Radoslav Pavazza, Fakultet elektrotehnike, strojarstva i brodogradnje, Ruđera Boškovića bb, 210000 Split
Branko Blagojević, Fakultet elektrotehnike, strojarstva i brodogradnje, Ruđera Boškovića bb, 210000 Split
Bože Plazibat, Odjel za stručne studije Sveučilišta u Splitu,Livanjska 5, 21000 Split

O DISTORZIJI POPREČNOG PRESJEKA VELIKIH TANKERA OPTEREĆENIH NA SAVIJANJE

SAŽETAK

U radu je razmatrana distorzija poprečnog presjeka velikih suvremenih brodova za prijevoz tekućeg tereta s jednom uzdužnom pregradom opterećenih na savijanje u vertikalnoj ravnini simetrije. Distorzija poprečnog presjeka razmatrana je u graničnom slučaju. Podrazumijeva se da su paneli brodskog trupa — dno, paluba, bokovi i uzdužne pregrade — međusobno zglobno vezani duž uzdužnih rubova. Dodatna naprezanja i pomaci zbog distorzije analizirani su u odnosu na naprezanje i pomake koje daje elementarna teorija savijanja. Pretpostavka o „zglobnim poprečnim presjecima“ suprotna je pretpostavci o „apsolutno krutim poprečnim presjecima. Obje pretpostavke granični su slučajevi stanja naprezanja i deformacije stvarnih konstrukcija, posebno u slučaju velikih brodova za prijevoz tekućeg tereta. Dana je usporedba s rezultatima analize s pomoću metode konačnih elemenata.

Ključne riječi: *distorzija, poprečni presjek, brodski trup, jedna uzdužna pregrada*

ON THE CROSS-SECTION DISTORTION OF LARGE TANKERS SUBJECTED TO BENDING

SUMMARY

The paper deals with distortion of hull cross-sections of large modern tankers with one longitudinal bulkhead subjected to bending in the vertical plane of symmetry. The cross-section distortion is considered in the limit case. It is assumed that hull panels — bottom, deck, sides and longitudinal bulkheads — are hinged along their longitudinal edges. The additional stresses and displacements due to distortion with respect to the stresses and displacements of the ordinary hull girder bending theory are analysed. The assumption of “hinged cross-sections” is opposite to the assumption of “rigid cross-sections”. Both are limiting cases of stresses and deformations of actual hull structures, particularly in the case of large tankers. A comparison with the results of the finite element analysis is provided.

Key words: distortion, cross-section, ship hull, single longitudinal bulkhead

1. Introduction

The assumption that the cross-section shape of thin-walled beams is maintained during beam bending depends on the transverse stiffening of the actual structure; in some cases on the load distribution in the transverse direction as well [1, 2, and 3]. In the limit, it may be assumed that the thin-walled beam panels are "hinged" along their longitudinal edges; if the panels are considered as plane stress elements, the compatibility of longitudinal displacement at hinged connections is assumed only [1]. The two different types of structural behaviour—with "rigid" cross-sections and "hinged" cross-section—may be considered as two limiting cases of stresses and deformations of the actual structure [1].

In the case of cross-sections of large tankers with double skin and one longitudinal bulkhead, the load distribution in the transverse direction can significantly be non-uniform [4,5,6]. In this work, the distortion effect on the stresses and displacements of such structures will be analysed, by assuming that the structure is assembled of thin plates hinged along their longitudinal edges.

2. Thin-walled beams subjected to bending with the cross-section distortion

2.1. Beam components

It is assumed that a prismatic thin-walled beam, with two closed cells, is assembled of thin walls that are hinged along the longitudinal edges; the loads act in vertical walls only (Fig. 1a).

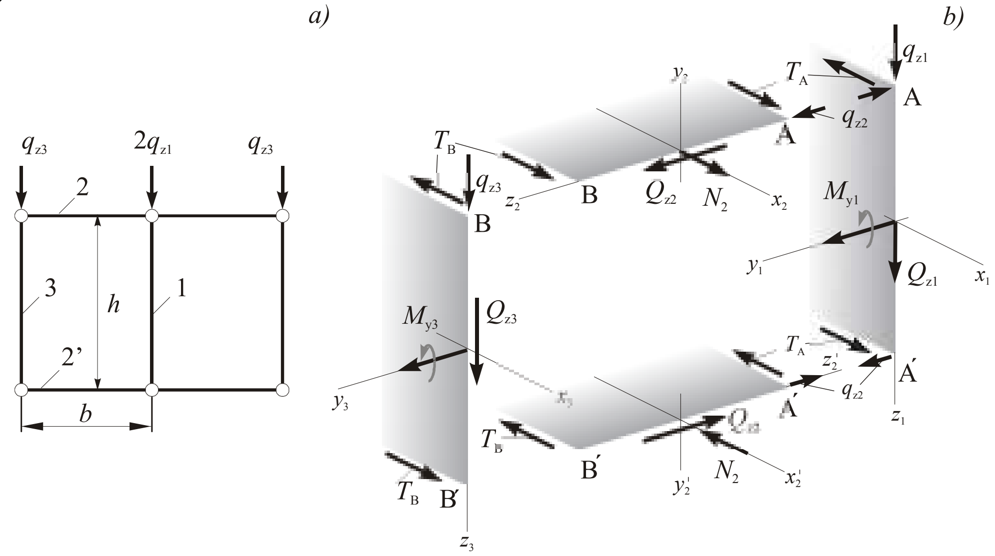

Figure 1 a) Cross-section with hinged walls; b) beam components
Slika 1 a) presjek sa zglobno vezanim stijenama; b) komponente grede

The walls can be treated as simple thin-walled beams [6,7,8]; the beam components 1 and 3 subjected to bending with influence of shear and the beams 2 and $2'$ to tension and shear due to cross-section distortion (Fig. 1b).

