High Performance Predictive Current Control for Active Shunt Filters

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***Abstract*—A discrete-time predictive current controller, for the application of active power filtering, is designed and analyzed in this paper. A modified algorithm of reference current prediction in the conditions of step change in amplitude and/or frequency is proposed. The theoretical analysis and the validity of the proposed method are verified by simulations. Comparison for the 5th and the 7th harmonic current control is made with the results achieved using synchronous PI plus lead compensator.**

I. INTRODUCTION

The use of active power filters (APF) in distribution systems represents the best solution, in terms of performance and effectiveness, for elimination of harmonic distortion as well as power factor correction, balancing of loads, voltage regulation and flicker compensation. The shunt APF, connected in parallel with the non-linear load, is commonly utilized to compensate for current disturbances while the series APF is utilized to compensate for voltage disturbances.

The shunt APF generates the harmonic currents required by the non-linear load so that the supply feeds only the fundamental component. The effectiveness of shunt active filters greatly depends both on the method to determine the current references and the current control strategy.

The main focus of this paper is the development of an efficient current control design. Most commonly, active power filters inject the 5th and the 7th harmonics as well as the reactive current component of the nonlinear load at the point of connection with step change even in steady state.

Classical control approach using PI controllers eliminates steady state error if reference signals are dc quantities. So multiple reference frames rotating at the harmonic frequencies to be compensated need to be implemented if the application of a traditional proportional-integral control is required. Unfortunately the interactions among these different frames and the presence of suitable band-pass filtering stages make the design and tuning of these loops a quite complicated task.

A single dq frame controller, synchronous with the 50Hz supply voltage, is commonly found on commercial products, as it provides a good compromise between implementation simplicity, limited commissioning and control robustness and stability. The PI controller characteristic of tracking dc quantities can be used in this case also for sinusoidal currents which frequency is equal to fundamental supply voltage frequency. In the case of active power filtering systems, reference currents consist of higher harmonics, whose d-q components in this frame are pulsating quantities and steady state error cannot be eliminated.

Predictive current control [1]-[5] is a linear control technique suitable for APF applications offering the advantage of precise current tracking over a wide frequency range. A predictive current controller is model-based controller; therefore knowledge of system parameters is essential for satisfactory performance. In particular, stability problems occur in ac drive applications [6] when back emf needs to be estimated. This drawback doesn't apply for active power filtering systems because line voltage is available and can be measured.

In this paper, dynamic as well as static performance of a predictive current control, which incorporates computational delays, is analyzed. Since here the main goal is focused on tracking performance of the current control loop, it's assumed that the reference current can be arbitrary given. To take into account a more realistic case, it is assumed that the waveform of the reference current is not known, so a prediction of the reference current in the control algorithm should be independent of the specific waveform.

A modified algorithm for reference current prediction to achieve good dynamic performance during transient states, is here also proposed. The validity of the whole prediction algorithm is verified by simulations developed in Matlab/Simulink environment; SimPowerSystems blocks were used for the power part of the simulation model.

II. MATHEMATICAL MODEL

The circuit topology of the pulse width modulation voltage-source converter (PWM-VSC) used in the application as active power filtering systems is presented in Fig. 1. The converter consists of six fully controllable switches (S_a, S_b, S_c) and it is connected to the grid via input inductors. The output power stage of APF (single phase equivalent) can be described with the following linear first order differential equation:

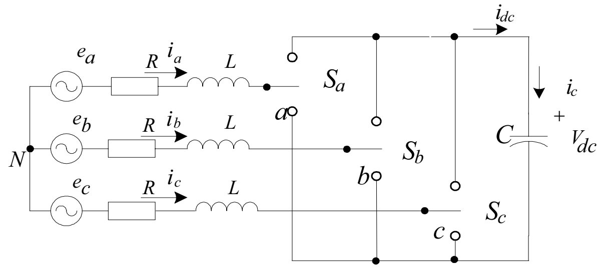


Fig.1 Main topology of the analyzed system

$$\frac{di}{dt}+\frac{R}{L}\cdot i=\frac{e-v}{L} \quad (1)$$

where e - supply voltage, i – active filter current, v – active filter voltage, R, L –resistance and inductance of input inductors. The discretized solution [2] of the system differential equation (1) is:

