

# Design of a coordinated control system for marine vehicles

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**Abstract** — The field of control of marine vessels is seeing new advances in control algorithms, and new technologies introduced both in the field of sensors (MEMS, nano-), as well as the actuators. All these influences have made the prospect of using fleets of unmanned marine vehicles (surface and/or underwater) very attractive. In such a fleet, individual craft need to be equipped with a control system that guides them to behave in a coordinated manner and thereby elevate the mission performance indicators to new, previously unobtainable levels.

**Keywords:** cooperative navigation and control, survey, AUV

## I. INTRODUCTION

The immediate, plant-level control of actuators, supplementary systems and sensors aboard modern unmanned marine craft, is a well researched topic. Next level of a control hierarchy of both manned and unmanned craft is the servo-control of directly actuated (or rarely, dynamically coupled to the actuation of other) degrees of freedom (DOF). The dynamics of at least some, if not all of the DOF of marine craft are nonlinear due to the physics of moving partly or wholly through water. In spite of that, good methodologies and rules-of-thumb have been applied in the programming of both linear time-invariant and relatively simple nonlinear controllers. The most critical DOFs that need to be controlled in marine craft are course (yaw) and surge. Other DOFs of special interest, for whom controllers are regularly programmed in the craft control system depend on the hull engineering, category and envisioned mission specifications of a particular craft. In surface-going craft, the third most important DOF is the roll, whose ill control can cause the capsizing of the ship. In underwater craft, the third most important DOF is either the heave, or pitch. The heave is critical to work-grade underwater marine craft, usually unmanned, with poor hydrodynamic lines compensated for by the overabundance of installed power for the heave DOF actuator – also called the z-thruster. The pitch, on the other side, is critical to underwater craft of good hydrodynamic lines that only have significant installed power in the surge DOF, and actuate other DOFs by using actuated elevons, fins and rudders. A typical representative is a modern military nuclear submarine or a torpedo-type unmanned marine craft of the cruise AUV type.

The DOF servo-controllers currently employed and researched achieve good results (in the sense of the

natural time constants of the actual craft themselves) in controlling the state of the craft in the said DOF so as to conform to a signal provided by either a human operator or some other automatic system.

The reference signal for all controlled DOFs is the third level of the unmanned craft control system. Manned craft do not have this control level, substituted by a human operator / pilot / skipper. It is by this level that coordinated control of a group of like unmanned marine craft can be achieved. Such coordinated control requires this third level, the autopilot-level to provide the craft with the abilities of formation-keeping, collision avoidance, terrain following and localization.

The constraints and requirements on any system that endeavors to solve the coordinated control problem for unmanned marine craft, in order for it to be of real use are:

- The system needs to be stable,
- The system needs to be autonomous and human-operator-independent,
- The system needs to be robust, fault tolerant and reconfigurable,
- The system needs to be energetically conservative,
- The system needs to be spatially conservative,
- The system needs to be computationally conservative. It needs to be programmable, feasible and of practical formal computational complexity,
- The system needs to be relatively cheap and easily serviceable,
- The system needs to be tractable and deterministic,
- The system should not experience loss of generality, applicability and efficiency in an unstructured dynamically changing environment,
- The system needs to contain collision-avoidance.

Due to this fairly large number of very conservative requirements, the coordinated control system is usually hierarchical. If the functionalities of such a generally complex system are layered into a hierarchy, it becomes much easier to assure the conformation to the given relatively large set of requirements. Such hierarchies are usually a variant of the one presented in Figure 1.

