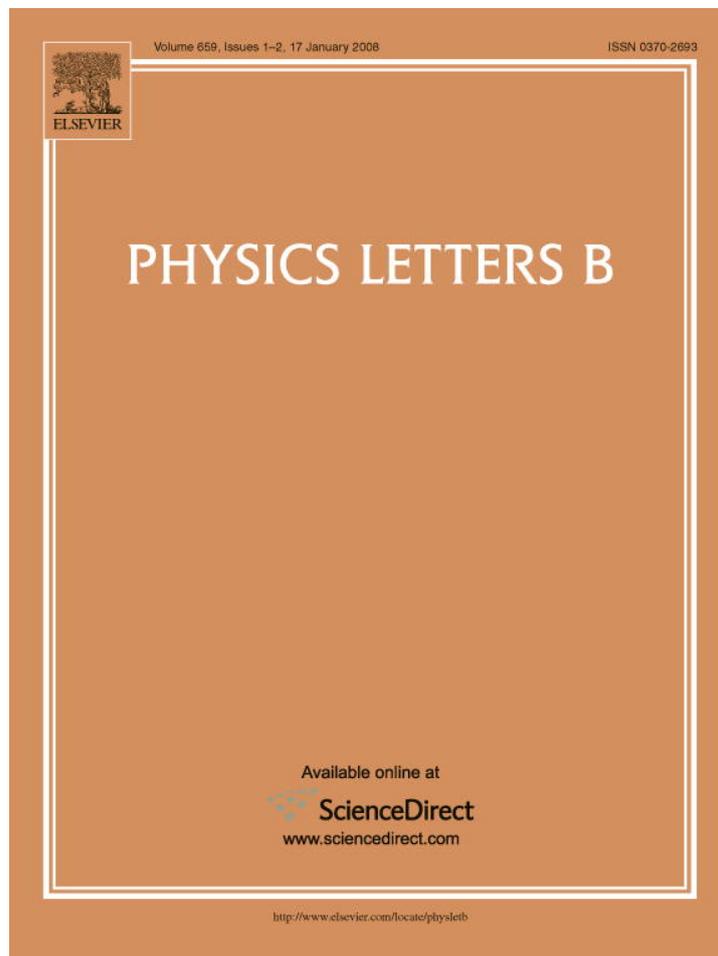


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## Model-independent resonance parameter extraction from the trace of $K$ and $T$ matrices

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### Abstract

A model-independent method for the determination of resonance parameters as  $K$ -matrix pole parameters from a  $T$  matrix is presented. The method is based on eliminating the dependence on the choice of channel basis by analyzing the trace of the  $K$  and  $T$  matrices in the coupled-channel formalism, rather than individual matrix elements of the multichannel scattering matrix.

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### 1. Introduction

A general problem in theoretical baryon physics is to make a connection between resonances that are predicted by various quark models and experiment. A reasonable way to proceed is by identifying the poles of analytic functions that are able to describe simultaneously all experimental data in a multiplicity of existing channels with theoretically predicted resonant states. Therefore, properly and uniquely extracting resonance parameters from experiment is a task of primary importance. We emphasize the problem of uniqueness. The work described here is motivated by the need to extract resonance parameters from multichannel partial-wave analyses (PWAs) in a way which has the least model dependence. Many PWAs produce similar partial waves using similar experimental data, while the extracted resonance parameters are often quite different. This fact can easily be seen in the *Review of Particle Physics* [1] by the Particle Data Group (PDG). Each resonance in the *Review*

has been parametrized in two ways: first, with Breit–Wigner parameters usually defined when the Breit–Wigner form is used to parameterize the partial wave  $T$  matrix, i.e., the resonance mass  $M^R$ , decay width  $\Gamma^R$ , and branching fractions  $x_a$  (the ratio between the partial width into channel  $a$  and the total width), and alternatively with  $T$ -matrix complex pole positions ( $\text{Re } W_p$  and  $-2 \text{Im } W_p$ ) and related complex residues (moduli  $|r|$ , and phases  $\theta$ ). Unlike the  $T$ -matrix pole positions, Breit–Wigner parameters obtained in various partial-wave analyses vary quite substantially because details of those analyses are different: the number and character of the included channels, different parameterization schemes, analyticity constraints for scattering amplitudes, the choice of background models, the method of unitarization (if at all) of the  $S$  matrix, etc. However, it is also the case that the methods for extracting resonance parameters are different: Argand-plot fits [2], Breit–Wigner fits with background [3], direct fits of analysis parameters [4,5], or model specific schemes which extract  $T$ -matrix poles [6–8]. It seems to us that the problem is that Breit–Wigner parameters are conventionally model dependent by nature.

