Simple Transmission Line Representation of Tesla Coil and Tesla's Wave Propagation Concept

Zoran Blažević[1], Dragan Poljak[2], Mario Cvetković[3]

Abstract – **In this paper, we shall propose a simple transmission line model for a Tesla coil that includes distributed voltage source along secondary of Tesla transformer, which is able to explain some of the effects that cannot be fully simulated by the model with the lumped source. Also, a transmission line representation of "propagation of current-through-Earth wave" will be presented upon.**

Keywords – **Transmission line modelling, Tesla's resonator, Magnifying effect, Tesla's current wave.**

I. Introduction

Ever since Dr. Nikola Tesla started to unfold the principles of electricity through the results of his legendary experiments and propositions, there is a level talk about it that actually never ceased, even to these days. These discussions, even a number of them have been just in order to neglect Tesla's theories and to take over the credits that in reality belong to Tesla, eventually have contributed to a widespread interest for studying his achievements and consequently to a fast development of many branches of science. One of the most representative examples for this is the development of the radio. It took several decades for Tesla to prove an extremely obvious fact that this invention belongs to him, with respect to the recognized great contribution of Dr. Hertz. However, maybe the main problem of Tesla was that he disputed Hertz's theory and considered the radio waves produced by a Hertzian apparatus as longitudinal shock waves in space rather than transversal [1 *et al*]. This and other related claims about the nature of radio waves put him against the majority of scientific community, which caused that Tesla's work about radio has never been fully recognised as it should be. Nevertheless, his intriguing ideas and results live even today in every scientist or engineer who touched his work.

In order to build a more efficient radio transmitter than Hertzian apparatus he designed his magnifying transmitter (TMT) that he defined in [2,3], truly more than a remarkable device even for our time. With that apparatus, according to his own words, he could efficiently emit Hertzian radiations or "currents through the earth" depending on the design [4,5], and is able to transmit efficiently both power and communication signals at great distances. However, he considered transmissions by means of Hertzian waves as a much less efficient method than his, which he compared with cutting the butter with blunt and sharp edge of a knife.

[1,2,3]University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, Split, Croatia
[1]zblaz@fesb.hr, [2]dpoljak@fesb.hr, [3]mcvetkov@fesb.hr

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Many researchers of today in explanation of the Tesla's concept assign the term "Hertzian wave" to the term "space wave" and "current wave" to "surface wave" justifying this with the difference in terminology of $19^{th}/20^{th}$ century and nowadays. However Tesla's surface waves, if we could be allowed to call them that way, have some different properties from, for instance, the Zenneck surface waves. Namely, they travel with a variable velocity in range from infinity at the electric poles down to the light speed at the electric equator as a consequence of the charges pushed by the resonator to tunnel through Earth along her diameter with speed equal or close to the speed of light in vacuum. And its effectiveness does not depend on polarization [1]. Regardless of the fact that science considers this theory as fallacious, which is well explained in [6], it is also true that his conclusions are based on the results of extensive measurements and considerations. Moreover, Tesla was exceptionally successful in every significant project that he was in position to finish, thus shadow of doubt lingers on. For that reason at least, it sure is one of the saddest things in the history of science that Tesla never completely finished his Long Island TMT station and put it in full operation. Then, he would have had a fair chance to prove that "he was right and that profession was completely misled" [4]. If he had managed to achieve this goal, that would surely have a vast influence to science and technology. Nevertheless, research of the Tesla's work in radio continues and there are many tries to explain his results from Colorado Springs and Long Island. Some of the hypotheses are even in grey areas of science and require a great deal of study and experiments. Nonetheless, the most acceptable theory that is in accordance to our present state of knowledge is one that relays on electromagnetic coupling to the Schumann cavity [7]. This standpoint is strengthened by the fact that the results of measurements of cavity parameters such as the resonant frequencies, coherence time, etc. made by Tesla are close to measurements conducted latter [7]. However, two facts must be stressed out. First that there are claims that there is actually a difference between the so called Tesla resonance and Schumann resonance and that Tesla was aware of this fact [8], and second that the resonances of the cavity have nowhere a sufficient Q-factor to support an efficient transmission of energy [6]. However, as stated in [9], the low Q factor does not necessarily limit the practicability of wireless power and, furthermore, there seems to be some measurements that dispute these results and report the Q-factors in excess of 1000. These are only a fair number of reasons that confirm the necessity of further studies on that matter. Therefore, for a researcher it is necessary to have an insight into the principle of the operation of the Tesla's transmitters. A remarkable work about this was made by K. Corum and J. Corum and some of their papers are listed in references. Of course, the model of a TMT secondary as a transmission line (TL) is the model that best suits the operation mode of the transmitter. According to the model of a slow-wave helical resonator TL [10] the magnifying effect is achieved by the standing waves in the secondary loosely coupled to the primary of the transmitter. In the same work the TL mode T_0 is analysed and standing wave formation explained. The TL compatible explanations can be found in many Tesla's papers and it is obvious that the TL principles were very well applied by him.

