Fault Analysis of Structures with Low Sensitivity in Realisation of Active Filters

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Abstract - The components values in electrical circuits deviate from nominal ones due to different reasons: component tolerances, aging etc. If deviations are inside tolerance bounds, sensitivity analysis could be performed in order to asses network's characteristic changes. If component values deteriorate outside tolerance bounds, than we speak about failures of the network. In the latter case it is necessary to carry out the fault isolation procedure and one need to perform the testability analysis first in order to find optimal testing conditions. This paper tries to find correlation between sensitivity (Shoeffler's) of a network characteristic and testability on single component failures. The emphasis is on the analog filters. In one example the testability and sensitivity analyses were done on the eight order Chebyshev narrow band pass filter which was realised by different structures, i.e. CAS, FLF, CBQ and LF. The main conclusion is that those structures having lower sensitivities, have lower testability too, in the corresponding frequency bands. Therefore, Shoeffler's sensitivity could be proposed as the measure of testability.

I INTRODUCTION

How accurate would electric filter satisfy given specifications, depends on accuracy of elements which consist the filter and the way the filter is constructed. There are different reasons for the components values deviations from nominal such as component tolerances, aging, parasitic effects, external influences (temperature, humidity), and so on. By performing sensitivity analysis [1, 2] it is possible to asses filter characteristic changes due to component values deviations. The main assumption is that deviations are within component tolerances. Active filters are more sensitive on these changes than passive filters. Many authors in recent years has tried to offer such solutions in filter design that could decrease sensitivities to component values deviations.

On the other hand, the component value can deteriorate in much more drastical sense, when deviation comes out of tolerance boundaries. Then we speak about component failures in circuits. In these cases it is useful to carry out the fault isolation procedure. Many authors has proposed efficient methods for fault analysis [3, 4, 5].

In this paper we try to find the correlation between filter characteristic sensitivity to component parameters deviations and feasibility of fault isolation. The emphasis is on the fault analysis of analog filters.

II DEFINITIONS OF SENSITIVITY AND TESTABILITY

A. Sensitivity of active filters

Active filters with transfer functions of higher order are realised as a set of mutually interconnected blocks. The way this connection is accomplished is said to be the filter structure. The widest application has cascade structure because of its simplicity. However, it has been shown that some multiple feedback structures have significantly lower sensitivities than the cascade structure. Therefore they became an object of interest of many researchers. Narrow band pass filters especially have notable sensitivity problem [1].

Given the network function $F(s, r_1, \ldots, r_n)$, where s is complex variable and r_k; k=1, …,n are real parameters, the relative function deviation $\Delta F/F$ due to one parameter value relative deviation $\Delta r_k/r_k$ in the first approximation can be given by

$$\frac{\Delta F}{F} \cong \frac{r_k}{F}\frac{dF}{dr_k} \cdot \frac{\Delta r_k}{r_k}, \tag{1}$$

where

$$S_{r_k}^{F} = \frac{r_k}{F}\frac{dF}{dr_k} \tag{2}$$

represents relative sensitivity of function F to single parameter r_k.

In the real situation more than one component can change from nominal value. Hence, we define the criterion to asses function deviation due to change of many parameters. Suppose $\Delta r_k/r_k$ are independent normal random variables with zero means and standard deviations equal to σ_r. The squared standard deviation σ_F of relative function change $\Delta F/F$ is

$$\sigma_F^2 = \sigma_r^2 \sum_{k=1}^{n}\left(S_{r_k}^{F}\right)^2 = \sigma_r^2 S_2, \tag{3}$$

where sum of squares of function sensitivities to all parameters is called Shoeffler's sensitivity and defined as

$$S_2 = \sum_{k=1}^{n}\left(S_{r_k}^{F}\right)^2 . \tag{4}$$

In majority of cases Shoeffler's sensitivity provide good enough criterion for assessing filter's quality. Concerning structures with feedback loops it is very complicated to calculate (4) analytically, therefore we shall use numerical methods in this paper. We calculate standard deviation σ_F of relative change of function $\Delta F/F$ which equals to square root of Shoeffler's sensitivity [1, 2].