In this Letter we present a method for extracting resonance parameters defined as  $K$ -matrix mass and width, exactly at the

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energies of the  $K$ -matrix poles. To avoid confusion with Breit–Wigner resonance parameters, our parameters will simply be called “ $K$ -matrix pole parameters”. The proposed method is applicable to any unitary analysis able to provide the full multichannel  $T$  matrix. To eliminate ambiguities caused by the multichannel character of the formalism, we propose to use the trace of corresponding  $K$  and  $T$  matrices.

Because the resonance parameters we extract are defined at the energy of the  $K$ -matrix poles, they are independent of specific parameterizations of the  $K$  matrix, and are therefore unique, and should be directly compared to those from quark models and lattice QCD. In order to connect the results of a model-independent  $K$ -matrix extraction with those of a model-dependent analysis, e.g. based on the  $T$  matrix, we shall keep the relations defining multichannel  $T$  and  $K$  matrices as general as possible. It turns out that the  $T$ -matrix trace simplifies the formalism without loss of generality, and shows resonant behavior more prominently than any  $T$ -matrix element does. To illustrate this, we shall take the  $T$  matrix from an earlier analysis [7] and recalculate the resonance parameters. The  $T$ -matrix trace does show resonant behavior at energies matching those of the  $K$ -matrix poles.

## 2. Multichannel scattering

The essence of any multichannel theory is the fact that the evolution of a system is no longer described by scalars, but by operators acting in an orthonormal wave-function space, and the transition probabilities for physical (measurable) processes are given by the matrix elements of their representation in the chosen basis. Once this basis is specified, the evolution of the system is described by solving equations which are matrices in the multichannel space, rather than scalar equations.

All equations given here are considered to be matrix relations, unless matrix indices are explicitly stated. The transition probability  $P_{a \rightarrow b}$  that a two-body system from initial channel  $|a; q\rangle$  ends up in the final two-body (or quasi-two-body) channel  $|b; q\rangle$  is given by the absolute square of the scattering  $S^q$ -matrix element  $P_{a \rightarrow b} = |\langle b; q | \hat{S}^q | a; q \rangle|^2$ , where  $q$  designates all quantum numbers conserved in the scattering reaction, and  $a$  and  $b$  are channels. In the case of  $\pi N$  scattering we have conserved spin, parity, and almost conserved isospin (charge symmetry is only slightly violated). Conservation of probability is ensured if the  $S$  matrix (for simplicity, we drop  $q$  henceforth) is unitary. Therefore, the  $S$  matrix can be written as  $S = e^{2i\delta}$ , where  $\delta$  is some matrix Hermitian in the channel indices.

Because Hermitian matrices have real eigenvalues and are diagonalized by unitary matrices, there exists a particular orthonormal wave-function basis, that of the scattering matrix eigenstates, in which the scattering operators are diagonal matrices. The  $\delta$  matrix in an arbitrary basis is related to a real, diagonal matrix  $\delta_D$  by a unitary transformation  $\delta = U^\dagger \delta_D U$ , where  $U$  is a unitary matrix. The  $S$  matrix in this basis is clearly diagonalized by the same transformation, so  $S = U^\dagger e^{2i\delta_D} U$ .

The  $K$  matrix [6,9] is defined as  $K = i(I - S)/(I + S)$ , where  $I$  is the unit matrix. The  $K$  matrix in the eigenstate basis can be written using the diagonal matrix  $\delta_D$  as  $K =$

$U^\dagger \tan \delta_D U$ . The  $K$  matrix is Hermitian because  $S$  is unitary, and symmetric because of time-reversal invariance, so  $K$  is, in fact, a real matrix, and this implies that  $U$  is a real orthogonal matrix which we designate as  $O$  in what follows.