For the beam components 1 and 3, one has

$$\frac{\mathrm{d}w_i}{\mathrm{d}x} = -\beta_i\,, \qquad EI_{yi}\frac{\mathrm{d}^2 w_i}{\mathrm{d}x^2} = -M_{yi}\,, \qquad EI_{yi}\frac{\mathrm{d}^3 w_i}{\mathrm{d}x^3} = -\frac{\mathrm{d}M_{yi}}{\mathrm{d}x} = -(Q_{zi} - m_{yi})\,,$$

$$EI_{yi}\frac{\mathrm{d}^4 w_i}{\mathrm{d}x^4} = -\frac{\mathrm{d}^2 M_{yi}}{\mathrm{d}x^2} = -\frac{\mathrm{d}}{\mathrm{d}x}(Q_{zi} - m_{yi}) = q_{zi} + \frac{\mathrm{d}m_{yi}}{\mathrm{d}x}\,, \tag{1}$$

where $w_i = w_i(x)$ is the displacement in the z_i-direction, $\beta_i = \beta_i(x)$ is the angular displacement with respect to the y_i-axis, I_{yi} is the cross-section moment of inertia with respect to the y_i-axis, $M_{yi} = M_{yi}(x)$ is the bending moment with respect to the y_i-axis, $Q_{zi} = Q_{zi}(x)$ is the shearing force with respect to the z_i-axis, $q_{zi} = q_{zi}(x)$ is the force per unit length in z_i-direction, $m_{yi} = m_{yi}(x)$ is the moment per unit length, $O_i x_i y_i z_i$ is the rectangular co-ordinate system, where the y_i, z_i-axes coincide with principal axes of the corresponding cross-sections.

The stresses may be expressed as follows

$$\sigma_{xi} = \frac{M_{yi}}{I_{yi}} z_i, \quad \tau_{xsi} = \frac{T_{xsi}}{t_i}, \quad T_{xsi} = \frac{(Q_{zi} - m_{yi})}{I_{yi}} S_{yi}^* + T_i, \tag{2}$$

where $\sigma_{xi} = \sigma_{xi}(x,z)$ is the normal stress in the x_i direction, $\tau_{xi} = \tau_{xi}(x,z)$ is the shear stress in the z_i direction, $T_{xzi} = T_{xzi}(x,z)$ is the shear flow in the z_i direction, t_i is the wall thickness;

$$m_{y1} = T_A h, \; m_{y3} = T_B h, \tag{3}$$

where $T_A = T_A(x)$ and $T_B = T_B(x)$ are the force per unit length along the longitudinal edges *A* and *B*, respectively, and *h* is the height of the cross-section

For the beam component 2, it may be written

$$EA_2 \frac{\mathrm{d}u_{2p}}{\mathrm{d}x} = N_2, \quad EA_2 \frac{\mathrm{d}^2 u_{2p}}{\mathrm{d}x^2} = \frac{\mathrm{d}N_2}{\mathrm{d}x} = -q_{x2}, \quad q_{x2} = T_A + T_B, \tag{4}$$

where A_2 is the cross-section area of the beam component 2, $u_{2p} = u_{2p}(x)$ is the displacement of the cross-section in the x_2-direction as a plane section and $N_2 = N_2(x)$ is the normal force;

$$Q_{z2} = m_{y2}, \quad \frac{\mathrm{d}Q_{z2}}{\mathrm{d}x} = -q_{z2} = \frac{\mathrm{d}m_{y2}}{\mathrm{d}x}, \quad m_{y2} = -\left(T_A - T_B\right)\frac{b}{2}; \tag{5}$$

$$u_2 = u_{2P} - \left[T_A - T_B - k\left(T_A + T_B\right)\right] \frac{b^2}{2GA_2}\left[\frac{2G}{kE}\left(\frac{z_2}{b}\right)^2 + \frac{z_2}{b} - \frac{G}{6kE}\right], \tag{6}$$

where $u_2 = u_2(x,z)$ is the displacement in the x_2 direction. For $w_1 = w_3$:

$$u_2 = u_{2P}, \quad T_A - T_B = k\left(T_A + T_B\right), \quad k = \frac{2z_{20}}{b} = \frac{S_{yA}^* - S_{yB}^*}{S_{yA}^* + S_{yB}^*},$$

$$S_{yA}^* = \frac{A_2 h}{4}\left(1 + k\right), \quad S_{yB}^* = \frac{A_2 h}{4}\left(1 - k\right), \tag{7}$$