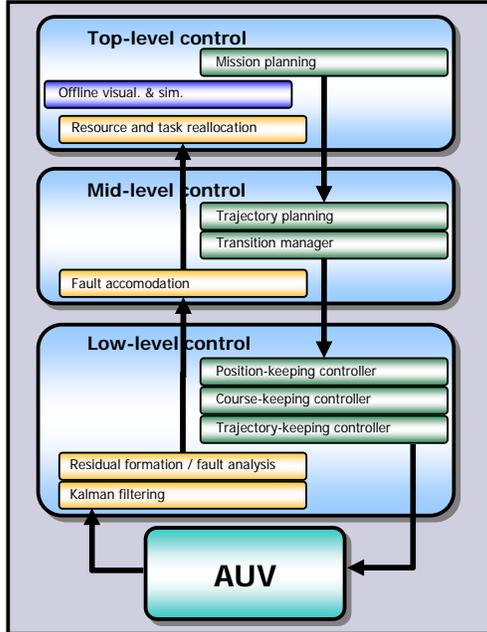


Figure 1: The hierarchical topology of a generalized coordinated control system

After this Introduction, this paper proceeds to survey and comment on the methodologies and algorithms for coordinated control of a group of unmanned marine vessels. This is covered in Section II. However, the main purpose of the paper is achieved in Section III, by showing through a specific example how to choose, apply and link together specific instances of the listed and surveyed approaches to the unmanned marine vehicle coordinated control problem. A single example of a coordinated control hierarchy, presently being engineered at the Faculty of Electrical Engineering and Computing of the University of Zagreb, is followed through from inception to the implementation layout.

## II. THE COORDINATED CONTROL PARADIGM AND CURRENT APPROACHES

### A. The Coordinated Control Paradigm

The solution of a control problem for a given group of AUVs that obeys the coordinated control paradigm begins by the choice of approach to the following problems / analysis tasks:

- Selecting and choosing a modeling approach to individual AUVs (dictating the nature and number of parameters that will be obtained by the modeling process itself and the physical scope of validity of the model),

- Selecting and choosing an approach to model possible AUV interactions (dictating the nature and number of parameters that will be obtained by the modeling process itself and the physical scope of validity of the model)
- Selecting a particular technique (this is the first choice that clearly eliminates or heavily penalizes certain coordinated control paradigm features and requirements in favor of others),
- Applying appropriate initial and boundary conditions, and possible constraints, to the selected technique, to arrive at a first-principles algorithm of online real-time motion planning.

After the first-principles algorithm is calculated and constructed, some “post-production” problems remain. These need to be addressed before the algorithm is to be expected to behave up to specifications in a real world, real-time environment. These “post-production” problems mostly include some or all of the following:

- real-time, effectiveness, complexity and other optimizations,
- actual implementation issues relating to the current state-of-the-art of command and control electronics, motherboards, operating systems, APIs, SDKs and development suites, embedded aboard modern AUVs.

A more detailed overview of these problems and issues actually represents the structured coordinated control paradigm, in Figure 2.

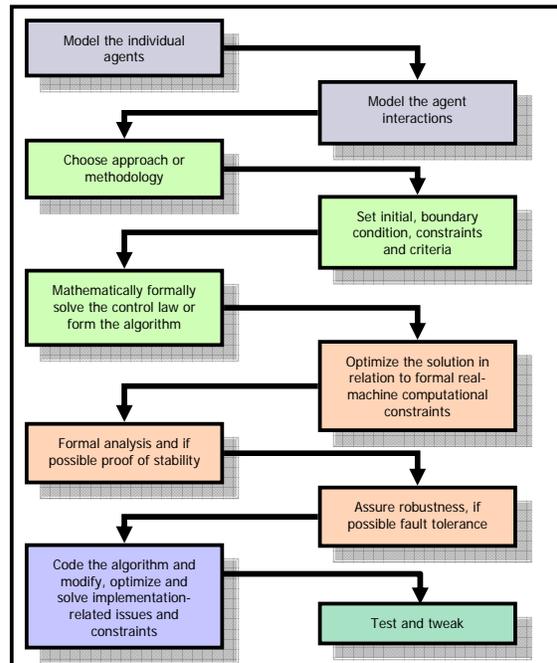


Figure 2: The meta-algorithm that encapsulates the coordinated control paradigm

## B. The Layout of Current Approaches

The approaches that have been pursued in this field in the last few years can be broadly subdivided according to the following categories:

- graph-theoretical approaches, [8, 14, 18, 19, 22],
- the virtual structure approaches, [2, 12, 13, 25],
- the virtual potential method, [9, 16, 20],
- the general iterative methods, based on receding horizon MPC, mixed integer programming, dynamic programming, or simulation of state machines – most notably in the field of coordinated control of a formation of unmanned aerial vehicles, [3, 4, 5, 7, 10, 11, 21, 23, 24, 26].
- Behaviouristic approaches, [1, 6]

## C. The Graph-theoretical Approaches

Graph-theoretical approaches mostly deal with the proof of stability. They have been instrumental in reducing the scale and complexity of the proof-of-stability problem in coordinated control. The work in [8, 18, 22] deals with using the mathematical term of graph rigidity with the Lyapunov approach to determine the stability of a coordinated control system. The work in [8, 19] gives crucial insights into the constraints and necessities of AUV interaction. These must be effected if complex formation features are to be achieved – namely formation split into stable sub-formations and a complementary operation – rejoin. Both [8, 19] and [22] establish the fact that very simple AUV-local algorithms that function well as a coordinated control system can be implemented. These, although individually simple, when coordinating in a fashion that can be modeled as a graph, produce a complex and intelligent behavior.