We introduce the ortho-normal vector basis  $\{E^1, \dots, E^N\}$ , where

$$E^1 = \begin{pmatrix} 1 & 0 & \cdot & 0 \\ 0 & 0 & \cdot & \cdot \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdot & \cdot & 0 \end{pmatrix}, \quad \dots, \quad E^N = \begin{pmatrix} 0 & \cdot & \cdot & 0 \\ \vdots & \ddots & \vdots & \vdots \\ \cdot & \cdot & 0 & 0 \\ 0 & \cdot & 0 & 1 \end{pmatrix}. \quad (1)$$

Every diagonal  $N \times N$  matrix can be spanned in this basis, so we have

$$\tan \delta_D = \sum_{i=1}^N E^i \tan \delta^i, \quad (2)$$

where  $\delta^i$  is the  $i$ th diagonal element of  $\delta_D$ , also known as the *eigenphase shift*, and  $N$  is the number of channels. The coupling matrices  $\chi^i$ , defined by

$$\chi^i = O^T E^i O, \quad (3)$$

connect the scattering operators in any basis with their diagonal form in the eigenstate basis, and turn out to be ortho-normal projectors, satisfying

$$\sum_{i=1}^N \chi^i = I, \quad \chi^i \chi^j = \chi^j \delta_{ij}, \quad (4)$$

where  $\delta_{ij}$  is the Kronecker  $\delta$  symbol. The  $K$  and  $T$  matrices can be written as the sums

$$K = \sum_{j=1}^N \chi^j \tan \delta^j, \quad T = \sum_{j=1}^N \chi^j e^{i\delta^j} \sin \delta^j, \quad (5)$$

where the connection between them is given by the relation

$$K = T/(I + iT). \quad (6)$$

The trace of a matrix is, by definition, a sum of its diagonal elements. A trace has two particularly important properties: (i) the trace of a product of matrices is invariant with respect to cyclic permutations,  $\text{Tr}[ABC] = \text{Tr}[BCA]$ ; and (ii) the trace is a distributive function with respect to scalars  $\alpha$  and  $\beta$ ,  $\text{Tr}[\alpha A + \beta B] = \alpha \text{Tr}[A] + \beta \text{Tr}[B]$ . In what follows we will demonstrate that the trace of a scattering matrix allows a direct link to be made between a general basis where scattering operators are represented by complicated matrices, and the eigenstate basis where they maintain diagonal form. Note that the orthogonal transformation in definition (3) preserves the trace of a matrix, so

$$\text{Tr} \chi^i = 1. \quad (7)$$

## 3. Extraction procedure

Elements of  $\tan \delta_D$ , as well as the  $\chi^j$ , are functions of energy or a corresponding kinematical variable, and their description

requires modeling of the energy dependence of numerous functions. We see resonances in scattering reactions as real poles of the  $K$  matrix. The  $r$ th element of the diagonal matrix  $\tan \delta_D$  can be written as [9]

$$\tan \delta^r = \frac{\Gamma_r/2}{M_r - W} + \tan \delta_B^r, \quad (8)$$

where the first term on the RHS is written as a simple pole term, and it is singled out from other contributions (including other resonances), designated collectively as the background term at resonance  $\tan \delta_B^r$ . The  $K$ -matrix mass ( $M_r$ ) and total width ( $\Gamma_r$ ) are allowed to be functions of the center-of-mass total energy  $W$ . The reported  $K$ -matrix pole parameters  $M_r^R$  and  $\Gamma_r^R$  are given by the values of  $M_r(W)$  and  $\Gamma_r(W)$  evaluated at an energy equal to the corresponding resonance mass  $M_r^R$ :

$$M_r^R = M_r(M_r^R), \quad \Gamma_r^R = \Gamma_r(M_r^R), \quad (9)$$

where we have explicitly written  $M_r$  and  $\Gamma_r$  from Eq. (8) as functions of energy  $W$ .