The ordinary TL models that work with a lumped source on one end cannot explain some of the effects that Tesla has been discussing, i.e. the standing wave pattern presented in [1]. However, a TL model that, instead of a lumped voltage source at the base of the secondary, involves distributed voltage along the line better suits the description of a resonating secondary and we shall try to present this in the paper. We will derive by the model the most important parameters of TL resonator and provide, from that standpoint, an explanation for some of the effects that Tesla has reported. Main TL aspects of his propagation theory will be commented upon.

II. TL Model of Tesla coil

The following TL model basically relays on the assumption that the alternate current induced in the secondary wire by the oscillations in the primary is constant along the wire which results in linear increments of potential along the wire. This model is presented in Fig. 1, and it can be easily upgraded by introducing radiation components as an imaginary inductivity and capacity. In relation to a Tesla coil, the impedance Z_1 is the impedance of Earth or the area of the ground comprising the ground plate in the zone of influence of the transmitter represented as an ideal or no ideal condenser, whereas Z_2 represents a capacitive impedance of the top spheroid. Besides, the same relations for the characteristic impedance and propagation constant apply as in [10]. This all means that no radiation into the surrounding space is assumed, T_0 mode is propagating through the helix (higher modes are neglected even they are unavoidable to some extent), voltage is uniformly distributed along the line and there is no escape of electromagnetic energy from the structure (no sparks). Although generally different load impedances could be settled on the ends of the line, for obvious reasons we shall concentrate our analysis mainly to ideal capacitive loads.

The derivation of the TL model starts with the circuit analysis of a small peace of the line of length Δx (Fig. 1). Taking that:

$$u_g(t) = U_g e^{j\omega t}, \Delta u_g = U_g \frac{\Delta x}{l} e^{j\omega t} \tag{1}$$

$$\Delta u = \frac{\partial u}{\partial x}\Delta x,\ \Delta i = \frac{\partial i}{\partial x}\Delta x \tag{2}$$

$$I_0 = (G + j\omega C)\frac{U_g}{l} = \frac{\gamma^2 U_g}{l(R + j\omega L)} \tag{3}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)} \tag{4}$$

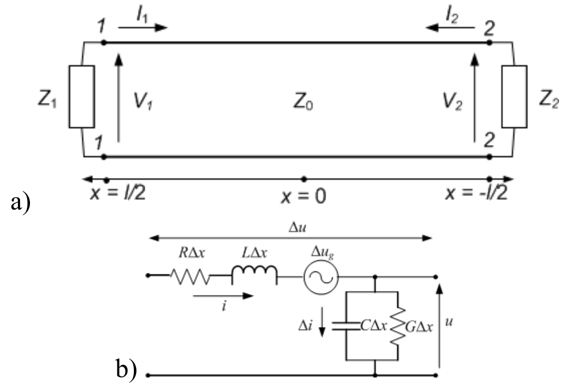

Fig. 1. a) TL model with distributed voltage source; b) element of an infinitesimal length Δx of the line

where u_g is the voltage along the TL, due to the alternate current I_0 induced in the secondary wire of length l, γ is the complex propagation constant, $\omega = 2\pi f$ angular frequency and the rest of parameters are as indicated in Fig. 1.b, it is easy to derive following equations for the voltage u and current i on the line:

$$\frac{\partial^2 u}{\partial x^2} + \gamma^2 u = 0 \tag{5}$$

$$\frac{\partial^2 i}{\partial x^2} + \gamma^2 i = I_0, \tag{6}$$

which, by introducing the gauge conditions for currents I_1, I_2 and voltages V_1, V_2 on ends of the line:

$$V_1 = u\left(\frac{l}{2}\right) = -i\left(\frac{l}{2}\right)Z_1,\ V_2 = u\left(-\frac{l}{2}\right) = i\left(-\frac{l}{2}\right)Z_2 \tag{7}$$

and defining the reflection coefficients of load impedances for a TL of characteristic impedance Z_0 as:

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} = |\Gamma_1| e^{j\varphi_1},\ \Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} = |\Gamma_2| e^{j\varphi_2}, \tag{8}$$

lead to solutions for current and potential on the line:

$$u(x,t) = \frac{-I_0 Z_0}{2}\frac{Ae^{\gamma x} - Be^{-\gamma x}}{1 - \Gamma_1\Gamma_2 e^{-2\gamma l}} e^{-\gamma\frac{l}{2}} e^{j\omega t} \tag{9}$$

$$i(x,t) = I_0\left(1 - \frac{1}{2}\frac{Ae^{\gamma x} + Be^{-\gamma x}}{1 - \Gamma_1\Gamma_2 e^{-2\gamma l}}\right) e^{-\gamma\frac{l}{2}} e^{j\omega t}, \tag{10}$$

$$A = (1 + \Gamma_1) - \Gamma_1(1 + \Gamma_2)e^{-\gamma l}, \tag{11}$$

$$B = (1 + \Gamma_2) - \Gamma_2(1 + \Gamma_1)e^{-\gamma l}. \tag{12}$$

Clearly, the necessary condition for the magnifying effect is accomplished when the parameter:

$$\Upsilon = 1 - \Gamma_1\Gamma_2 e^{-2\gamma l} \tag{13}$$

reaches the minimum value. (It could easily be shown that the same condition applies for TL with a lumped power source instead of the distributed one.) Theoretically in a lossless TL resonator, voltage as well as current can reach an infinite value. In that case assuming capacity loads on TL ends 1 and 2, the exact position x_0 in wavelengths λ of the zero voltage can be calculated as:

$$x_0 = \frac{\varphi_2 - \varphi_1}{8\pi}\lambda, \tag{14}$$

where angles φ_1 and φ_2 in radians can be red out from Smith chart for a given characteristic impedance of the resonator. It is easily shown by (9) that it is the position on the line that divides the resonator on two parts, each of them represented in Fig. 2 as two LC circuits resonating at the same frequency.

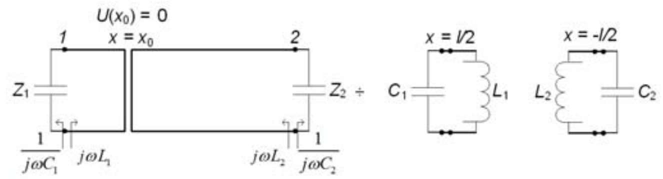

Fig. 2. Lossless TL resonator as two resonators seen from opposite ends that resonate at same frequency

This case is illustrated by an example from [1] in Fig. 3, where similar diagrams obtained by this TL model are given.

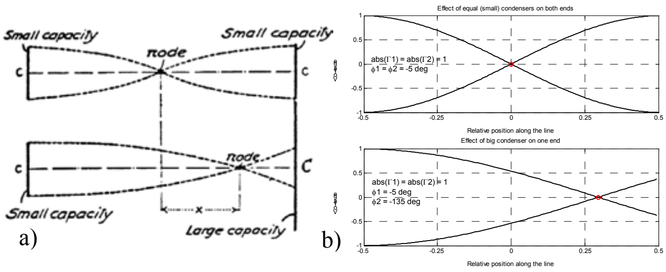

Fig. 4. a) Diagram elucidating effect of large capacity on one end [1]; b) A similar diagram obtained by model of TL in resonance

In order to derive the Tesla Equation [11], let us propose somewhat different form of the magnifying ratio M definition for this TL model. Here the top voltage V_{TOP} for an arbitrary load is going to be related with the top voltage in the case of a perfectly matched top load. By using (9)-(12) and certain mathematical manipulations the following relation applies:

$$\frac{V_{TOP}}{V_{TOP}(\Gamma_2 = 0)} = \frac{u\left(x = -\frac{l}{2}; \Gamma_1, \Gamma_2\right)}{u\left(x = -\frac{l}{2}; \Gamma_1, \Gamma_2 = 0\right)} = \frac{1+\Gamma_2}{1-\Gamma_1\Gamma_2 e^{-2\gamma l}}. \tag{15}$$