where z_{20} is the coordinate of the shear flow zero point for the rigid cross-section, S_{yA}^* and S_{yB}^* are the statical moments of the cut-off portion of the rigid cross-section for the points *A* and *B*, respectively;

$$\sigma_{x2} = E\varepsilon_x = \frac{\partial u_2}{\partial x} = \frac{N_2}{A_2} - \frac{Eb^2}{2GA_2}\left[\frac{2G}{kE}\left(\frac{z_2}{b}\right)^2 + \frac{z_2}{b} - \frac{G}{6kE}\right]\frac{\mathrm{d}}{\mathrm{d}x}\left[T_A - T_B - k\left(T_A + T_B\right)\right],$$

$$\tau_{xz2} = \frac{T_{xz2}}{t_2}, \quad T_{xz2} = -\frac{1}{2}(T_A - T_B) + (T_A + T_B)\frac{z_2}{b}, \tag{8}$$

where $\sigma_{x2} = \sigma_{x2}(x,z)$ is the normal stress in the x_2 direction , $\tau_{xz2} = \tau_{xz2}(x,z)$ is the shear stress, $T_{xz2} = T_{xz2}(x,z)$ is the shear flow, *G* is the shear modulus.

For the beam as an assembly of the beam components rigidly connected along the longitudinal edges, i.e. for the beam bending by elementary theory, the relation (1) may be used, where $w_i = w$

2.2. Compatibility conditions

The compatibility conditions, along the edges A and B, according (1) and (6), can be expressed as

$$-\beta_1 \frac{h}{2} = u_{2p} + \frac{b^2}{4GA_2}\left(1 - \frac{2G}{3kE}\right)\left[T_A - T_B - k\left(T_A + T_B\right)\right],$$
$$-\beta_3 \frac{h}{2} = u_{2p} - \frac{b^2}{4GA_2}\left(1 + \frac{2G}{3kE}\right)\left[T_A - T_B - k\left(T_A + T_B\right)\right]. \tag{9}$$

Hence, referring to (1) and (4)

$$N_2 = -\left(\frac{M_{y1}}{I_{y1}} + \frac{M_{y3}}{I_{y3}}\right)\frac{A_2 h}{4} - \left(\frac{M_{y1}}{I_{y1}} - \frac{M_{y3}}{I_{y3}}\right)\frac{GA_2 h}{6kE}, \tag{10}$$

i.e.

$$T_A = -\frac{EA_2 h}{8}(1+k)\frac{\mathrm{d}^3}{\mathrm{d}x^3}\left(w_1 + w_3\right) - \frac{GA_2 h}{12}\left(1 + \frac{1}{k}\right)\frac{\mathrm{d}^3}{\mathrm{d}x^3}\left(w_1 - w_3\right) + \frac{GA_2 h}{2b^2}\frac{\mathrm{d}}{\mathrm{d}x}\left(w_1 - w_3\right)$$
$$T_B = -\frac{EA_2 h}{8}(1-k)\frac{\mathrm{d}^3}{\mathrm{d}x^3}\left(w_1 + w_3\right) + \frac{GA_2 h}{12}\left(1 - \frac{1}{k}\right)\frac{\mathrm{d}^3}{\mathrm{d}x^3}\left(w_1 - w_3\right) - \frac{GA_2 h}{2b^2}\frac{\mathrm{d}}{\mathrm{d}x}\left(w_1 - w_3\right). \tag{11}$$

2.3. Internal forces and displacements

By substituting (11) into the fourth equation of (1), taking onto account (3), the following differential equations can be written

$$EI_{yc}\frac{\mathrm{d}^4 w_1}{\mathrm{d}x^4} - \frac{EA_2 h^2}{8}(1+k)\left(1 - \frac{2G}{3kE}\right)\frac{\mathrm{d}^4}{\mathrm{d}x^4}\left(w_1 - w_3\right) - \frac{GA_2 h^2}{2b^2}\times\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(w_1 - w_3\right) = q_{z1},$$
$$EI_{ys}\frac{\mathrm{d}^4 w_1}{\mathrm{d}x^4} + \frac{EA_2 h^2}{8}(1-k)\left(1 + \frac{2G}{3kE}\right)\frac{\mathrm{d}^4}{\mathrm{d}x^4}\left(w_1 - w_3\right) + \frac{GA_2 h^2}{2b^2}\times\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(w_1 - w_3\right) = q_{z3},$$
$$I_{yc} = I_{y1} + (1+k)\frac{A_2 h^2}{4}, \qquad I_{ys} = I_{y3} + (1-k)\frac{A_2 h^2}{4}. \tag{12}$$