The corresponding  $K$  and  $T$  matrices are given by the equations

$$K = \chi^r \frac{\Gamma_r'/2}{M_r - W} + \sum_{j \neq r} \chi^j \tan \delta^j, \quad (10)$$

$$T = \chi^r \frac{\Gamma_r'/2}{M_r - W - i\Gamma_r'/2} + \sum_{j \neq r} \chi^j e^{i\delta^j} \sin \delta^j, \quad (11)$$

where the second term in each equation is the coupled-channel background contribution, and  $\Gamma_r'/2$  represents  $\Gamma_r/2 + (M_r - W) \tan \delta_B^r$ . When  $W$  equals the mass of the resonance,  $\Gamma_r'$  is manifestly equal to  $\Gamma_r$ . If there is a pole in the  $K$  matrix at some energy  $M_r^R$ , then the matrix element  $\chi_{ab}^r$  at that energy gives the coupling strength of the resonance with mass  $M_r^R$  and total decay width  $\Gamma_r^R$  from channel  $a$  to channel  $b$ . The diagonal element of the matrix  $\chi^r$  is the branching fraction  $x_a^r$  of a given resonance to the channel  $a$

$$x_a^r = \chi_{aa}^r. \quad (12)$$

Although Eqs. (10) and (11) are in general a sum over several resonances  $r$ , here they are written for one resonance for simplicity.

The channel dependence of resonance parameters can be reduced significantly by using only diagonal elements of the  $T$  and  $K$  matrices. In practice, these matrices can be obtained either by unitary coupled-channel partial-wave analyses, or by using partial-wave  $T$  matrices obtained in diverse single-channel PWAs as input to a unitary coupled-channel formalism, and refitting them to obtain a unitary set of all coupled-channel  $T$ -matrix elements.

Channel-resonance mixing is completely removed from the sums

$$\text{Tr}(K) = \sum_{j=1}^N \tan \delta^j, \quad \text{Tr}(T) = \sum_{j=1}^N e^{i\delta^j} \sin \delta^j, \quad (13)$$

because the traces of the  $K$  and  $T$  matrices are the same as the traces of their similar diagonal partners  $\tan \delta_D$  and  $e^{i\delta_D} \sin \delta_D$ ,

respectively. The same is also evident from Eqs. (5) and (7). Consequently, Eqs. (10) and (11) are simplified by taking the traces

$$\text{Tr}(K) = \frac{\Gamma_r'/2}{M_r - W} + \sum_{j \neq r}^N \tan \delta^j, \quad (14)$$

$$\text{Tr}(T) = \frac{\Gamma_r'/2}{M_r - W - i\Gamma_r'/2} + \sum_{j \neq r}^N e^{i\delta^j} \sin \delta^j. \quad (15)$$

The last relation, i.e., the  $T$ -matrix trace, would be a good starting point for model-dependent extraction methods.

However, instead of putting considerable effort into modeling the background and energy- and channel-dependent resonance parameters, we use the following procedure to extract  $K$ -matrix pole parameters (i.e.,  $M_r$  and  $\Gamma_r$  at the energy of the  $K$ -matrix pole):

(i) The parameter extraction procedure starts when a full  $T$  matrix has been obtained from an energy-dependent partial-wave analysis of experimental data.

(ii) Contrary to the usual prescription, where Eq. (11) is used to obtain resonance parameters from the  $T$  matrix in a model-dependent way, we use Eq. (6) to obtain the full  $K$  matrix from the known  $T$  matrix.

(iii) Poles of  $\text{Tr} K$  are found in order to obtain the masses of a set of  $N_R$  resonances  $M_1^R, \dots, M_{N_R}^R$ , defined by Eq. (9) at the position of the pole.

(iv) Multiplying both sides of Eq. (14) by  $(M_k^R - W)$  and setting the energy  $W$  to the value of the  $k$ th resonance mass ( $M_k^R$ ), the corresponding resonance width at the pole energy is isolated:

$$\Gamma_k^R = 2 \lim_{W \rightarrow M_k^R} [(M_k^R - W) \text{Tr}(K)]. \quad (16)$$

All other contributions to the  $K$ -matrix trace, i.e., background, other resonances, and channel-couplings, are removed in this limiting process (this relation turns out to be similar to Eq. (16) in Ref. [10] for the case of the various  $\pi N$  isospin channels).

