RADIO SCIENCE, VOL. 42, RS2S90, doi:10.1029/2004RS003147, 2007

A new root-based direction-finding algorithm

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Received 9 August 2004; revised 24 August 2006; accepted 6 December 2006; published 10 April 2007.

[1] Polynomial rooting direction-finding (DF) algorithms are a computationally efficient alternative to search-based DF algorithms and are particularly suitable for uniform linear arrays of physically identical elements provided that mutual interaction among the array elements can be either neglected or compensated for. A popular algorithm in such situations is Root Multiple Signal Classification (Root MUSIC (RM)), wherein the estimation of the directions of arrivals (DOA) requires the computation of the roots of a $(2N - 2)$ -order polynomial, where N represents number of array elements. The DOA are estimated from the L pairs of roots closest to the unit circle, where L represents number of sources. In this paper we derive a modified root polynomial (MRP) algorithm requiring the calculation of only L roots in order to estimate the L DOA. We evaluate the performance of the MRP algorithm numerically and show that it is as accurate as the RM algorithm but with a significantly simpler algebraic structure. In order to demonstrate that the theoretically predicted performance can be achieved in an experimental setting, a decoupled array is emulated in hardware using phase shifters. The results are in excellent agreement with theory.

Citation: Wasylkiwskyj, W., I. Kopriva, M. Doroslovački, and A. I. Zaghloul (2007), A new root-based direction-finding algorithm, *Radio Sci.*, *42*, RS2S90, doi:10.1029/2004RS003147.

1. Introduction

[2] Superresolution direction-finding (DF) algorithms for linear arrays fall into two broad categories: search-based algorithms, as exemplified by MUSIC [*Schmidt*, 1981; *Roy and Kailath*, 1989] and root-based algorithms such as Root-MUSIC [*Barabell*, 1983; *Rao and Hari*, 1989], Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [*Roy and Kailath*, 1989]. Search algorithms make no assumptions about the algebraic structure of the array steering vectors but require that they be known to great accuracy, especially if a high degree of angular resolution is called for. In that case they can also be computationally quite demanding. In practice the determination of the array steering vector amounts to an accurate measurement of the magnitude and phase of the array element patterns, sometimes referred to as array manifold calibration. Normal accuracies attained in such measurements are a few tenths of a dB in amplitude and about 1 degree in phase, which generally is insufficient for the design of high-resolution DF systems. Admittedly an alternative technique would be to rely on numerical computer simulations (either computing the element patterns directly or inferring them from the array geometry and the computed impedance or scattering matrix). However, our experience with comparisons of numerical simulations using the latest commercially available software with experimental data indicates that presently this is not yet a fruitful approach [*Abdallah et al.*, 2004].

[3] Root-based algorithms on the other hand require no array calibration and afford substantial computational efficiency over search algorithms. They require that the elements be uniformly spaced and physically identical, which a search algorithm such as MUSIC does not. The more significant restriction however is that the array steering vector must have the form of an array factor of a linear array of uniformly spaced elements. Unfortunately, because of interelement mutual coupling this idealized form of the steering vector is practically unattainable. Indeed when root-based DF algorithms are applied to a real array without some form of compensation significant angle estimation errors can result (W. Wasylkiwskyj et al., Direction finding using root algorithms with mutual

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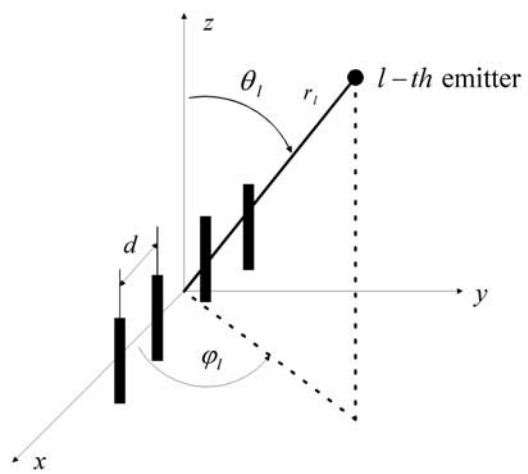

Figure 1. Coordinate system of the linear antenna array.

coupling compensation, submitted to *IEEE Transactions on Antennas and Propagation*, 2004, hereinafter referred to as Wasylkiwskyj et al., submitted manuscript, 2004). Compensation for the effects of mutual coupling can be realized either through the use of a decoupling transform (Wasylkiwskyj et al., submitted manuscript, 2004) or by employing extra "dummy" elements to equalize the active element radiation patterns [*Wasylkiwskyj and Kopriva*, 2004; *Lundgren*, 1996]. Under the assumption that element radiation patterns are sufficiently equalized the nonnegative pseudospectrum function becomes a polynomial and the DF problem is reduced to polynomial rooting problem [*Barabell*, 1983; *Rao and Hari*, 1989], ESPRIT [*Roy and Kailath*, 1989]. In the case of the Root Multiple Signal Classification (Root MUSIC (RM)) algorithm the degree of the polynomial equals $2N - 2$, where N represents number of array elements and $2N - 2$ roots have to be calculated. The directions of arrivals (DOA) are calculated from the L roots closest to the unit circle where L represents number of emitters. This selection process can introduce serious errors especially in low SNR and coupled array environments. Here we formulate a root algorithm with the property that the polynomial that follows from the nonnegative pseudospectrum function has degree $2L$. Unlike of the case of RM algorithm there are no extraneous roots produced by our modified root polynomial (MRP) algorithm. Its chief advantage over the classical RM algorithm is that it yields only roots corresponding to the actual DOA. The algorithm is derived in section 3. Results of comparative performance evaluation of the MRP and RM algorithms are presented in section 4. The hardware emulation of the decoupled array environment shows excellent agreement with theory. The conclusion is given in section 5.