B. Fault analysis and testability

By fault, in general, we mean any change in the value of an element with respect to its nominal value which can cause the failure of the whole circuit. The faults could be hard (catastrophic) or soft (deviation faults) depending on amount of element value deviation. When faulty element deviates from its nominal value outside tolerance bounds, but without reaching its extreme values, it produces soft fault of circuit. Open and short circuit of elements produce hard faults [3, 4, 5].

When only one parameter causes the fault, it is referred to as a single fault. When several parameters simultaneously change they produce multiple fault.

To perform fault analysis on an analog filter we choose frequency domain, because magnitude of its transfer function is of major interest. The choice of test frequencies is then very important.

To find the most informative set of measurements for fault analysis many authors have defined various testability measures. Optimal testing conditions can be found maximising such a measure.

Using probabilistic theoretical approach, Freeman [4] presented one criterion for selecting optimum set of measures. It is based on statistical model of network and assumes occurrences of noncatastrophic faults.

The actual value of voltage measurement due to component failures and measurement errors is given by

$$V_i^m = V_i^0 + \sum_k \frac{\partial V_i^0}{\partial r_k}\Delta r_k + \varepsilon_i \quad ; i=1, \ldots, I_M, \qquad (5)$$

where Δr_k represents deviation from nominal value of the kth component, ε_i represents the error in measurement, V_i^0 is the nominal voltage at the ith measurement (i.e. ith frequency) and the partial derivatives are evaluated at nominal values for all parameters. Freeman assumed that Δr_k and ε_i are independent random normal variables with means equal to zeros and the standard deviations equal to w_k and σ_i, respectively. The probability of measuring vector $\mathbf{V}^m$ is given by multivariate normal distribution with vector of mean values $\mathbf{V}^0$ and the variance-covariance matrix $\mathbf{U}$ whose element U_{ij}, evaluated for nominal component parameters, is

$$\sum_k \left(\frac{\partial V_i^0}{\partial r_k}\right)\left(\frac{\partial V_j^0}{\partial r_k}\right) w_k^2 \text{ when } i \neq j \text{ and} \qquad (6a)$$

$$\sigma_i^2 + \sum_k \left(\frac{\partial V_i^0}{\partial r_k}\right)^2 w_k^2 \text{ when } i = j. \qquad (6b)$$

For independent measures matrix $\mathbf{U}$ is diagonal.

Freeman has shown, that the maximum of information could be given for the set of measurements I_M which maximise the determinant of $\mathbf{U}$ [4]. This way Freeman defined testability measure

$$T(\omega) = |\mathbf{U}(\omega)| \qquad (7)$$

Testability T depends on measured system, test points and test stimuli. The frequencies that give greater value for T have to be chosen because they provide more information.

Assume that r_k is the only element that could be faulty. Using network transfer function $F(s)$, unit input voltage, $\sigma_{i2}=0$, $w_{k2}=1$ the determinant $|\mathbf{U}(\omega)|$ is just a scalar value given by

$$T(\omega) = |\mathbf{U}(\omega)| = \left(\frac{\partial F(\omega)}{\partial r_k}\right)^2 \qquad (8)$$

and represents square absolute sensitivity of function F on single parameter r_k.

The catastrophic faults are sort of noncatastrophic faults with extreme shift in parameter values. Thus maximising T could also be criterion for obtaining the optimal set of measurements for catastrophic fault detection.

III STRUCTURES WITH LOW SENSITIVITY

As we stressed earlier, band pass filters have greater sensitivity [2], therefore they will be the object of analysis. All blocks, the filters in the Fig. 1. consist of, are second order and have band pass transfer function

$$T_i(s) = \frac{k_i s(\omega_{pi}/Q_{pi})}{s^2 + s(\omega_{pi}/Q_{pi}) + \omega_{pi}^2} \quad ; i=1, \ldots, N, \qquad (9)$$

where ω_{pi} and Q_{pi} represent pole frequency and Q-pole factor, respectively. Blocks form structures and thus realise filters. Choice of the structure influences the filters transfer function sensitivity.

The widest implementation has the cascade structure shown on the Fig. 1.a, because of its simplicity. Its sensitivity has been most researched [1].