By multiplying the first equation of (12) by $I_{ys}/\left(I_{yc} + I_{ys}\right)$ and the second by $I_{yc}/\left(I_{yc} + I_{ys}\right)$, and subtracting the second equation from the first, the following equation may be written

$$EI_{ycs}\frac{\mathrm{d}^4 w_{1-3}}{\mathrm{d}x^4} - k_\beta \frac{\mathrm{d}^2 w_{1-3}}{\mathrm{d}x^2} = q_{zcs}, \tag{13}$$

where

$$w_{1-3} = w_1 - w_3, \qquad q_{zcs} = \frac{q_{z1} I_{ys} - q_{z3} I_{yc}}{I_{yc} + I_{ys}}, \qquad k_\beta = \frac{GA_2}{2}\left(\frac{h}{b}\right)^2,$$

$$I_{ycs} = \frac{I_{yc} I_{ys}}{I_{yc} + I_{ys}}\left[1 - \frac{(1+k)\left(1 - \frac{2G}{3kE}\right)I_{ys} + (1-k)\left(1 + \frac{2G}{3kE}\right)I_{yc}}{I_{yc} I_{ys}} \cdot \frac{A_2 h^2}{8}\right]; \tag{14}$$

$$\frac{\mathrm{d}w_{1-3}}{\mathrm{d}x} = -\beta_{1-3}, \quad EI_{ycs}\frac{\mathrm{d}^2 w_{1-3}}{\mathrm{d}x^2} = -M_{ycs}, \quad EI_{ycs}\frac{\mathrm{d}^3 w_{1-3}}{\mathrm{d}x^3} = -\frac{\mathrm{d}M_{ycs}}{\mathrm{d}x} = -\left(Q_{zcs} - m_{ycs}\right),$$

$$EI_{ycs}\frac{\mathrm{d}^4 w_{1-3}}{\mathrm{d}x^4} = -\frac{\mathrm{d}^2 M_{ycs}}{\mathrm{d}x^2} = -\frac{\mathrm{d}}{\mathrm{d}x}\left(Q_{zcs} - m_{ycs}\right) = q_{zcs} + \frac{\mathrm{d}m_{ycs}}{\mathrm{d}x};$$

$$m_{ycs} = k_\beta \frac{\mathrm{d}w_{1-3}}{\mathrm{d}x} = -k_\beta \beta_{1-3}. \tag{15}$$

Referring to (1), one may write

$$\frac{M_{y1}}{I_{y1}} - \frac{M_{y3}}{I_{y3}} = \frac{M_{ycs}}{I_{ycs}}. \tag{16}$$

From the equilibrium, it follows

$$M_{y1} + M_{y3} - N_2 h = \frac{1}{2}M_y, \qquad Q_{z1} + Q_{z3} = \frac{1}{2}Q_z. \tag{17}$$

By substituting (10), one obtains

$$\frac{M_{y1}}{I_{y1}}I'_{yc} + \frac{M_{y3}}{I_{y3}}I'_{ys} = \frac{1}{2}M_y, \tag{18}$$

where

$$I'_{yc} = I_{y1} + \frac{A_2 h^2}{4}\left(1 + \frac{2G}{3kE}\right), \quad I'_{ys} = I_{y3} + \frac{A_2 h^2}{4}\left(1 - \frac{2G}{3kE}\right), \quad I'_{yc} + I'_{ys} = \frac{1}{2}I_y. \tag{19}$$

From (16) and (18), one has

$$M_{y1} = \frac{M_y}{I_y}I_{y1} + 2\frac{I'_{ys}}{I_{ycs}} \times \frac{M_{ycs}}{I_y}I_{y1}, \qquad M_{y3} = \frac{M_y}{I_y}I_{y3} - 2\frac{I'_{yc}}{I_{ycs}} \times \frac{M_{ycs}}{I_y}I_{y3}, \tag{20}$$

and, and according to (1) and (15),

$$Q_{z1} - m_{y1} = \frac{Q_z}{I_y}I_{y1} + 2\frac{I'_{ys}}{I_{ycs}} \times \frac{Q_{zcs} - m_{ycs}}{I_y}I_{y1},$$

$$Q_{z3} - m_{y3} = \frac{Q_z}{I_y}I_{y3} - 2\frac{I'_{yc}}{I_{ycs}} \times \frac{Q_{zcs} - m_{ycs}}{I_y}I_{y3}. \tag{21}$$

Referring to (20), taking into account (1) and (15), it may be written

$$w_1 = w + \Delta w_1, \quad w_3 = w + \Delta w_3; \quad \beta_1 = \beta + \Delta\beta_1, \quad \beta_3 = \beta + \Delta\beta_3. \tag{22}$$

where

$$\Delta w_1 = 2\frac{I'_{ys}}{I_y}w_{1-3}, \quad \Delta w_3 = -2\frac{I'_{yc}}{I_y}w_{1-3}; \quad \Delta\beta_1 = 2\frac{I'_{ys}}{I_y}\beta_{1-3}, \quad \Delta\beta_3 = -2\frac{I'_{yc}}{I_y}\beta_{1-3}; \tag{23}$$

$$w = \frac{1}{2}\left(w_1 + w_3\right) + \frac{I'_{yc} - I'_{ys}}{I_y}w_{1-3}. \tag{24}$$