However, the necessity of any communication in an underwater environment is reduced primarily to hydro-acoustic ultrasound communication. The characteristics of such communication are broadcast nature, low bandwidth and low signal strength drop-off. Ultrasound communications are almost always continuous, since, as they are low-bandwidth it is extremely difficult for short bursts to contain the amount of information necessary. Being continuous, and there being many craft trying to communicate significant simultaneity of communications occurs. Combined with low drop-off and the inexistence or severe limitations on multiplexing, the communication channels inevitably become clogged. In other words, communications between pairs of craft represent critical noise to other craft expecting communications.

## D. The Virtual Structure Approaches

The approaches to treating a formation as a virtual rigid body in [13, 24] deal with an AUV-local algorithm which produces a control output according to an iterative mathematical procedure. The distinct property of this algorithm is that while not being explicitly programmed

in such a way, it has the function of maintaining formation pose around a faulty or error-state AUV. Another significant approach in both [12, 17] also treats a formation as a virtual rigid body. The work in the references concentrates on the distinct mathematical features and isomorphism of such an approach. These contribute to a significantly decreased number of necessary control signals i.e. the reduction in the number of DOFs of such an ideally rigid formation. It also shows that if some other initial control is implemented that achieves the initial formation rigidity, the described approach maintains the same measure of rigidity.

## E. The Virtual Potential Method

The virtual potential methods, described in [9, 16, 20] have a distinct benefit of low computational overhead and good scalability. However, the virtual potential method also suffers from built-in problems. These are most acute in the form of limitations of optimality and usability with certain sets, combinations, forms and poses of obstacles in the staging area.

This is most obvious through a (virtual, software-generated) phenomenon called obstacle-goal shadowing. An ill-scaled continually dropping-off potential repulsive function of an obstacle can function as an irrationally posed “no go” area. This area is actually free of obstacles, and additionally, sometimes the only route to a goal point. If this set-up occurs, the trajectory planning algorithm produced by the virtual potential function will fail to produce a valid trajectory towards the goal point, although an actual obstacle-free route exists.

Stability of virtual potential methods must be assured by intervention into the mechanics of calculation of the reference signal to be forwarded to the DOF actuators. These mechanics must include some mode of “shedding” or attenuating the energy desired of the craft’s actuators (thrusters) in order for the planned trajectory to stably terminate. Without that, un-damped oscillatory motion can be produced that fails to bring the craft to an all-stop at a desired goal point.

## F. The General Iterative Approaches

This includes the approaches of planning and iteratively arriving at a set of trajectories by either dynamic programming [26], mixed integer programming [7] or receding horizon approaches [5, 21]. Such methods provide a body of knowledge based upon which an optimal algorithm for the use with a specific fleet of AUVs for a specific mission profile can be built.

Research and analysis of differences between AUVs and UAVs (unmanned aerial vehicles) is needed in this field since most of the work done is on platoons or squadrons of flying agents. However, most algorithms are easily portable. This is due to the hierarchical structure of the coordinated control system. It allows for the trajectory

planner's insensitivity and generality in relation to the low-level control of a particular agent – UAV or AUV.

The strength of this group of approaches is that they are easily understood and rely on regular, graduate-level knowledge and techniques. Being well researched, there is sufficient breadth for tweaking and adapting an approach covered in the literature to a new problem. The weaknesses of this group of approaches are twofold. The first is that the mathematics and system analysis necessary to express the coordinated control program in the syntax that dynamic programming, mixed integer programming or receding horizon approaches are designed to solve is nontrivial. The second is that sometimes, tradeoffs and simplifications occur in this system analysis and mathematical representations which, although the stability of the solution is assured, contribute towards either instability of the final, actual behavior of the controlled craft, or intractability of the approach. The former occurs when sets of parameters, states of the coordinated group system and internal program values achieve values that produce numerically intractable values in the on-line receding horizon algorithm, which then shuts down and produces an error state.