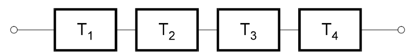

Fig. 1.a CAS structure

Follow the Leader Feedback structure (FLF) is presented in the Fig. 1.b.

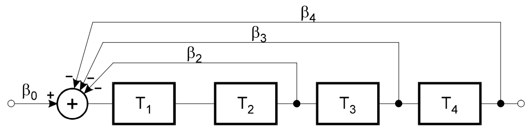

Fig. 1.b FLF structure

The Leap Frog structure (LF) in Fig. 1.c, also known as active ladder structure, simulates flow chart of passive ladder LC network. It is known that those passive networks have low sensitivity, therefore the LF structure has low sensitivity, too. The only inconvenient thing is that those structures have inner blocks with infinite Q-pole factor.

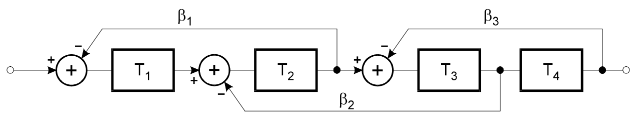

Fig. 1.c LF structure

Finally, one combination of minimal FLF and cascade structures is cascade of biquartic sections (CBQ). It has improved features in pass band and in stop band, too. The two blocks inside each feedback loop are identical. The structure is shown on Fig. 1.d.

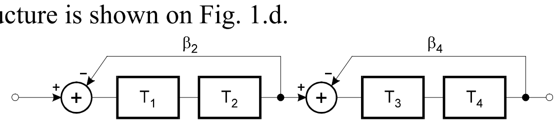

Fig. 1.d CBQ structure

IV EXAMPLE

Analysis of transfer function sensitivity on component values deviations and testability analysis were performed on the 8th order Chebishev band pass filter with narrow pass band width of 0.1 and pass band ripple defined by reflection factor equal to 0.1. Magnitude $|H(j\omega)|$ [dB] of its transfer function is shown in the Fig. 2.

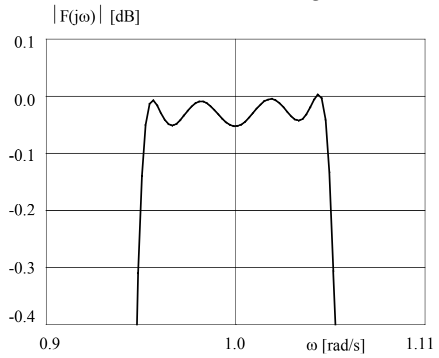

Fig. 2. Magnitude of narrow BP-filter

CAS, FLF, LF and CBQ structures presented in the Fig. 1. have been realised. Their coefficients are given in the Table I [1].

Table I Coefficients of structures on Fig. 1.					
	i	Q_p	ω_p	k	β
CAS	1	13.202	0.9755	1.0000	
	2	13.202	1.0251	1.4274	
	3	31.919	0.9420	2.1733	
	4	31.919	1.0615	7.1188	
FLF*	1	18.665	1.0000	2.5933	0.0000
	2	18.665	1.0000	2.4669	0.8571
	3	18.665	1.0000	2.3572	0.2263
	4	18.665	1.0000	2.1840	0.1601
CBQ	1	13.198	1.0000	1.4482	
	2	13.198	1.0000	1.0363	0.2843
	3	31.862	1.0000	3.8230	
	4	31.862	1.0000	3.4848	1.0880
LF	1	9.357	1.0000	1.2085	0.8116
	2	∞	1.0000	0.07892**	0.7508
	3	∞	1.0000	0.08267**	0.6810
	4	9.307	1.0000	1.3737	

*There is coefficient β_0=0.4582
**Instead of value of k in the table is $k\omega_p/Q_p$

All 2nd order blocks T_i ; i=1, …,4 in (9) are realised with general purpose section (GP) shown oh the Fig. 3.