$$i(k+1)=i(k)\cdot e^{-T_s/\tau}+\frac{E-V}{R}\cdot\left[1-e^{-T_s/\tau}\right] \quad (2)$$

$$i(k+1)=i(k)\cdot a+(E-V)\cdot b \quad (3)$$

$$a=e^{\frac{-R}{L}\cdot T_s}=1-\frac{R}{L}\cdot T_s \quad (4)$$

$$b=\frac{1-e^{-T_s/\tau}}{R}\approx\frac{1-1+\frac{R}{L}\cdot T}{R}=\frac{T}{L} \quad (5)$$

where the coefficients a and b are approximated by Taylor series, active filter and supply voltage are assumed to be constant and equal to V and E respectively during one sampling time T_s. The time constant of the output stage of APF is denoted with $\tau=L/R$. APF current at time instants k and k+1 is denoted by $i(k)$ and $i(k+1)$ respectively.

III. Control Strategy

The proposed predictive current control is based on the discretization of the system differential equation so the discretized model in (3) is used for current controller design. The aim of this control is to calculate such an APF voltage $v=v_i$ ($i=a$, b, c) that the current error at the end of the sampling period is eliminated (8). For the purposes of control, current error at the time instants k and k+1 can be introduced:

$$\Delta i(k)=i^*(k)-i(k) \quad (6)$$

$$\Delta i(k+1)=i^*(k+1)-i(k+1) \quad (7)$$

$$\Delta i(k+1)=0 \quad (8)$$

where $i^*(k)$ and $i^*(k+1)$ denote the reference currents at time instants k and k+1 respectively. From (3), (6), (7) and (8) the desired active filter voltage is:

$$V(k)=E(k)-\frac{1}{b}\left[i^*(k+1)-i(k)\cdot a\right] \quad (9)$$

Including a computational delay, which in this case is kept constant and equal to one sampling period, equation (9) becomes:

$$V(k+1)=E(k+1)-\frac{1}{b}\left[i^*(k+2)-i(k+1)\cdot a\right] \quad (10)$$

In this analysis it is assumed, at first, that the waveform of reference current is known. This is a case when each of the harmonics is detected separately, so two-ahead prediction of reference current is straightforward.

The current prediction at time instant k+1 is:

$$i_{pred}(k+1)=i(k)+(i^*(k+1)-i^*(k))-error \quad (11)$$

$$error=i_{pred}(k+1)-i^*(k+1) \quad (12)$$

This control approach eliminates the steady sate error and the transient inside one sampling period without oscillations as shown in (13).

$$\frac{i(k)\cdot}{i^*(k)}=\frac{z^2-z+0.5}{z^2-z+0.5}=1 \quad (13)$$

To take into account changes in the supply voltage during one sampling period, an integral of supply voltage between the time instants k+1 and k+2, in the solution of system differential equation (1), is modeled by average value of voltage assuming it changes linearly. The prediction of the supply voltage at time instants k+1 and k+2 is made using a linear-type prediction:

$$E(k+1)=2\cdot E(k)-E(k-1) \quad (14)$$

$$E(k+2)=3\cdot E(k)-2E(k-1) \quad (15)$$

If the waveform of the reference current $i^*(k+2)$ is not known two-ahead prediction first need to be applied. Most commonly in active filter applications the reference currents consist of the 5th and the 7th harmonics, so a simple linear prediction does not give a satisfactory accuracy; at least a second order polynomial extrapolation needs to be applied. A Lagrange second order polynomial-type prediction here is used:

$$i^*(k+1)=3\cdot i^*(k)-3\cdot i^*(k-1)+i^*(k-2) \quad (16)$$

$$i^*(k+2)=6\cdot i^*(k)-8\cdot i^*(k-1)+3\cdot i^*(k-2) \quad (17)$$

The second order Lagrange extrapolation formula uses the current value and two recent previous values with suitable coefficients to predict the value of the signal at the instant k+2. During amplitude and/or frequency transients this method gives incorrect results for next two sampling periods, as shown in Fig. 2.