#### G. *The Behavioristic Approaches*

Finally, behavioral approaches also present an interesting possibility. There don't seem to be many approaches other than [1], combining the behavioral algorithms with a coordinated control paradigm. This reference researches applications of single-agent control based on the behavioral structure, such as the one described in [6] to a coordinated control system. However, further research is necessary and might provide a solution to many problems encountered when pursuing other methods.

The advantage of the behavioristic approaches is an inherent logic and applicability since behavioristic approaches mimic nature – the reasoning and behavior of highly agile, maneuverable and fast animals. This logic is often self-evident and easily understandable to the control engineer since it draws from real-life, real-world experience untied to any formal knowledge.

The disadvantage of the behavioristic approaches is that the proof-of-stability can be either hard or unobtainable. This occurs due to the fact that competing behavioral modules can trigger off chaotic behaviors when two or more modules are operating close to their respective thresholds of sensitivity to either some state of the environment or of the craft. The marked difference between such modes of a behavioristic coordinated control system and the behavior of real animals lies in the fact that all current models of behavior are by their very nature simplistic. Primarily, this simplicity manifests itself by behaviors not being interactive enough with each other nor with measures of the internal "animal" or "craft" state.

### III. APPLICATION TO THE PROBLEM OF AUV FLEET CONTROL

Due to the different nature and different position within the typical AUV control hierarchy of various methods that is discussed in section II, a definite choice of a cooperative control system in our research group, applied to a problem of a small fleet of AUVs is a combination of the following:

- A. a GUI mission preparation package which is user-friendly, uses visualization (standard nautical maps and 3D models extracted off them), and is easily extendible and modular in functionality,
- B. a locally implemented rule-based system for relative placing of the waypoints and individual AUV locations within a formation at a given leg or stage of the mission, which interacts with...
- C. the virtual potential method trajectory planner forwarding trajectories to a lower-level adaptive linear AUV course and depth controller,
- D. the low-level adaptive linear course and depth controller.

#### A. *The GUI Mission Preparation Package*

The GUI mission planning package is insofar as time, linearity, mathematical tractability and other usual control theory issues are considered, not a control algorithm. It is an intuitive, non-expert human-machine interface, a visualization tool and a meta-programming package. Its purpose is to produce a time-ordered sequence of highly contextual and abstract top-level instructions (a mission schedule).

#### B. *The Rule System*

This is a locally implemented rule-based system built using a semantics-handling language such as CLIPS. It translates the general semantic description of the mission schedule into more detailed and less abstract rules for each phase or leg of the mission. As its link to the lower control level, it features an evaluation and defuzzification layer. The evaluation layer forwards the numerical values to the lower-level layers of the coordinated control hierarchy. These numerical values are mostly mathematical representations of the goal- or waypoints.

The information that needs to be retrieved from the actual physical mission taking place, into the Rule System are the states of the craft itself, other craft in the vicinity, and the environs. This state data needs to address existence of possible in-system faults, or a general incapability to perform up to required parameters. Additionally, qualitative, semantic information on the completeness and quality of achievement of mission goals must also be present.

The Rule System uses this information to short-list: unfulfilled requirements of this phase of the mission, present capabilities and status of the fleet, and operator-

input preferences regarding handling of sub-optimal situations. Out of these three lists, using semantic rules, the System determines the requirements for a single craft’s maneuvers. The completed list of semantic “orders” is forwarded to the defuzzification layer. This “output interface” produces a set of parameters that is made available to the lower control levels – the DOF servo-controllers.

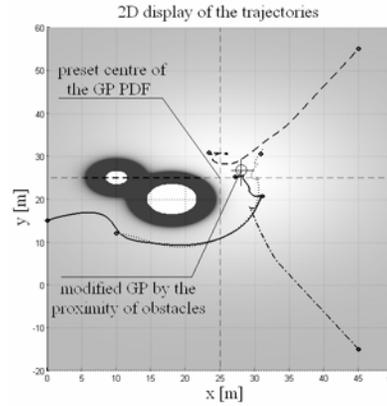
### C. The Virtual Potential Method Trajectory Planner

The virtual potential method trajectory planner builds a reference signal for the set of DOF controllers (in our case, the surge, course and heave). It operates in an online mode contemporaneously but not necessarily simultaneously (in the sense of synchronized sample time) with the DOF servo-controller. All features of the environment, meaning regions of the mission space that represent obstacles, or regions of the mission space that represent vicinities of other cooperative craft, are represented by virtual potential distribution functions (PDFs). This representation relies upon two channels of information:

1. The feature extractor operating on a signal from a “main navigational” sensor of the environment (sonar). Extracted features include the class of obstacle (a geometric body), and, depending on that, craft-relative measures of position, pose and dimensions of the obstacles.
2. A parameterization policy, or set of values, forwarded from the Rule System, which affixes all “free” parameters – those that regulate the drop-off rate, spatial density, homogeneity, smoothness, number of maxima and minima, and “contribution” to repulsive or attractive action of the extracted features.