Thus, taking into account (1), (14) and (15), T_A and T_B given by (11) may be written as

$$T_A = \frac{Q_z}{I_y}S^*_{yA} - \frac{Q_{zcs} - m_{ycs}}{I_y} \times \frac{I'_{yc} - I'_{ys} - \frac{G}{3EK}I_y}{I_{ycs}}S^*_{yA} - k_\beta \frac{\beta_{1-3}}{h},$$

$$T_B = \frac{Q_z}{I_y} S_{yB}^* - \frac{Q_{zcs} - m_{ycs}}{I_y} \times \frac{I'_{yc} - I'_{ys} - \frac{G}{3Ek} I_y}{I_{ycs}} S_{yB}^* + k_\beta \frac{\beta_{1-3}}{h} . \qquad (25)$$

By substituting (25) into (21), referring to (2) and (12), (7) and (19), one may write

$$Q_{z1} = \frac{Q_z}{I_y} I_{yc} + Q_{zcs} - m_{ycs} , \qquad Q_{z3} = \frac{Q_z}{I_y} I_{ys} - Q_{zcs} + m_{ycs} . \qquad (26)$$

and according to (17)

$$Q_{zcs} - m_{ycs} = \frac{Q_{z1} I_{ys} - Q_{z3} I_{yc}}{I_{yc} + I_{ys}} , \qquad (27)$$

where

$$2 I_{yc} I_3 + \left(I'_{yc} - I'_{ys} - \frac{G}{3kE} I_y \right) S_{yB}^* = 2 I_{ys} I_1 - \left(I'_{yc} - I'_{ys} - \frac{G}{3kE} I_y \right) S_{yA}^* ,$$

2.4. Stresses

The stresses for the beam component 1 and 3 may, finally, be obtained by substituting (20) and (21) into (2):

$$\sigma_{x1} = \frac{M_{y1}}{I_y} z_1 + \Delta\sigma_{x1} , \qquad T_{xs1} = \frac{Q_z}{I_y} S_{y1}^* + \frac{Q_z}{I_y} S_{yA}^* + \Delta T_{xs1} ,$$
$$\sigma_{x3} = \frac{M_{y3}}{I_y} z_3 + \Delta\sigma_{x3} , \qquad T_{xs3} = \frac{Q_z}{I_y} S_{y3}^* + \frac{Q_z}{I_y} S_{yB}^* + \Delta T_{xs3} , \qquad (28)$$

where

$$\Delta\sigma_{x1} = \frac{2 I'_{ys}}{I_{ycs}} \times \frac{M_{ycs}}{I_y} z_1 , \qquad \Delta\sigma_{x3} = -\frac{2 I'_{yc}}{I_{ycs}} \times \frac{M_{ycs}}{I_y} z_3 ,$$

$$\Delta T_{xs1} = \frac{2 I'_{ys}}{I_{ycs}} \times \frac{Q_{zcs} - m_{ycs}}{I_y} S_{y1}^* - \frac{Q_{zcs} - m_{ycs}}{I_y} \times \frac{I'_{yc} - I'_{ys} - \frac{G}{3kE} I_y}{I_{ycs}} S_{yA}^* + \left. \frac{m_{ycs}}{h} \right|_A^{A'} ,$$

$$\Delta T_{xs3} = -\frac{2 I'_{yc}}{I_{ycs}} \times \frac{Q_{zcs} - m_{ycs}}{I_y} S_{y3}^* - \frac{Q_{zcs} - m_{ycs}}{I_y} \frac{I'_{yc} - I'_{ys} - \frac{G}{3kE} I_y}{I_{ycs}} S_{yB}^* - \frac{m_{ycs}}{h} . \qquad (29)$$

2.5. Boundary conditions

The boundary conditions at the beam ends may be formulated as

$$w_1 = w_1^* , \quad w_3 = w_3^* ; \qquad \beta_1 = \beta_1^* , \quad \beta_3 = \beta_3^* . \qquad (30)$$

Then, according to the first equation of (14)

$$w_{1-3} = w_1^* - w_3^* ; \qquad \beta_{1-3} = \beta_1^* - \beta_3^* ; \qquad (31)$$

and according to (24)

$$w = \frac{1}{2}\left(w_1^* + w_3^* \right) + \frac{I'_{yc} - I'_{ys}}{I_y} \left(w_1^* - w_3^* \right) ; \;\; \beta = \frac{1}{2}\left(\beta_1^* + \beta_3^* \right) + \frac{I'_{yc} - I'_{ys}}{I_y} \left(\beta_1^* - \beta_3^* \right) . \; (32)$$

In terms of forces, the boundary conditions may be formulated as follows

$$Q_{z1} = Q_{z1}^* , \quad Q_{z3} = Q_{z3}^* ; \qquad M_{y1} = M_{y1}^* , \quad M_{y3} = M_{y3}^* . \qquad (33)$$

Then, according to (27) and (16)

$$Q_{zcs} - m_{ycs} = \frac{Q_{z1}^* I_{ys} - Q_{z3}^* I_{yc}}{I_{yc} + I_{ys}}; \qquad M_{ycs} = \left(\frac{M_{y1}^*}{I_{y1}} - \frac{M_{y3}^*}{I_{y3}} \right) I_{ycs}; \tag{34}$$

and from (26) and (18),

$$Q_z = 2\left(Q_{z1}^* + Q_{z3}^*\right); \qquad M_y = 2\left(\frac{I'_{yc}}{I_{y1}} M_{y1}^* + \frac{I'_{ys}}{I_{y3}} M_{y3}^* \right). \tag{35}$$