To avoid this problem here it is proposed a modified algorithm for reference generation in transient conditions. At first a transient state needs to be identified. The identification is based on a maximum allowable prediction error. To define this limit prediction error independently from the value of current amplitude, a maximum amplitude of reference current need to be defined. Third-order extrapolation is used for the reference current prediction error calculation. The maximum error of the one-step-ahead Lagrange third-order extrapolation is 0.25%, as shown in Fig. 3. The third-order one-step-ahead extrapolation of reference current is:

$$i^*(k+1)=4i^*(k)-6i^*(k-1)+4i^*(k-2)-i^*(k-3) \quad (18)$$

After detecting the transient state, during next two sampling periods, reference is generated using the value $i^*(k)$ instead of the two-ahead predicted value $i^*(k+2)$. If the reference current $i^*(k)$ is used instead of the two-ahead predicted one, the delay between the reference and real current is two sampling periods.

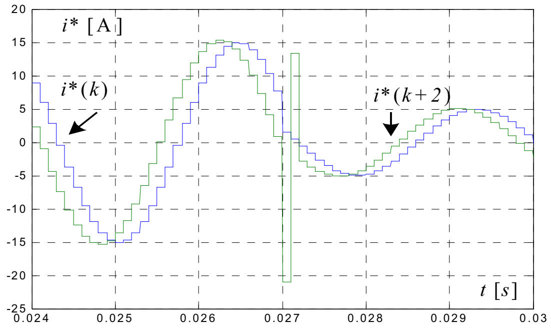


Fig. 2 Comparison of the measured reference current (blue) and two-ahead predicted reference current (green) using second order Lagrange formula (at t=0.027s step change from 5A to 15A)

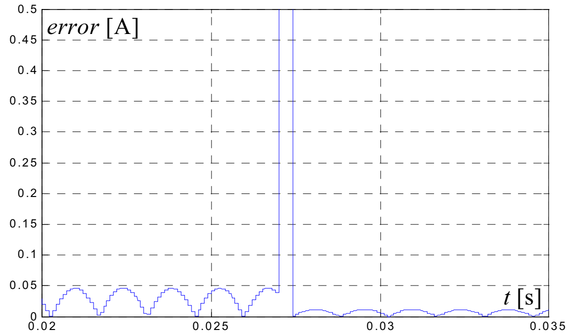


Fig. 3 Absolute value of the prediction error using one-step-ahead Lagrange third-order extrapolation (at t=0.027s amplitude change from 20A to 5A)

The system control structure is cascaded, current control as inner and voltage control as outer control loop. The time constant of the voltage control loop is at least ten times higher then the current control loop time constant so the design of these two loops can be independent. The block diagram of the proposed control strategy is presented in Fig. 4. The output of voltage PI controller presents the active component of the active power filter current to cover losses of switching devices and parasitic resistance in the circuit. The active component need to be added to the harmonic reference current demanded by the nonlinear loads at the point of connection. The predictive current controller is made in a-b-c coordinate frame so to add the active component to harmonic reference current, the supply voltage needs to be measured, Fig 4.

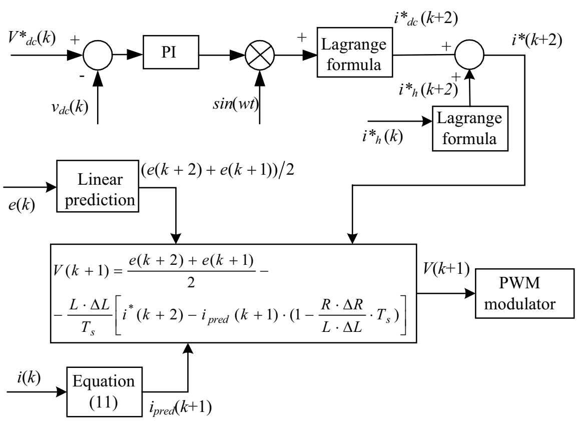


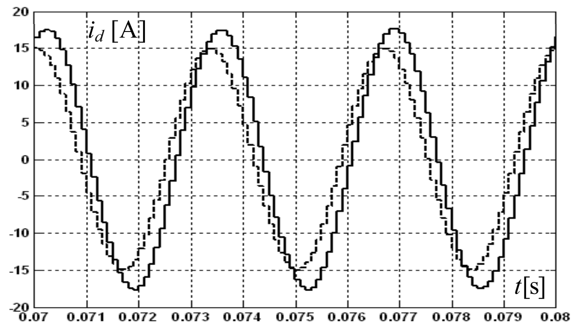
Fig. 4 Block diagram of the proposed control strategy (single phase representation)

