}_k + k^2\omega^2\tilde{z}_k = Q_k(\dot{\tilde{q}}+\alpha\tilde{q}), \qquad k=1,...,N, \qquad (30)$$

$$\dot{\tilde{z}}_0 = \dot{\tilde{q}} + \alpha\tilde{q}, \qquad (31)$$

where we used the Fourier series expansion (18) of the function $f(q_d,\dot{q}_d,\ddot{q}_d)$ and property $\ddot{z}_k^* + k^2\omega^2 z_k^* = 0$. The function $w(t;N+1)$ in (28) is the error in estimation of the desired robot inverse dynamics which consists harmonics of $N+1$ order

$$w(t;N+1) = \sum_{k=N+1}^{\infty}[\bar{a}_k\cos(k\omega t) + \bar{b}_k\sin(k\omega t)]. \quad (32)$$

Remark 2. From the equation (32) we can conclude that the tracking error has zero harmonic content at the repetitive frequency and its harmonics up to N (where N is the number of harmonic oscillators in the controller). Also, the bound on the tracking error decreases with N. In the limit $N \to \infty$ the above model of the repetitive controller works as well as the ideal infinity dimensional model in achieving perfect tracking of periodic reference signals [18]. Note that this conclusion is valid only for twice continuously differentiable periodic reference signals, and cannot be generalized for arbitrary periodic reference signals, as shown in [2]. In that ideal case the stationary state of the system (15), (11)-(13) is $q = q_d$, $\dot{q} = \dot{q}_d$, $z_k = z_k^*$, $k = 0,1,...,N$.

IV. Stability analysis

We consider stability of the unperturbed systems (28)-(31), where $w(t;N+1) = 0$, by Lyapunov's direct method. First, we propose appropriate Lyapunov function candidate. Then, global stability conditions on the controller gains are established. Finally, LaSalle invariance principle is invoked to guarantee the asymptotic stability.

A. Construction of Lyapunov Function

Multiplying equation (28) with output variable $y_1 = (\dot{\tilde{q}} + \alpha\tilde{q})$, equation (30) with output variable $y_2 = \dot{z}_k$, equation (31) with output variable $y_3 = K_I z_0$ and summing all together, we get the nonlinear differential form which can be separated in the following way

$$\frac{d}{dt}V(\tilde{q},\dot{q},\tilde{z}_0,\tilde{z}_1,\dot{\tilde{z}}_1,...,\tilde{z}_N,\dot{\tilde{z}}_N) = -W(\tilde{q},\dot{q}), \qquad (33)$$

(for more details see e.g. [13]) where $V = V_1 + V_2$ is the Lyapunov function candidate

$$V_1 = \frac{1}{2}\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} + \alpha\tilde{q}^T M(q)\dot{\tilde{q}} + \frac{1}{2}\tilde{q}^T K_P\tilde{q} + \\ + \frac{1}{2}\alpha\tilde{q}^T K_D\tilde{q} + \frac{1}{3}\alpha k_D^{(1)}\|\tilde{q}\|^3, \quad (34)$$

$$V_2 = \frac{1}{2}\tilde{z}_0^T K_I\tilde{z}_0 + \frac{1}{2}\sum_{k=1}^{N}\dot{\tilde{z}}_k^T\dot{\tilde{z}}_k + \frac{1}{2}\omega^2\sum_{k=1}^{N}k^2\tilde{z}_k^T\tilde{z}_k, \quad (35)$$

and $-W$ is its time derivative

$$W = \dot{\tilde{q}}^T K_D\dot{\tilde{q}} + k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}}^T\dot{\tilde{q}} - \alpha\dot{\tilde{q}}^T M(q)\dot{\tilde{q}} - \alpha\dot{\tilde{q}}^T C(q,\dot{q})\tilde{q} \\ + \alpha\tilde{q}^T K_P\tilde{q} + (\dot{\tilde{q}} + \alpha\tilde{q})^T h(\tilde{q},\dot{\tilde{q}}). \quad (36)$$

B. Stability Conditions

The following step is determination of conditions for positive definiteness of the function V and positive semi-definiteness of the function W.