3. Tanker hull subjected to bending with the cross-section distortion

The prismatic tanker hull with one longitudinal bulkhead within $l = 0,7L$ is considered, where L is the hull length. The load q_z is uniformly distributed in the longitudinal direction, where: $Q_z(0) = -Q_{zA}(l) = Q$, $M_y(0) = M_y(l) = M$; in the transverse direction: $q_{z1}/q_{z3} = 4$. The model corresponds approximately to the double skin tanker with full cargo tanks within 0,7 L, in the still water condition. At the boundaries the cross-section distortion will be neglected:

$$w_{1-3}(0) = w_{1-3}(l) = 0, \quad \beta_{1-3}(0) = \beta_{1-3}(l) = 0. \tag{36}$$

The internal forces and displacement, given by (15), at $x = 0$ read

$$Q_{zcs} = \frac{q_{zcs} l}{2} = \frac{q_z l}{2} \cdot \frac{q_{zcs}}{q_z} = Q_z \frac{q_{zcs}}{q_z},$$

$$M_{ycs} = -\frac{q_{zcs} l^2}{12} \chi_2 = -\frac{q_z l^2}{12} \cdot \frac{q_{zcs}}{q_z} \chi_2 = M_y \frac{q_{zcs}}{q_z} \chi_2, \quad \chi_2 = \frac{3}{v^2}\left(\frac{v}{\operatorname{th} v} - 1 \right); \tag{37}$$

and at $x = l/2$

$$M_{ycs} = \frac{q_{zcs} l^2}{24} \chi_1 = \frac{q_z l^2}{24} \cdot \frac{q_{zcs}}{q_z} \chi_1 = M_y \frac{q_{zcs}}{q_z} \chi_1, \qquad \chi_1 = \frac{6}{v^2}\left(1 - \frac{v}{\operatorname{sh} v} \right), \tag{38}$$

where

$$v = \frac{g}{2} = \frac{l}{2}\sqrt{\frac{k_\beta}{EI_{ycs}}}. \tag{39}$$

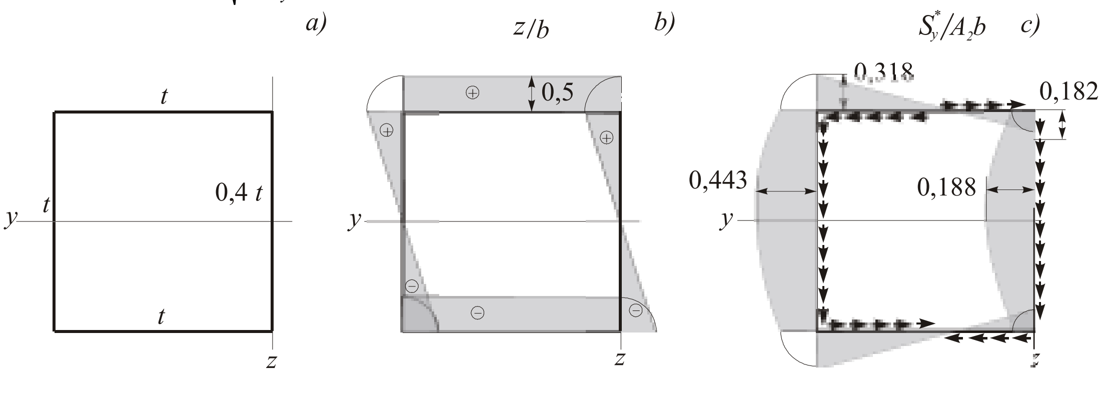

Figure 2 a) **s**implified section; b) z-coordinate; c) statical moment S_y^* (half of the section)

Slika 2 a) pojednostavnjeni presjek; b) z-koordinata; c) statički moment S_y^* (polovica presjeka)

The cross-section of a tanker: $L = 270$ m, $B = 48,2$ m, $D = 23$ m, $d = 16$ m, is simplified as shown in Fig. 2a; the properties are (Fig 2b,c):

$$l = 189\,\text{m}, \; h = b = 22,5\,\text{m}, \quad I_y = 1,233\; A_2 b^2, \quad S_{yA}^* = 0,182 A_2 b, \quad S_{yB}^* = 0,318 A_2 b,$$

$$k = -0,272\,,\quad I_{y1} = 0,0333 A_2 b^2\,,\quad I_{y3} = 0,0833 A_2 b^2\,,\quad I_{yc} = 0,2153 A_2 b^2\,,$$

$$I_{ys} = 0,4013 A_2 b^2\,,\quad I'_{yc} = 0,0476 A_2 b^2\,,\quad I'_{ys} = 0,5690 A_2 b^2\,,\quad I_{ycs} = 0,02189 A_2 b^2\,,$$

$$k_\beta = 0,5 G A_2\,,\quad \frac{E}{G} = 2,6\,,\quad \frac{2I'_{yc}}{I_{ycs}} = 4,349\,,\quad \frac{2I'_{ys}}{I_{ycs}} = 51,99\,,\quad \frac{q_{zcs}}{q} = 0,2254\,.$$