IV. Simulation Results

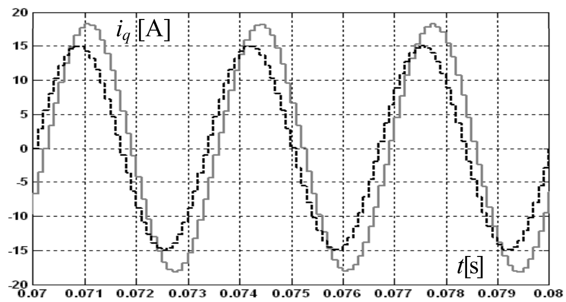
The theoretical analysis is verified on the simulation model developed in MATLAB/SIMULINK environment. This model includes real switches (IGBT plus anti-parallel diode) and supply voltage distortion (3% of the 5th and the 7th harmonics) as well as 3% of measuring noise. The parameters of input inductors and output capacitor are (Fig. 1): inductance L=3.75μH, resistance R=0.3Ω and capacitance C=1000μF. Asymmetrical centre aligned pulse width modulation with switching frequency f_s=5kHz is applied. The asymmetrical PWM is achieved by updating the duty cycle twice per switching period (sampling frequency is f=10kHz). Using this approach the need for anti-aliasing filters is eliminated. Current sampling is applied at the beginning and at the end of PWM cycle. It must be assured that all calculations are done inside one sampling period.

At first some simulation results using a synchronous PI plus lead compensator are given in Fig. 5. In this reference frame the 5th and the 7th harmonics appear as signals at 300 Hz. So the bandwidth of the current control loop needs to be higher then 300Hz. Although to obtain acceptable results the achieved bandwidth of this current control loop is almost 1kHz, the control of the 7th harmonic current results in amplitude discrepancy of 13% while the phase delay is 24°, Fig. 5. A steady state current error always exists because the current reference varies in time and cannot be eliminated by PI controller.

The performance of the predictive current controller is tested at first for the case when the waveform of the reference current is known, excluding the impact of the reference prediction error on the controller behavior. A dynamic response can be seen in Fig.6 and Fig.7. Transient response is reduced inside one sampling period without overshoot.



a) d component-7th harmonic



b) q component-7th harmonic

Fig. 5 Comparison of the d and q current components (full line) and the current reference (broken line)

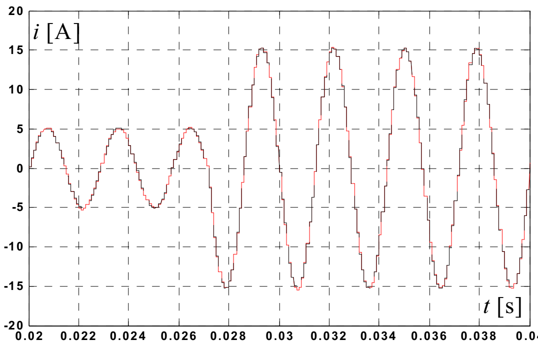


Fig. 6 Comparison of the phase current (red) and the current reference (black) assuming that the waveform of the reference current is known (at t = 0.027 s step change in amplitude from 5A to 15A)

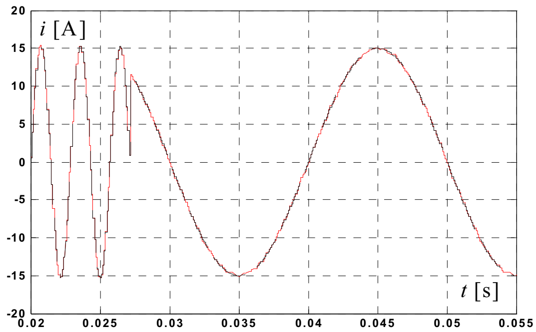


Fig. 7 Comparison of the phase current (red) and the current reference (black) assuming that the waveform of the reference current is known (at t = 0.027 s step change in frequency from 350Hz to 50Hz)