1) Positive definiteness of Lyapunov function: The function V_2 is positive definite for any positive definite matrix K_I, so that

$$V \geq V_1 = \frac{1}{2}\left(\dot{\tilde{q}} + \alpha\tilde{q}\right)^T M(q)\left(\dot{\tilde{q}} + \alpha\tilde{q}\right) - \frac{1}{2}\alpha^2\tilde{q}^T M(q)\tilde{q} + \\ + \frac{1}{2}\tilde{q}^T(K_P + \alpha K_D)\tilde{q} + \frac{1}{3}\alpha k_D^{(1)}\|\tilde{q}\|^3. \quad (37)$$

Using property (5) we get

$$V \geq \frac{1}{2}(\lambda_m\{K_P\} + \alpha\lambda_m\{K_D\} - \alpha^2\lambda_M\{M\})\|\tilde{q}\|^2, \quad (38)$$

that will be satisfied when

$$\lambda_m\{K_P\} + \alpha\lambda_m\{K_D\} > \alpha^2\lambda_M\{M\}. \quad (39)$$

2) Negative semi-definiteness of time derivative of Lyapunov function: The following step is derivation of the condition which ensure that time derivative of Lyapunov function is negative semi-definite function, i.e., $W \geq 0$.

First, notice that the upper bound on term $\dot{\tilde{q}}^T C(q,\dot{q})\tilde{q}$ in (36) can be estimated by

$$\dot{\tilde{q}}^T C(q,\dot{q})\tilde{q} \leq \alpha k_{C1}\|\tilde{q}\|\,\|\dot{q}\|\,\left\|\dot{\tilde{q}}\right\| \leq \\ \leq k_{C1}\|\dot{q}_d\|_M\,\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\| + k_{C1}\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\|^2, \quad (40)$$

where we used triangle inequality $\|\dot{q}\| \leq \left\|\dot{\tilde{q}}\right\| + \|\dot{q}_d\|$.

Applying properties (5), (22) and (40) we get

$$W \geq (\lambda_m\{K_D\} - \alpha\lambda_M\{M\} - c_2)\left\|\dot{\tilde{q}}\right\|^2 + \\ + \alpha(\lambda_m\{K_P\} - c_1)\|\tilde{q}\|^2 - (c_1 + 2\alpha c_2)\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\| + \\ + (k_D^{(1)} - \alpha k_{C1})\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\|^2 \geq 0. \quad (41)$$

Finally, W can be bounded by a quadratic plus a cubic function

$$W \geq \begin{bmatrix}\|\tilde{q}\| \\ \|\dot{\tilde{q}}\|\end{bmatrix}^T R\begin{bmatrix}\|\tilde{q}\| \\ \|\dot{\tilde{q}}\|\end{bmatrix} + (k_D^{(1)} - \alpha k_{C1})\|\tilde{q}\|\,\left\|\dot{\tilde{q}}\right\|^2, \quad (42)$$

with the matrix R given by

$$R = \begin{bmatrix}\alpha(\lambda_m\{K_P\} - c_1) & -\frac{1}{2}(c_1 + 2\alpha c_2) \\ -\frac{1}{2}(c_1 + 2\alpha c_2) & (\lambda_m\{K_D\} - \alpha\lambda_M\{M\} - c_2)\end{bmatrix}.$$

The function W is positive definite if the following conditions are satisfied

$$k_D^{(1)} > \alpha k_{C1}, \quad (43)$$

$$\lambda_m\{K_P\} > c_1, \quad (44)$$

$$\lambda_m\{K_D\} > \frac{(c_1 + 2\alpha c_2)^2}{4\alpha(\lambda_m\{K_P\} - c_1)} + \alpha\lambda_M\{M\} + c_2. \quad (45)$$

We can see that condition (39) is trivially implied by conditions (44)-(45). So, the conditions (43)-(45) are the final stability criterions which guaranty global stability. Finally, by invoking the LaSalle invariance principle, we conclude asymptotic stability.

Note that stability conditions (44)-(45), for $\alpha = 1$, are exactly the same as local stability conditions in [8]. The global stability of repetitive controller (11)-(13) is achieved by the nonlinear derivative term whose gain $k_D^{(1)}$ satisfies condition (43). So, the proposed repetitive controller practically has no any influence on stability margin of the closed-loop system what is obvious from stability conditions which does not contain interaction gains Q_k. This fact is a consequence of passive interconnection of robot dynamics (1) with repetitive controller (11)-(13).

V. Passivity Properties of Repetitive Controller

Consider dynamical systems represented by

$$\dot{x} = f(x,u), \quad (46)$$

$$y = h(x,u), \quad (47)$$

where $x \in \mathbb{R}^n$, $y,u \in \mathbb{R}^m$, $f(0,0) = 0$ and $h(0,0) = 0$. Moreover $f(x,u)$ and $h(x,u)$ are supposed sufficiently smooth such that the system is well-defined.