The, additional stresses due to distortion, given by (29), then are, at $x = 0$, according to (37)

$$\Delta\sigma_{x1} = 51,99 \frac{M_{ycs}}{I_y} z_1 = 51,99 \frac{M_y}{I_y} z_1 \frac{q_{zcs}}{q_z} \chi_2\,,$$

$$\Delta\sigma_{x3} = -4,349 \frac{M_{ycs}}{I_y} z_3 = -4,349 \frac{M_y}{I_y} z_3 \frac{q_{zcs}}{q_z} \chi_2\,,$$

$$\Delta T_{xs1} = \frac{Q_{zcs}}{I_y}\left(51,99 S^*_{y1} - 2,71 S^*_{yA}\right) = \frac{Q_z}{I_y}\left(51,99 S^*_{y1} - 2,71 S^*_{yA}\right)\frac{q_{zcs}}{q_z}\,,$$

$$\Delta T_{xs3} = \frac{Q_{zcs}}{I_y}\left(-4,349 S^*_{y3} + 2,71 S^*_{yB}\right) = \frac{Q_z}{I_y}\left(-4,349 S^*_{y3} - 2,71 S^*_{yB}\right)\frac{q_{zcs}}{q_z}\,,$$

and at $x = l/2$, according to (38)

$$\Delta\sigma_{x1} = 51,99 \frac{M_{ycs}}{I_y} z_1 = 51,99 \frac{M_y}{I_y} z_1 \frac{q_{zcs}}{q_z} \chi_1\,,$$

$$\Delta\sigma_{x3} = -4,349 \frac{M_{ycs}}{I_y} z_3 = -4,349 \frac{M_y}{I_y} z_3 \frac{q_{zcs}}{q_z} \chi_1\,.$$

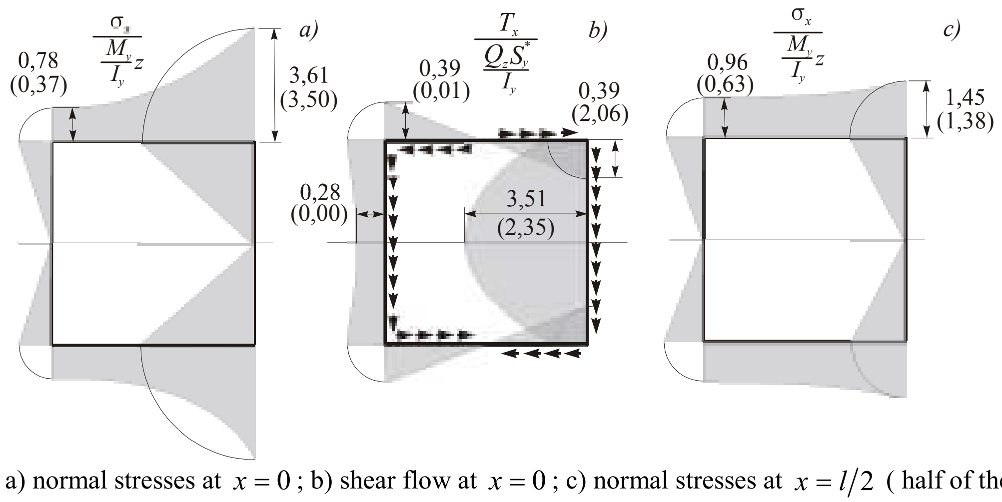

Figure 3 a) normal stresses at $x = 0$; b) shear flow at $x = 0$; c) normal stresses at $x = l/2$ (half of the section; FEM in brackets, shear flow at $x = 0,02l$)

Slika 3 a) normalno naprezanje na $x = 0$; b) posmično naprezanje na $x = 0$; c) normalno naprezanje na $x = l/2$ (polovica presjeka; MKE u zagradama, tok posmičnih naprezanja na $x = 0,02l$)

For the argument v given by (40), one has $v = 12,449$, and by (38) and (39) $\chi_1 = 0,0387$, $\chi_2 = 0,222$. Then, for $x = 0$

$$\Delta\sigma_{x1} = 2,61 \frac{M_y}{I_y} z_1\,,\quad \Delta\sigma_{x3} = -0,78 \frac{M_y}{I_y} z_3\,,$$

$$\Delta T_{xs1} = \frac{Q_z}{I_y}\left(51,99 S^*_{y1} - 2,71 S^*_{yA}\right) 0,2254\,,$$

$$\Delta T_{xs3} = \frac{Q_z}{I_y}\left(-4,349 S^{*}_{y3} - 2,71 S^{*}_{yB}\right) 0,2254 ;$$

and for $x = l/2$

$$\Delta \sigma_{x1} = 0,454 \frac{M_y}{I_y} z_1 , \quad \Delta \sigma_{x3} = -0,038 \frac{M_y}{I_y} z_3 .$$

Stresses at $x = 0$ (at the ends of the region 0,7*L*) and at $x = l/2$ (amidships) are shown in Fig. 3.