These two channels of information contain all parameters necessary to compute the trajectory on-line. A few “predictive” samples of this trajectory, valid for a “still” environment sampled at the trajectory planner sample time, are forwarded to the servo-control of relevant DOFs. Implementation-wise this sample time depends on the time required for an “environment snapshot” off of the “main navigational” sensor. Since this time is usually longer than the sample time at which it is possible to drive the DOF servo-controller control level, this in effect produces a finite, multi-sample horizon of the “predicted” trajectory. Due to this, results of experiments and research in receding horizon approaches to the solution of the coordinated control problem [5, 12], are of great importance.

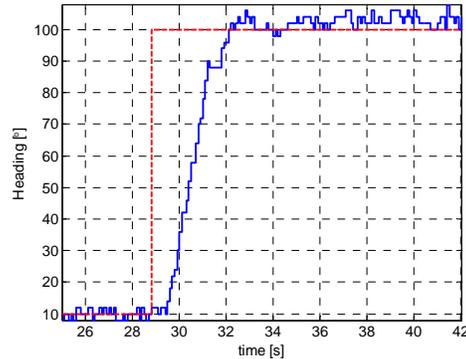
Ideally, the trajectory planner level would be driven at the same sample time as the servo-control tier and the horizon would recede to one sample of “predicted” trajectory ahead of the actual control action. The action of the virtual potential method trajectory planner in a four-craft formation with significant obstacles is displayed in Figure 3.



**Figure 3:** Formation problem for a group of 4 agents in the vicinity of obstacle solved – sub-optimal but feasible solution found

### D. Thruster-level Adaptive PI-D Controller

The lowest, thruster-level course control is performed by an adaptive PI-D controller developed on the foundation of [15]. It is used to reduce the dynamics of the AUV to some non-ideal linear time invariant dynamics as close as possible to the presupposed double-integrator action. A set of tests relying on self-oscillations along controlled DOFs (course, depth) introduced by relay action in the feed-forward branch are hard-coded into the algorithm. These are performed at any instance of the change of craft’s handling parameters: installation of additional sensors, redesign and installation of different thrusters, reconfiguration of thrusters, etc. The result of the tuning process is displayed in Figure 4.



**Figure 4:** The result of the autotuning procedure on the improvement of course servo-control on a cheap, commercially available micro-ROV VideoRay Pro

## IV. CONCLUSION

In conclusion, we envision this system to be the one to allow operators of fleets of unmanned marine vehicles to easily have control over the mission. The GUI package that is under development should allow for precise setting of mission goals, good visualization of the environment in which mission is to take place and good awareness of the necessary maneuvers and formation changes that need

to be included in the mission schedule. As a result, the operators will have elevated awareness about the time, energy and effort expenditures that need to be made to ensure the success of the mission. The Rule Based System (RBS) that is also under development should provide for an easy reprogramming and tweaking of maneuvers to different types of craft included in the “fleet”. Due to the fact that the rule base is semantic (simplified English sentences according to fixed syntax), adapting or extending the rule base when including heterogeneous craft in a same mission is natural. We regard this as critical since we aim to construct a system that will enable the operators to use USVs working in cooperation with AUVs, different types of AUVs: cruise torpedo-hulled ones and work-class AUVs etc. The described hierarchical and modular structure of the coordinated control system takes into account this possibility and precludes any necessity of rebuilding hard programming code.

The results under the Virtual potential method trajectory planner are 2D for purposes of clarity and easy display of the potential field that is the direct cause of the trajectory, but the planner works with 3D obstacles and way- and goal-points.

Further work will include the programming, building, linking and embedding of the GUI Mission Planner and the RBS. After these are finished as stand-alone server applications, the work will proceed to the construction of a multi-motherboard, networked open computing structure which will have different levels of the coordinated control systems running on parallel processors and communicating via Ethernet.

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