(v) The branching fraction of a resonance to a given channel can be obtained in similar manner, but this time using the diagonal  $K$ -matrix element,  $K_{aa}$  from Eqs. (10) and (12)

$$x_a^k = \frac{2}{\Gamma_k^R} \lim_{W \rightarrow M_k^R} [(M_k^R - W) K_{aa}], \quad (17)$$

where, as before, all undesired contributions vanish.

(vi) Steps (iv) and (v) are then repeated for all resonances found in (iii).

Up to now, all of our considerations have avoided the complications of the multi-channel aspect of the problem by introducing trace and by singling out contributions from individual resonances. At this point we should mention that adding resonances in an unitary way is another problem altogether. It is well known that in general there is no one-to-one correspondence between the location and number of the  $K$ -matrix and

$T$ -matrix poles; indeed, in some cases, these quantities may differ significantly. A very good example of such a behavior can be found in meson physics. Namely, the  $\pi\pi \rightarrow \pi\pi$  scattering data obtained by the GAMS Collaboration [11] are completely consistent with the  $T$  matrix having a pole at 980 MeV defining a  $f_0(980)$  resonance, while at the same time the lowest  $K$ -matrix poles that describe the  $\pi\pi$  scattering amplitude were found in the region 600–800 MeV and 1150–1300 MeV. In our nomenclature, the  $K$  matrix would indeed have poles with masses around 650 and 1200 MeV; however the corresponding  $T$  matrix would have two poles at the same mass of 980 MeV, one broad and one narrow.

#### 4. Results and discussions

To illustrate the usefulness of our method, resonance parameters from a unitary, multi-resonance, coupled-channel analysis [7] have been extracted. As the intent of our Letter is limited to proposing the extraction method itself, error analysis will not be presented.

The channels used in the analysis were  $\pi N$ ,  $\eta N$ , and an effective two-body channel designated as  $\pi^2 N$ . Extracted resonance parameters are given in Table 1. With minimal calculation, the proposed model gives resonance parameters very close to the values obtained in the original publication, where a complicated method of diagonalizing the matrix of the generalized Breit–Wigner function denominator has been used.

We have also compared the  $K$ -matrix poles to the trace of the  $T$  matrix. It can be seen in Fig. 1 that the resonance positions obtained by looking for the poles in  $\text{Tr} K$  (indicated by gray vertical lines) directly correspond to the positions of peaks in  $\text{Im}(\text{Tr} T)$ , and of zeros in  $\text{Re}(\text{Tr} T)$ . The peaks of the  $T$ -matrix elements corresponding to individual channels, however, show a certain deviation from that behavior. This suggests that fitting individual channels in order to obtain resonance parameters introduces an uncontrolled error, which is avoided if the trace of the  $T$  matrix is used.

The resonance parameters produced by the  $K$ -matrix extraction method presented here, are in accordance with values obtained by the original analysis as well as with the  $T$ -matrix trace. The procedure involves no fitting, diagonalizing, or modeling of the energy dependence of the resonance parameters and background. Furthermore, a model-independent procedure cannot be given with the  $T$ -matrix formalism, because background makes a substantial contribution to the  $T$  matrix, even at an energy equal to the resonance mass,  $M^R$ . The  $T$ -matrix background is removed at a complex energy equal to the  $T$ -matrix pole position. This might be the reason why extractions of  $T$ -matrix poles work much better than  $T$ -matrix extractions of Breit–Wigner parameters. By using the trace of the  $K$  matrix, background has been completely removed from consideration at the resonance energies.