The 3D membrane model of one-quarter of the hull is considered with 34020 rectangular elements and 34200 nodes (Fig. 4) [9]. At $x = 0$, the node displacements of the vertical walls are restricted in both directions, while the node displacements of the horizontal walls are restricted in the longitudinal direction only. At $x = l/2$, the node displacements are restricted in longitudinal direction only. At $y = 0$ (in the plane of symmetry) the displacements of the horizontal walls are restricted in the transverse direction only. The results obtained by the finite element method are shown in Fig. 3 in brackets (the shear stresses are given for $x = 0,02l$ since at $x = 0$ these stresses are equal to zero due to boundary condition, as shown in Fig.4).

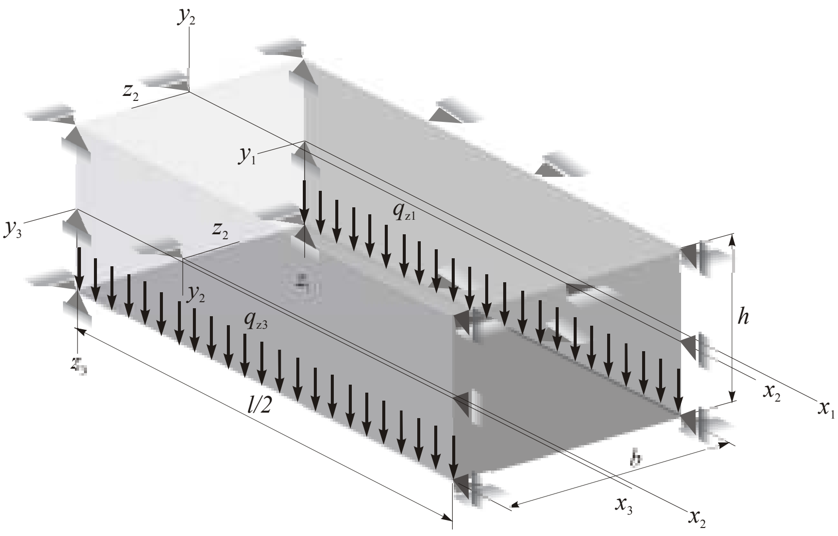

Figure 4 Finite element model (one quarter of the hull)
Slika 4 Model s konačnim elementima (četvrtina trupa)

The shear stresses are almost equal to zero for the side walls (ship sides) due to the loading condition ($q_{z1}/q_{z3} = 4$).

The differences between the results for the shear stresses are significant due to the assumed approximate solution for the displacement field for the beam flanges (ship deck/bottom plating), given by (2), as well as due to the boundary conditions for the clamped ends, which are not identical for the finite element model and the analytical solution of the problem (Fig. 4).

For the given boundary conditions, as is expected, the maximum stresses due to the cross-section distortion occur at the clamped ends (at $x = 0$ and $x = l = 0,7L$).

4. Conclusion

The additional stresses due to the cross-section distortion of the tanker hull with one longitudinal bulkhead are compared to the stresses of the ordinary bending theory, where the cross-section distortion is ignored. It is assumed that the hull panels are hinged along their longitudinal edges.

The comparison with the finite element method has shown acceptable agreements of the obtained results.

Although the stiffness of the transverse frames and bulkheads is ignored, the model with hinged walls can be useful to further, more realistic, considerations, with the actual hull cross-section stiffening.

The real stiffness of the hull cross-section can be easily included in the consideration. The torsional stiffness of the plating together with bending stiffness of the framing and the shear stiffness of the transverse bulkheads should be taken into account.

REFERENCES

[1] Kollbrunner, C.F., Basler, K., Torsion in Structures, Springer-Verlag, Heildeberg, New York, 1969.

[2] Kim, J.H., Kim, Y.Y., Thin-walled multi-cell beam analysis for coupled torsion, distortion and warping deformation, ASME Journal of Applied mechanics **68** (2001) 260-269.

[3] Jönsson, J., Distortional theory of thin-walled beams, Thin-Walled Structures **33** (1999) 269-303. respect to the cross-section distortion, International Journal of Mechanical Sciences **44** (2002) 423-442.

[4] Pavazza, R., Plazibat. B., Matoković, A., 2001. On the effective breadth problem of deck plating of ships with two longitudinal bulkheads, International shipbuilding progress **48** (2001) 51-85.

[5] Pavazza, R., Matoković, A., On the shear flow due to distortion of hull cross-sections of tankers with two longitudinal bulkheads. Proceedings, IMAM 2000, Vol II, Ischia, Italy, pp. F32-39.

[6] Pavazza, R., On the load distribution of thin-walled beams subjected to bending with respect to the cross-section distortion, International Journal of Mechanical Sciences **44** (2002) 423-442.

[7] Pavazza, R., Blagojević, B., On the cross-section distortion of thin-walled beams with multi-cell cross-sections subjected to bending, International Journal of Solids and Structures **42** (2005) 901-9295.

[8] Hughes, O.F., Ship Structural Design. John Wiley & Sons, New York, 1983.

[9] ALGOR V15 User's Guide, Algor, 2004.