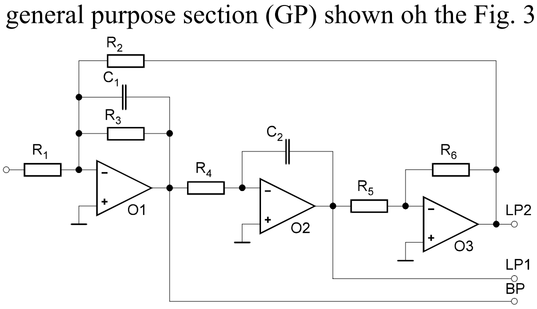

Fig. 3. GP section in realisation of filter blocks

$$k = \frac{R_3}{R_1} \; ; \; \omega_p = \frac{R_6}{R_2 R_4 R_5 C_1 C_2} \; ; \; Q_p = R_3 C_1 \omega_p \quad (10)$$

One can use (10) to calculate required parameters for GP section's transfer function. This section is suitable because it can realise infinite pole Q factors. Furthermore, pole frequency and pole Q factor can be tuned independently. One could tune ω_p with R_4, Q_p with R_3 and k with R_1.

As stated earlier, main assumption is that the relative element value deviations are independent normal variables with zero means and standard deviations σ_r. Let σ_r equals to 1%. Performing analysis on computer, the standard deviation σ_F of relative change of magnitude $|F(j\omega)|$ in [dB] defined in (3) has been calculated and presented on the Fig. 4. Influence of feedback elements and gain of active elements in overall sensitivity has not been included in consideration. The sensitivities of feedback elements could be neglected.

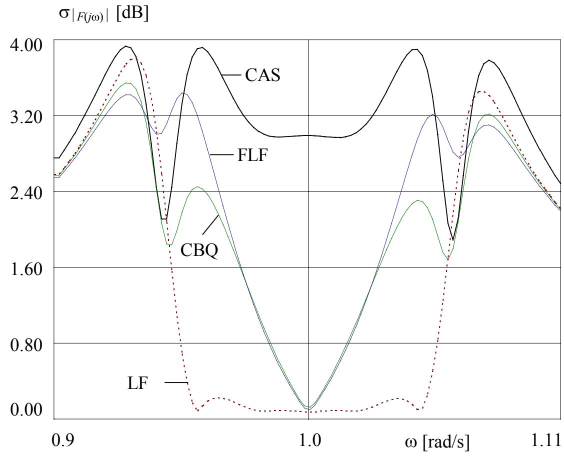

Fig. 4. Square root of Shoeffler's sensitivity of structures on Fig. 1.

Sensitivity analysis on Fig. 4. shows that CAS structure has the biggest sensitivity, and structures with feedbacks have lower sensitivities. The best results yields LF structure [1].

After all, the testability measure T defined in (8) has been applied for possible faults of every single element in all GP blocks. Elements in feedback and active elements are not taken into consideration. Analysing faults of R_2, R_4, R_5 and C_2 produced identical testability curves, furthermore, testability of R_6 and C_1 were close to them, too. We distinguish among three main groups of elements in GP section that could be tested under similar conditions. Those three representative elements are R_1, R_3 and R_4 (R_4 represents behaviour of R_2, R_5, R_6, C_1 and C_2). Using them, as we stated earlier we can tune k, Q_p and ω_p. Testability analysis [3] has been performed for all four blocks (noted with I, II, III, IV) of CAS, FLF, LF and CBQ structures. Results are presented on the Fig. 5., and give us comprehensive picture about testability of all elements in mentioned structures.

Analysing characteristics on the Fig. 5. one can see that the optimal region for fault analysis of elements in CAS structure is situated throughout all pass band. For structures with feedback loops (FLF, CBQ, LF) testability is lower in the middle of the pass band and higher in pass band edges. Blocks with higher Q have greater testability.

This all is similar with sensitivity analysis carried out on different structures, as shown on the Fig. 4. One can therefore use Shoeffler's sensitivity to find optimal frequency ranges for fault analysis instead of hardly calculating T.