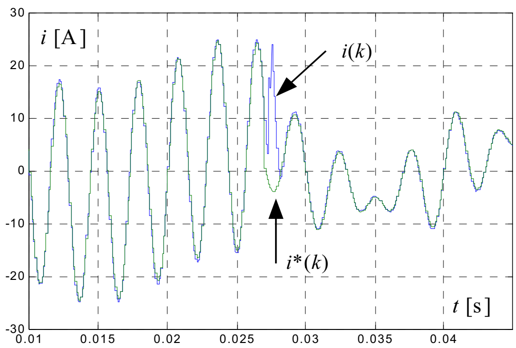


Fig. 8 Comparison of the phase current (blue) and the current reference (green) using Lagrange formula for the current reference prediction (at t = 0.027 s step change in amplitude and frequency)

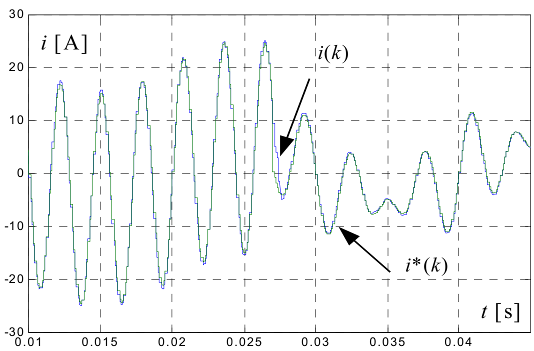


Fig. 9 Comparison of the phase current (blue) and the current reference (green) using proposed modified reference prediction (at t = 0.027 s step change in amplitude and frequency)

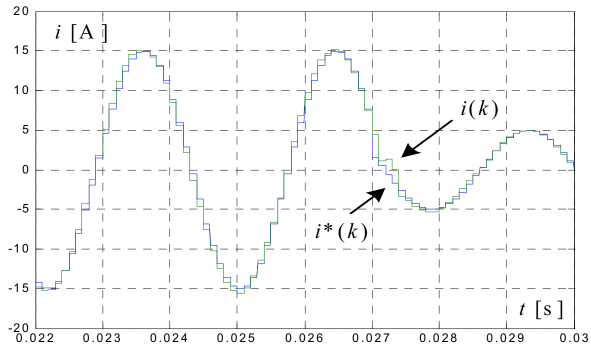


Fig. 10 Comparison of the phase current (green) and the current reference (blue) using proposed modified reference prediction

Including the reference prediction, the response of the analyzed system has instead a big oscillation in the transient state, Fig. 8.

The response of the system including the modification of reference prediction is presented in Fig. 9 and Fig. 10. Transient response is reduced inside four sampling periods with significantly reduced overshoot.

V. Stability Analysis

The performance of the analysed predictive current controller, which is a model-based controller, depends on the accuracy of the model parameters. The system stability according to inaccuracy of the supply impedance parameters is here examined.

The following discrete equations describe the presented predictive current controller:

$$V(k+1)=\frac{E(k+2)+E(k+1)}{2}-\frac{L\cdot\Delta L}{T_s}\left[i^*(k+2)-i_{pred}(k+1)\cdot(1-\frac{R\cdot\Delta R}{L\cdot\Delta L}\cdot T_s)\right] \quad (19)$$

$$V(k-1)=E(k-1)-\frac{L}{T_s}\left[i(k)-i(k-1)\cdot(1-\frac{R}{L}\cdot T_s)\right] \quad (20)$$

where the modeled input impedance is denoted by $R\Delta R$ and $L\Delta L$, where R and L present the actual values of the input impedance and ΔR and ΔL represent the inaccuracy of the input resistance and the input inductance respectivly. The integral of the supply voltage in the sampling period between the time instants k+1 and k+2 is modeled by the average value of voltage assuming it changes linearly. The prediction of the supply voltage at the time instants k+1 and k+2 is made using linear-type prediction (14) and (15) while for the prediction of the reference current the Lagrange interpolation formula (17) is used. Transforming the equations (14), (15), (17), (19) and (20) in the z-domain, the closed loop transfer functions can be obtained:

$$\frac{i(k)\cdot}{i^*(k)}=$$

$$=\frac{(3.5\cdot L\cdot\Delta L+2.5\cdot R\cdot\Delta R\cdot T_s)z^2}{z^4\cdot L+z^3\cdot(-L+R\cdot T_s)+z^2\cdot(L\cdot\Delta L\cdot 0.5-0.5\cdot R\cdot\Delta R\cdot T_s)} \quad (21)$$