With regard to the differences between the two approaches listed in Table 1, it is rather striking that all of them can be explained by arguments presented in the original analysis. Since an effective  $\pi^2 N$  channel was introduced in [7] to parametrize the first inelasticity in each partial wave, the parameters

Table 1

Resonance parameters extracted using the  $K$ -matrix procedure given in this Letter are listed in bold face. The original  $T$  matrix was taken from the first reference in [7] where the channels used were  $\pi N$ ,  $\eta N$ , and an effective two-body channel  $\pi^2 N$ . For comparison, resonance parameters from the original reference are shown below

$L2I2J \left( \begin{smallmatrix} x_{\pi N}/x_{\eta N}/x_{\pi^2 N} \\ M/\Gamma \end{smallmatrix} \right)$ PDG [1]	$M^R$ [MeV]	$\Gamma^R$ [MeV]	$x_{\pi N}$ [%]	$x_{\eta N}$ [%]	$x_{\pi^2 N}$ [%]
$S_{11} \left( \begin{smallmatrix} 35-55/30-55/1-10 \\ 1535 \pm_{15}^{20}/150 \pm_{50} \end{smallmatrix} \right)$	<b>1543</b> 1553	<b>165</b> 182	<b>39</b> 46	<b>54</b> 50	<b>7</b> 4
$S_{11} \left( \begin{smallmatrix} 55-90/3-10/10-20 \\ 1650 \pm_{10}^{30}/150 \pm_{5}^{40} \end{smallmatrix} \right)$	<b>1680</b> 1652	<b>233</b> 202	<b>64</b> 79	<b>16</b> 13	<b>20</b> 8
$S_{11} \left( \begin{smallmatrix} \text{UNKNOWN} \\ \approx 2090/\text{NE} \end{smallmatrix} \right)$	<b>2054</b> 1812	<b>1926</b> 405	<b>47</b> 32	<b>3</b> 22	<b>50</b> 46
$P_{11} \left( \begin{smallmatrix} 60-70/0/30-40 \\ 1440 \pm_{10}^{30}/350 \pm_{100} \end{smallmatrix} \right)$	<b>1482</b> 1439	<b>541</b> 437	<b>61</b> 62	<b>0</b> 0	<b>39</b> 38
$P_{11} \left( \begin{smallmatrix} 10-20/6/40-90 \\ 1710 \pm_{30}^{100}/100 \pm_{50}^{150} \end{smallmatrix} \right)$	<b>1738</b> 1740	<b>170</b> 140	<b>44</b> 28	<b>12</b> 12	<b>44</b> 60
$P_{11} \left( \begin{smallmatrix} \text{UNKNOWN} \\ \approx 2100/\text{NE} \end{smallmatrix} \right)$	<b>2123</b> 2157	<b>379</b> 355	<b>3</b> 16	<b>83</b> 83	<b>14</b> 1
$P_{13} \left( \begin{smallmatrix} 10-20/0/>70 \\ 1720 \pm_{70}^{30}/100 \pm_{50} \end{smallmatrix} \right)$	<b>1776</b> 1720	<b>409</b> 244	<b>20</b> 18	<b>0</b> 0	<b>80</b> 82
$D_{13} \left( \begin{smallmatrix} 50-60/0/40-50 \\ 1520 \pm_{5}^{10}/120 \pm_{10}^{15} \end{smallmatrix} \right)$	<b>1515</b> 1522	<b>121</b> 132	56 55	0 0	<b>44</b> 45
$D_{13} \left( \begin{smallmatrix} 50-60/0/40-50 \\ 1700 \pm_{50}^{100}/100 \pm_{50} \end{smallmatrix} \right)$	<b>1818</b> 1817	<b>126</b> 134	<b>15</b> 9	<b>15</b> 14	<b>70</b> 77
$D_{13} \left( \begin{smallmatrix} \text{UNKNOWN} \\ \approx 2080/\text{NE} \end{smallmatrix} \right)$	<b>2359</b> 2048	<b>1216</b> 529	<b>26</b> 17	<b>6</b> 8	<b>68</b> 75
$D_{15} \left( \begin{smallmatrix} 40-50/0/50-60 \\ 1675 \pm_{5}^{10}/150 \pm_{30}^{30} \end{smallmatrix} \right)$	<b>1674</b> 1679	<b>144</b> 152	<b>36</b> 35	<b>0</b> 0	<b>64</b> 65
$F_{15} \left( \begin{smallmatrix} 60-70/0/30-40 \\ 1680 \pm_{5}^{10}/130 \pm_{10} \end{smallmatrix} \right)$	<b>1682</b> 1680	<b>144</b> 142	<b>67</b> 67	<b>1</b> 0	<b>32</b> 33
$F_{17} \left( \begin{smallmatrix} \text{UNKNOWN} \\ \approx 1990/\text{NE} \end{smallmatrix} \right)$	<b>2139</b> 2262	<b>412</b> 2036	<b>7</b> 3	<b>3</b> 2	<b>90</b> 95
$G_{17} \left( \begin{smallmatrix} 10-20/\text{UNKNOWN} \\ 2190 \pm_{90}^{10}/450 \pm_{100} \end{smallmatrix} \right)$	<b>1806</b> 2125	<b>286</b> 381	<b>6</b> 18	<b>0</b> 0	<b>94</b> 82
$G_{17} \left( \begin{smallmatrix} -/-/- \\ -/- \end{smallmatrix} \right)$	<b>2397</b> 2125	<b>1217</b> 381	<b>16</b> 18	<b>0</b> 0	<b>84</b> 82