Definition 1. (see [19]) The system (46)-(47) is said to be passive if there exists a continuously differentiable positive semidefinite function $V(x)$ (called the storage function) such that

$$u^T y \geq \dot{V}(x) + \epsilon\|u\|^2 + \delta\|y\|^2 + \rho\psi(x), \quad (48)$$

where ϵ, δ, and ρ are nonnegative constants, and $\psi(x) : \mathbb{R}^n \to \mathbb{R}$ is a positive definite function of x. The term $\rho\psi(x)$ is called the state dissipation rate. Furthermore, the system is said to be: passive if $\epsilon = \delta = \rho = 0$; input strictly passive if $\delta = \rho = 0$ and $\epsilon > 0$; output strictly passive if $\epsilon = \rho = 0$ and $\delta > 0$; state strictly passive if $\epsilon = \delta = 0$ and $\rho > 0$.

Proposition 1. The robot dynamics

$$M(q)\ddot{\tilde{q}} + C(q,\dot{q})\dot{\tilde{q}} + h(\tilde{q},\dot{\tilde{q}}) = u_1 + w_1, \quad (49)$$

in closed-loop with nonlinear PD regulator

$$u_1 = -K_P\tilde{q} - K_D\dot{\tilde{q}} - k_D^{(1)}\|\tilde{q}\|\dot{\tilde{q}}, \quad (50)$$

is state strictly passive from the input torque w_1 to the output $y_1 = \dot{\tilde{q}} + \alpha\tilde{q}$, with a radially unbounded positive definite storage function V_1 defined by (34) and the state dissipation rate is given by W,

$$w_1^T y_1 \geq \dot{V}_1(\tilde{q},\dot{\tilde{q}}) + W(\tilde{q},\dot{\tilde{q}}). \quad (51)$$

Note that V_1 is positive definite and W is positive semidefinite function if conditions (44)-(45) are satisfied.

Proposition 2. The system

$$\ddot{\tilde{z}}_k + k^2\omega^2\tilde{z}_k = Q_k w_2, \qquad k = 1, ..., N, \tag{52}$$
$$\dot{\tilde{z}}_0 = w_2, \tag{53}$$

is passive from the input w_2 to the output $y_2 = K_I z_0 + \sum_{k=1}^{N} Q_k \dot{\tilde{z}}_k$ with a radially unbounded positive definite storage function V_2 defined by (35),

$$w_2^T y_2 \geq \dot{V}_2(\tilde{z}_0, \tilde{z}_1, \dot{\tilde{z}}_1, ..., \tilde{z}_N, \dot{\tilde{z}}_N). \tag{54}$$

Note that V_2 is positive definite for any positive definite matrix K_I.

Proposition 3. The feedback interconnection of the system (49)-(50) with the system (52)-(53),

$$w_1 = -y_2 + w, \qquad w_2 = y_1, \tag{55}$$

is output strictly passive from the input torque w to the output $y_1 = \dot{\tilde{q}} + \alpha\tilde{q}$, with a radially unbounded positive definite storage function $V = V_1 + V_2$,

$$w^T y_1 \geq \dot{V} + \delta\|y_1\|^2, \tag{56}$$

where

$$\delta \leq \frac{a_1 a_2 - \frac{1}{4}a_3^2}{a_2 + \alpha^2 a_1 + \alpha a_3}, \tag{57}$$

and

$$a_1 = \lambda_m\{K_D\} - \alpha\lambda_M\{M\} - c_2, \tag{58}$$
$$a_2 = \alpha(\lambda_m\{K_P\} - c_1), \tag{59}$$
$$a_3 = c_1 + 2\alpha c_2. \tag{60}$$

Proof. Inserting $w_1 = -y_2 + w$, $w_2 = y_1$ in (51) and (54) we get

$$w^T y_1 \geq \dot{V}(\tilde{q}, \dot{\tilde{q}}, \tilde{z}_0, \tilde{z}_1, \dot{\tilde{z}}_1, ..., \tilde{z}_N, \dot{\tilde{z}}_N) + W(\tilde{q}, \dot{\tilde{q}}). \tag{61}$$