$$+\frac{z(-5\cdot L\cdot\Delta L-3\cdot R\cdot\Delta R\cdot T_s)+2\cdot L\cdot\Delta L+R\cdot\Delta R\cdot T_s}{z^4\cdot L+z^3\cdot(-L+R\cdot T_s)+z^2\cdot(L\cdot\Delta L\cdot 0.5-0.5\cdot R\cdot\Delta R\cdot T_s)}$$

$$\frac{i(k)\cdot}{E(k)}= \tag{22}$$

$$=\frac{T_s\cdot z^3+z^2(-2.5\cdot T_s)+1.5\cdot T_s\cdot z}{z^4\cdot L+z^3\cdot(-L+R\cdot T_s)+z^2\cdot(L\cdot\Delta L\cdot 0.5-0.5\cdot R\cdot\Delta R\cdot T_s)}$$

where the supply voltage is considered as a disturbance.

The closed-loop characteristic equation is given:

$$z^4\cdot L+z^3\cdot(-L+R\cdot T_s)+z^2\cdot(L\cdot\Delta L\cdot 0.5-0.5\cdot R\cdot\Delta R\cdot T_s)=0 \tag{23}$$

To get root locus of the system as the parameters *ΔL* and *ΔR* vary, the characteristic equation is then rewritten into the desired form:

$$1+k\cdot G_o(z)=0 \tag{24}$$

where $G_o(z)$ denotes the open loop transfer function and k is the gain of the system. Rewritten equation (24) to get root locus as the parameter *ΔL* varies, is giving:

$$1+\Delta L\frac{L\cdot 0.5\cdot z^2}{z^4\cdot L+z^3\cdot(-L+R\cdot T_s)-z^2\cdot 0.5\cdot R\cdot\Delta R\cdot T_s}=0 \tag{25}$$

Similar procedure can be done to get root locus as the parameter *ΔR* varies. Fig. 11 presents placement of roots varying the *ΔL* parameter (25). The system is stabile, as it is well known, if all roots are inside the unit circle. As it can be seen from Fig. 11 and as could be expected, an overestimation of the input inductance is more critical situation. This stability analysis proves a good model robustness for parameters inaccuracy. The system is stable up to 100% error in the modeled inductance L. It is worth to mention that the system also goes into the instability region for 100% error in the modeled inductance L when the PI controller plus lead compensator is used. The accurate knowledge of the input resistance is less critical. The system is tested also for a step change in the supply voltage. The step variations in the supply voltage don't affect noticeably the current tracking (small value of the gain in the transfer function (22)). For the same conditions of the supply voltage step variations, the analyzed system with PI controller plus lead compensator is tested. A supply voltage step decrease of 30% introduces a transient response of about 20ms for the 7th harmonic of the active filter current showing therefore a not optimized behavior compared with the predictive approach.

VI. Conclusion

A detailed analysis of high performance predictive current control in active filter applications is given. A two-ahead prediction of reference current is obtained by Lagrange extrapolation formula using the fact that the sampling period is constant. Active filter current includes step variations even in steady state so the prediction of reference current in the conditions of amplitude and/or frequency change is of a great importance. Second order Lagrange polynomial-type prediction gives satisfactory steady state accuracy but fails in transient conditions. To overcome this drawback a modified prediction of reference generation is proposed. Using this modified prediction of reference current, the settling of transient response is achieved inside four sampling periods and overshoot is significantly reduced. Theoretical results are verified by simulations.

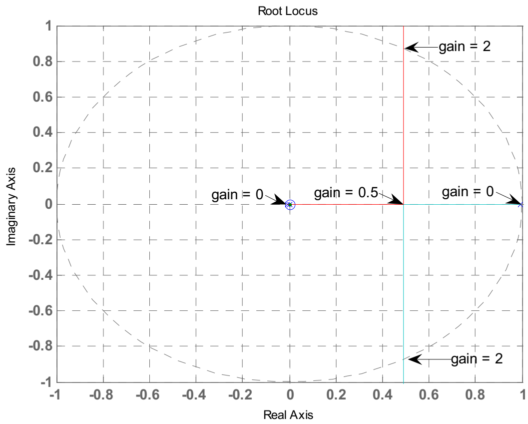


Fig. 11 Root locus of the system as the parameter *ΔL* (gain) varies

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