of low-lying resonances should be much better determined than those of heavier ones. A better quality of parameters is also expected for resonances that couple more strongly to the measured channels considered here. Therefore,  $N(1720)$   $P_{13}$  and the resonance(s) in  $G_{17}$  have unrealistic parameters since they are completely driven by the effective channel, as can be clearly seen from Fig. 1. The unusually large decay widths obtained for three heavier resonances (the third  $S_{11}$  and  $D_{13}$  resonances, and the second  $G_{17}$  resonance) reflect this issue. This effect is due to a problem with the original partial-wave analysis, where possibly spurious structures appear in the higher energy (above 1900 MeV) region, due to questionable partial-wave data and

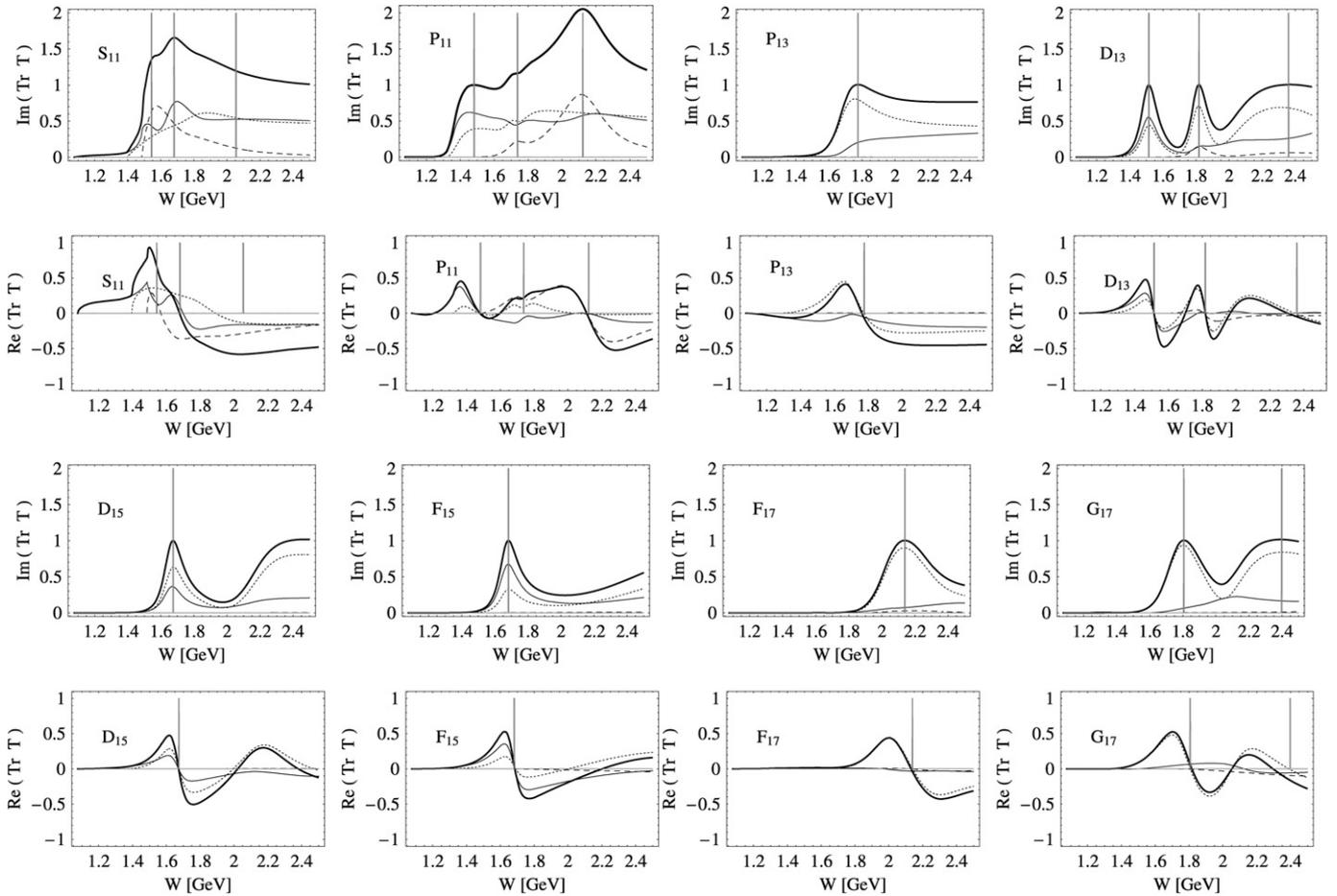


Fig. 1. The trace of the  $T$  matrix and its contributions for partial waves from  $S_{11}$  to  $G_{17}$ . The thick black line represents the imaginary part (upper graph) and real part (lower graph) of the trace of the  $T$  matrix. The thin line is the  $\pi N$  elastic contribution, the dashed line shows the contribution from  $\eta N$ , while the dotted line gives the effective-channel contribution (unitarity channel). Gray vertical lines are plotted at the  $K$ -matrix pole positions.

the use of an insufficient number of channels. These problems should be removed by the measurement and subsequent explicit inclusion of additional channels in the partial-wave analysis.

The parameters of the two lowest resonances in the  $S_{11}$  and  $D_{13}$  partial waves, as well as those of the  $D_{15}$ , second  $P_{11}$ , and  $F_{15}$  resonances, are in rough accordance with quark-model expectations for their masses and partial widths [12], with the exception of the mass of the second  $D_{13}$ , which is predicted to be roughly degenerate with the second  $S_{11}$  and the  $D_{15}$  resonance. This disagreement could be explained by the large coupling of this state to the effective channel. The large width and the somewhat larger mass of the first  $P_{11}$  (Roper) resonance extracted using the  $K$ -matrix procedure bring these parameters closer to those of the class of quark-model calculations based on one-gluon exchange potentials and pair creation for strong decays.