Further, inserting inequality (41) with notation (58)-(60) in (61), and comparing with (56) we get

$$\begin{aligned} w^T y_1 &\geq \dot{V} + a_1 \left\|\dot{\tilde{q}}\right\|^2 + a_2\|\tilde{q}\|^2 - a_3\|\tilde{q}\| \ \left\|\dot{\tilde{q}}\right\| \geq \\ &\geq \dot{V} + \delta\|\dot{\tilde{q}} + \alpha\tilde{q}\|^2, \end{aligned} \tag{62}$$

The final step is determination of the parameter δ which satisfies the above mentioned inequality. Rearranging inequality (62) and using property of scalar product $\tilde{q}^T\dot{\tilde{q}} \leq \|\tilde{q}\| \ \|\dot{\tilde{q}}\|$, we get the following inequality

$$\begin{aligned} &(a_1 - \delta)\left\|\dot{\tilde{q}}\right\|^2 + (a_2 - \alpha^2\delta)\|\tilde{q}\|^2 - (a_3 + 2\alpha\delta)\|\tilde{q}\| \ \left\|\dot{\tilde{q}}\right\| = \\ &= \begin{bmatrix} \|\tilde{q}\| \\ \|\dot{\tilde{q}}\| \end{bmatrix}^T \begin{bmatrix} (a_2 - \alpha^2\delta) & -\frac{1}{2}(a_3 + 2\alpha\delta) \\ -\frac{1}{2}(a_3 + 2\alpha\delta) & (a_1 - \delta) \end{bmatrix} \begin{bmatrix} \|\tilde{q}\| \\ \|\dot{\tilde{q}}\| \end{bmatrix} \geq 0 \end{aligned}$$

which is satisfied for

$$(a_1 - \delta)(a_2 - \alpha^2\delta) \geq \frac{1}{4}(a_3 + 2\alpha\delta)^2. \tag{63}$$

Solving the inequality (63) with respect to parameter δ we get (57). Note that, from the conditions of positive definiteness of the matrix R in (42), the nominator on the right side of inequality (57) is always positive.

Proposition 4. The feedback interconnection of the system (49)-(50) with the system (52)-(53) has finite $\mathcal{L}_2$ gain $\gamma \leq \frac{1}{\delta}$ where δ is defined by (57). For proof see e.g. [19].

From above mentioned propositions follows two important properties of the proposed repetitive controller. First, comparing expression (61) with (51) and (54) we can conclude that robot manipulator in closed loop with repetitive controller has the same stability margin as both subsystems separately. In other words, passive feedback interconnection of the proposed repetitive controller with robot dynamics doesn't decrease stability margin what is characteristics for classical internal model based repetitive controllers [2] and their finite dimensional representations [6]. Second, the map between the inverse dynamics estimation error w and output error $\dot{\tilde{q}} + \alpha\tilde{q}$ is strictly passive what means that the control system is robust to the imprecise estimation of the robot inverse dynamics.

VI. Simulation Example

The manipulator used for simulation is a two revolute jointed robot (planar elbow manipulator) with numerical values of robot parameters which have been taken from [20].

The desired periodic reference trajectories are

$$q_{d1} = \frac{1}{2} + \sum_{k=1}^{3}\frac{\sin(k\omega t)}{k+1}, \quad q_{d2} = 1 - 2\sum_{k=1}^{3}\frac{\cos(k\omega t)}{k^2+1}, \tag{64}$$

where $\omega = 2 \ rad/s$ and the number of oscillators is $N = 12$. The controller gains are chosen in accordance with stability conditions (43)-(45) as $K_P = \text{diag}\{50, 50\}$, $K_D = \text{diag}\{50, 50\}$, $k_D^{(1)} = 20$ and $\alpha = 0.5$. The matrix of integral gains and interconnection matrices are arbitrary chosen as $K_I = \text{diag}\{50, 50\}$ and $Q_k = \text{diag}\{20, 20\}$ for $k = 1, ..., N$.

In Fig. 1. is shown the comparison of positions of robot manipulators and the reference signals. In Fig. 2. we can see positions errors of the repetitive controllers (RC) with $Q_k = 0$ for $k = 1, ..., 12$ (which is actually a nonlinear PID controller) and positions errors of repetitive controllers with $Q_k = \text{diag}\{20, 20\}$. From the figure we can conclude that the PID controller can not asymptotically tracking the periodic reference signal. In contrast with PID controller, the repetitive controller shows exponential convergence toward an arbitrary small tracking error which depends on the number of oscillators N.