Clearly, for the short circuit on end 1 of the line (Γ_1 = -1), (15) takes the form of the Tesla equation. If furthermore a pure capacitance is assumed on end 2 ($|\Gamma_2|$ = 1), the square of this ratio could be interpreted as ratio of the power that a resonator is capable to preserve within the system and the power that can theoretically be transferred by the same resonator to the matched load and spent as heat, radiation or like. Basically, this relation is completely analogous with the one from [11], which is easy to prove if in the lumped source TL model one considers the influence of the source impedance and relates the input voltage to the voltage measured on the matched top load. For the case of end 2 of a low-loss quarter-wave line open (Γ_2 = 1), using (15) we obtain M as:

$$M = \frac{e^{\gamma l}}{\cosh \gamma l} = \frac{e^{\alpha l}}{\sinh(\alpha l)} \approx \frac{1}{\alpha l}. \tag{16}$$

Consider an example from Tesla's observations in Colorado Springs [12, pp. 351-356]. In a transmission of energy experiment, he put his resonating coil grounded on one side and open at the top at a distance from the transmitter. (In fact, it was deliberately not tuned perfectly.) Due to the magnifying effect a light bulb connected to the short secondary coupled to the primary was turned on. If, on contrary, the primary was disconnected from the ground, we would obtain a case of a quarter-wave TL open on both sides in which, for the sake of argument some voltage happens to build up but, due to the absence of the magnifying effect, incomparably smaller than in previous case. However, according to (9)-(12), if it were a half-wave low-loss coil this should be quite opposite. The latter concept may be used for explaining the antenna circuits on p. 198 in [12]. Figs. 2 and 3 are easily modelled as TL in Fig. 1 here, whereas the antenna in Fig. 1 needs a slightly different approach. For the s-c circuit in resonance we obtain an open circuit in this point, therefore it can be modelled as a TL with an ideal lumped voltage source in the node $x = x_0$. This last scheme associates very much to the Hertzian oscillator with difference that the standing wave ratio is probably significantly greater in the Tesla's scheme. Note that the magnifying effect cannot be produced in the case of short-circuit on both ends of the receiving coil ($A = B = 0$), despite the fact that condition $\Upsilon = 0$ can be fulfilled.

Finally, let us consider the effect of electrical shortening of the line. For that matter we assume the short circuit on end 1 of TL. Obviously, for a TL electrically shorter than $\lambda/4$, in order to maintain the magnifying effect the top capacity has to be increased accordingly. The diagrams of voltage and current on a lossless line normalised to the maximum values versus different TL lengths are presented in Fig. 4. Note that the top voltage, discrepancy between maximum and minimum current, and current slope decrease as TL becomes shorter.

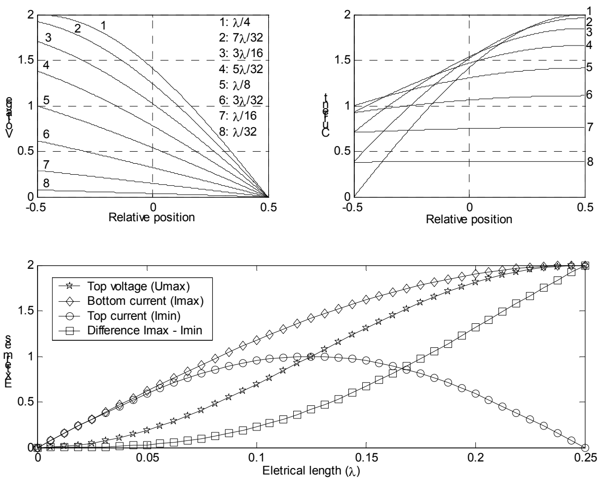

Fig. 4. Voltage, current and the extreme values for resonant lines of different lengths equal or less than quarter of wavelength