## 5. Conclusions

We have presented a model-independent method for extracting resonance parameters defined as  $K$ -matrix mass and width, exactly at the energies of the  $K$ -matrix poles, from a  $T$  matrix.

To eliminate problems originating in the multi-channel channel aspect of the formalism, our method uses the trace of the corresponding  $K$  and  $T$  matrices.

It is shown that real poles of the  $K$  matrix are related to the resonant behavior of the trace of the  $T$  matrix. Our resonance parameter extraction procedure is simple and straightforward once the full  $T$  matrix is known. Unrealistic extracted parameters for some higher mass resonances point to the need to include additional channels in partial-wave analyses.

At the energies of the  $K$ -matrix poles, the influence of background and channel mixing is eliminated, so only parameter values obtained at this particular energy should be compared directly to the predictions of quark model and lattice QCD calculations.

As long as a  $T$  matrix is model dependent, the *results* for pole parameters are model dependent as well. However, the *procedure* itself, which we present for extracting the  $K$ -matrix pole parameters from a  $T$  matrix, is independent of any model indeed.

This model-independent procedure cannot be extended to the  $T$ -matrix formalism because background makes a substantial contribution to the  $T$  matrix, even at the resonance energies  $M^R$ . This might be the reason why methods that ex-

tract  $T$ -matrix poles work much better than those which extract Breit–Wigner parameters from the  $T$  matrix. By using the trace of the  $K$  matrix, the background has been completely removed from consideration at resonance energies.

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