The dependence of tracking error and convergence rate on the number of oscillators N is illustrated in Fig. 3. From the figure we can see that the convergence rate is independent on the number of oscillators. It is expected because the time derivative of the Lyapunov function doesn't depend on the number of oscillators N. In other side, increasing in the number of oscillators decrease the tracking error. So, there are no trade-off between convergence and accuracy, which is characteristics for most of the internal model based repetitive controllers.

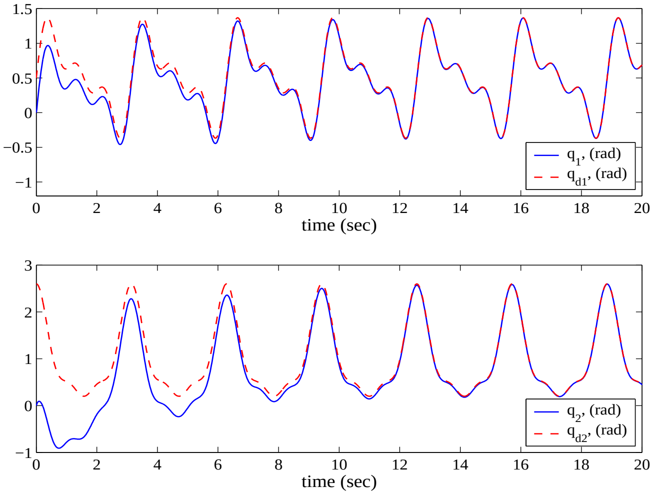

Fig. 1. The periodic reference signals and positions of robot manipulators.

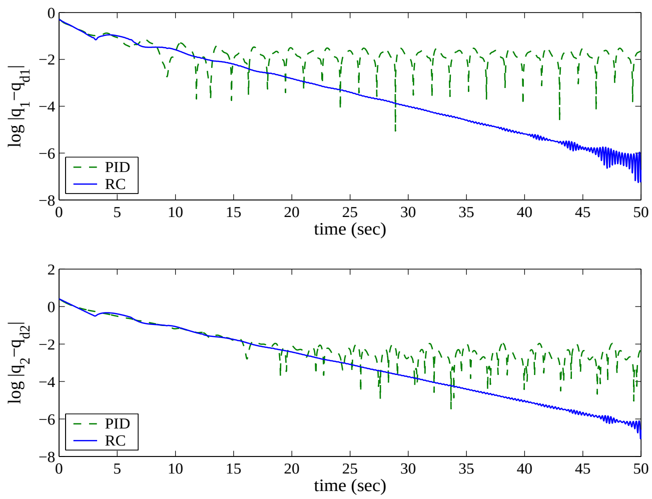

Fig. 2. The comparison of tracking errors for PID controller and repetitive controller (RC).

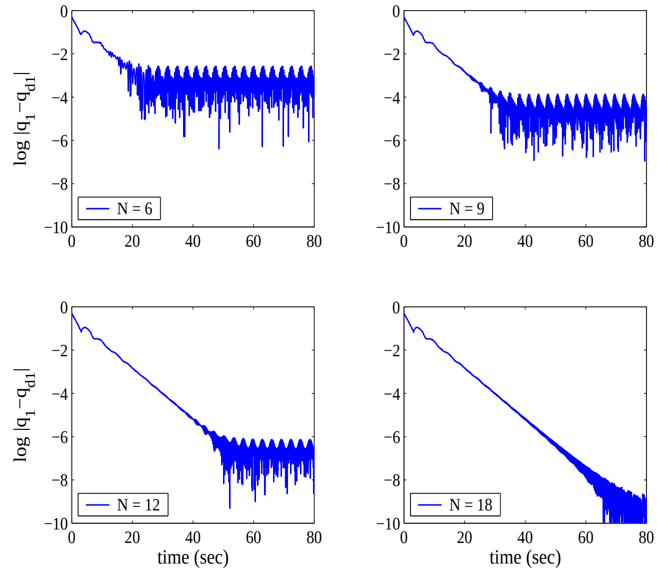

Fig. 3. The convergence rate and tracking errors depending on number of oscillators.

VII. Conclusions

In this paper a new class of finite dimensional repetitive controllers for robot manipulators is proposed. The proposed repetitive controller connects the main advantage of internal model controllers - implementation simplicity, with robustness based on passivity of external model controllers. Further research will be concentrated on the stability analysis of proposed repetitive controller in combination with the frequency estimators.

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