III. A TL Representation of Tesla's Current-Through-Earth Propagation Theory

The electromagnetic wave propagation concept of Tesla is presented in Fig. 5. It relays on the assumption of the charge redistribution across the globe, in which the earth current driven by the TMT voltage passes through the Earth along the diameter with a velocity equal or close to the speed of light c. The instant velocity of a Tesla surface wave is then determined from the simple geometry, assuming Earth as an ideal smooth sphere of diameter D as:

$$v = \frac{c}{\sin\phi} = c\csc\phi\,, \tag{17}$$

which means that the mean surface wave velocity is $c\cdot\pi/2$. Then, it is easy to show that "the waves on the terrestrial surface sweep in equal intervals over equal areas" [14]. Because the wave with the infinite velocity does not cross any distance, the infinity does not pose a theoretical problem.

Now if Earth could be seen in a form of TL as is presented in Fig. 5 (right side), the potential V and current I distribution and impedance Z would follow from the basic TL theory as:

$$V(x) = U_{TMT}\frac{\operatorname{ch}\gamma_e(d-x)}{\operatorname{ch}\gamma_e d} \tag{18}$$

$$I(x) = \frac{U_{TMT}}{Z_E}\frac{\operatorname{sh}\gamma_e(d-x)}{\operatorname{ch}\gamma_e d} \tag{19}$$

$$Z(x) = Z_E\operatorname{cth}\gamma_e(d-x)\,, \tag{20}$$

where d is the diameter D expressed in wavelength λ, $\gamma_e = \alpha_e + j\beta_e$ is the complex propagation constant of Earth and Z_E would be the characteristic impedance that, according to the values given in [13], equals to 58,3 Ω. The standing wave patterns of current and potential (Tesla condition $f = c/(4D)$ fulfilled) that would be formed on the surface of the lossless Earth are similar to the case 1 ($l = \lambda/4$) in Fig. 4. See that from (20) for frequencies below approximately 6 Hz, the Earth would behave as a capacity that increases with lowering the frequency (true short circuit for DC), as stated similarly in [5].

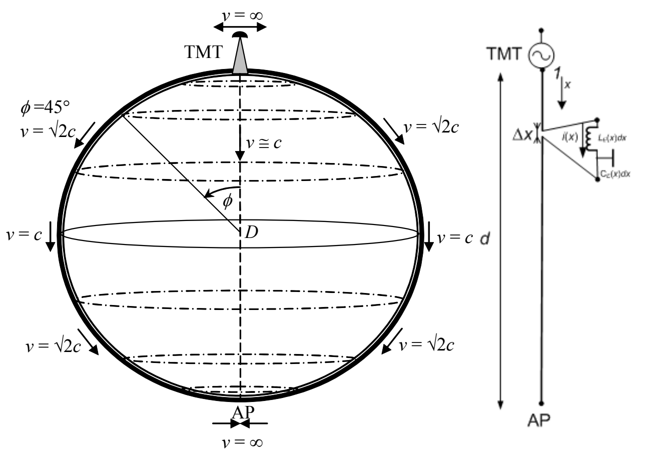

Fig. 5. Propagation of Tesla wave (left) and TL representation (right)

Furthermore, if there is no load present, only reactive energy exists in the system. Accordingly, between the TMT (source) and a loaded receiver (sink) the Poynting flux would be formed along an orthodromic line as explained in [14]. Accepting that the loss increases with frequency, it is preferably to use low resonant frequencies of the transmitter. Now reader should refer to Fig. 5 in [1], in which possible arrangements of radio receivers are explained.

IV. Conclusion

In this paper we presented a TL model of TMT and of propagation of the Tesla's ground current wave. We tried to show that the TL model with distributed voltage source better suits to a Tesla coil, one of the reasons being that it involves loads on both sides of the line equally. Unfortunately, Tesla did not explain in details how his "World System" would function; he only provided us with basic conditions, briefly explained the propagation mechanism and told us that he had done it; he sent a wave through Earth to the antipode from which it returned with almost undiminished strength. He surely left us something to dwell about seriously. Namely, in light of the usage of TMT principles in many areas of science and technology and of the fact that Tesla has really never been proven wrong in his experimental achievements, it seems reasonable to ask what if a TMT can do all that things that he claimed. How should our millennium look like if he really had accomplished this break on through to the other side?

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