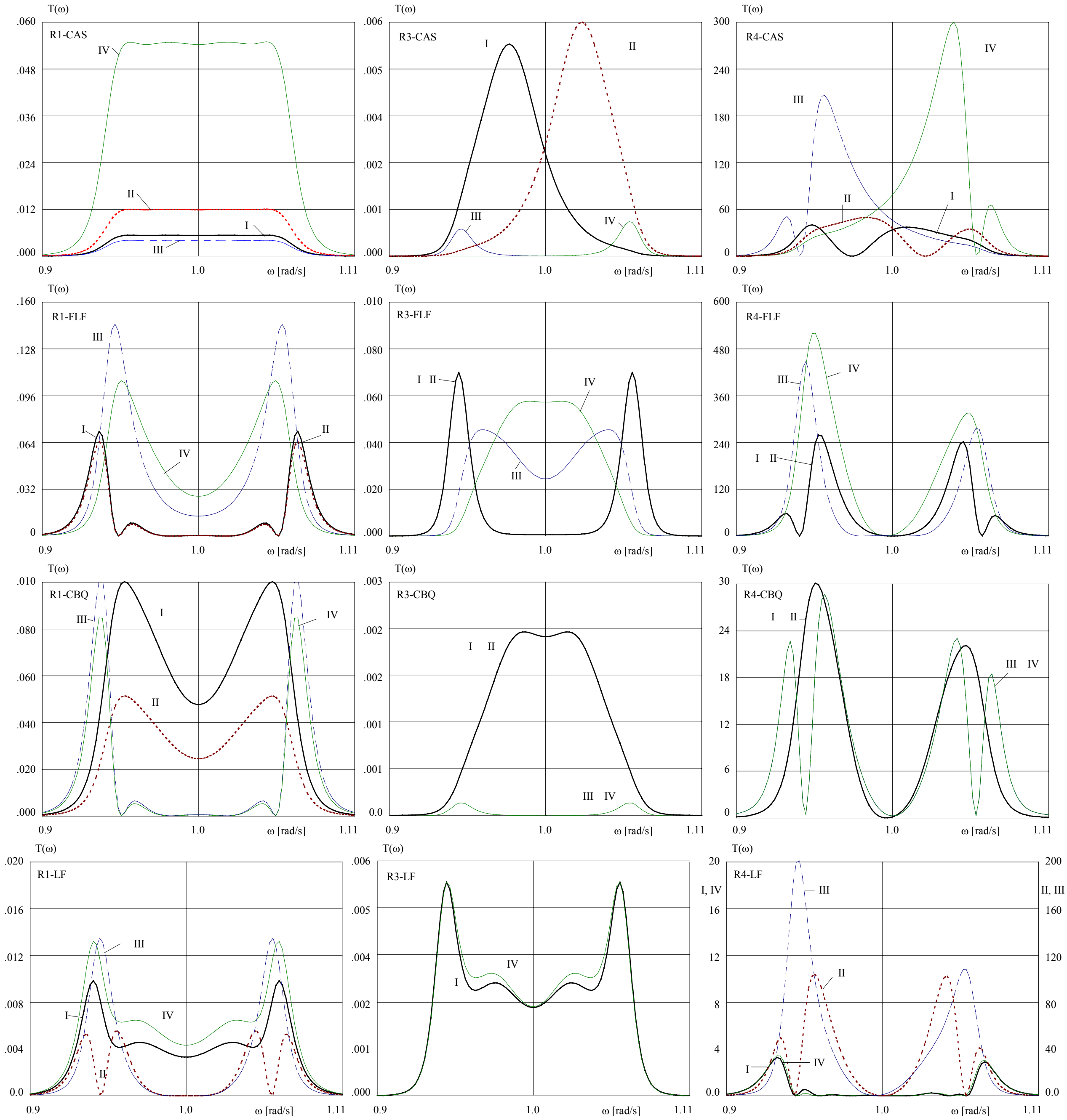

Fig. 5. Testability of faults caused by single elements R_1, R_3, R_4 in blocks I, II, III, IV (CAS, FLF,CBQ,LF)

V CONCLUSION

Obtained results show that there is correlation between sensitivity and testability. One can use Shoeffler's sensitivity analysis to determine optimal conditions for fault analysis. One can say that Shoeffler's sensitivity is good measure of testability. This was approved in comparing with Freeman's probabilistic testability measure.

Second conclusion is that structures with lower sensitivities have lower testabilities in corresponding ranges, and are less suitable for fault analysis.

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