2. Linear Antenna Array Model

[4] The estimation of the DOA of L mutually uncorrelated plane waves incident on an array of N sensors is described by the standard ("narrow band") model

$$\mathbf{z}(t) = \mathbf{A}\mathbf{s}(t) + \boldsymbol{\nu}(t), \tag{1}$$

where $\mathbf{z}(t)$ is a column vector comprising N signals that represent complex envelopes of the corresponding RF signals at the output of the linear array; $\mathbf{A}$ is an $N \times L$ steering matrix of the linear array comprising the L column vectors $\mathbf{a}(\theta_l, \varphi_l)$ $l = 1.L$ where the spherical coordinate θ_l ,φ_l corresponds to the DOA of the lth source signal, as indicated in Figure 1; $\mathbf{s}(t)$ is a column vector comprising the L signals incident on the array and $\boldsymbol{\nu}(t)$ represents the column vector of the N mutually uncorrelated receiver noise contributions. Most of the signal processing literature that treats DOA estimation problems assumes the steering vector $\mathbf{a}(\theta_l, \varphi_l)$ for the linear array of the form

$$\mathbf{a}(\theta_l, \varphi_l) = \frac{-j\lambda}{\sqrt{\varsigma_0}} \hat{f}(\theta_l, \varphi_l) \cdot \left[1 \quad e^{jk_0 d\xi_l} \quad e^{jk_0 2d\xi_l} \ldots \quad e^{jk_0(N-1)d\xi_l}\right]^T \tag{2}$$

where $\xi_\ell = \cos\alpha_\ell = \sin\theta_l \cos\varphi_l$, with α_ℓ the angle between the direction of incidence and the array axis, ς_0 is the characteristic impedance of free space, $k_0 = \frac{2\pi}{\lambda}$ is the free space wave number, d the interelement spacing and the $\hat{f}(\theta_l, \varphi_l)$ are element patterns, assumed identical for all elements. Implicit in the assumption of element pattern equality is the physical identity of the elementary radiators. Strict equality of the element patterns cannot be maintained for arrays of finite length even for identical elements because of mutual coupling [*Roller and Wasylkiwskyj*, 1992]. Instead, the steering vector $\mathbf{a}(\theta_l, \varphi_l)$ should more properly be represented by

$$\mathbf{a}(\theta_l, \varphi_l) = \frac{-j\lambda}{\sqrt{\zeta_0}} \left[\hat{f}_1(\theta_l, \varphi_l) \quad \hat{f}_2(\theta_l, \varphi_l) e^{jk_0 d\xi_l} \quad \hat{f}_3(\theta_l, \varphi_l) e^{jk_0 2d\xi_l} \ldots \quad \hat{f}_N(\theta_l, \varphi_l) e^{jk_0(N-1)d\xi_l}\right]^T, \tag{3}$$

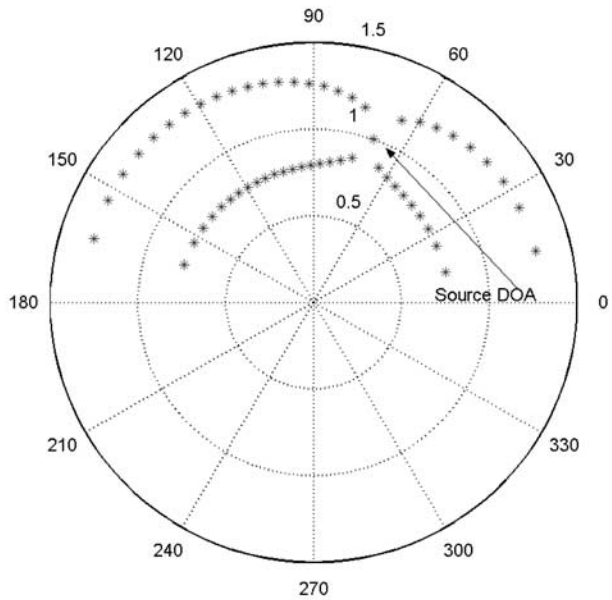

Figure 2. Root locus diagram of the RM algorithm for one source and 30-element ULA.

where the element patterns $\hat{f}_n(\theta_l,\varphi_l)$ are functions of element position in the array and are defined in the environment of (dissipatively) terminated elements (W. Wasylkiwskyj, Signal reception by array antennas with explicit electromagnetic constraints, submitted to *IEEE Transactions on Antennas and Propagation*, 2004, hereinafter referred to as Wasylkiwskyj, submitted manuscript, 2004). As already mentioned, compensation for the effects of mutual coupling has to be applied either through the use of a decoupling transform (Wasylkiwskyj et al., submitted manuscript, 2004) or by employing extra "dummy" elements to equalize the active element radiation patterns [*Wasylkiwskyj and Kopriva*, 2004; *Lundgren*, 1996].

3. MRP Algorithm

[5] Just like in MUSIC [*Roy and Kailath*, 1989] we first estimate the sample array covariance matrix

$$\hat{\mathbf{R}}_{\mathbf{zz}} = \frac{1}{K}\sum_{j=1}^{K} \mathbf{z}(t_j)\mathbf{z}^H(t_j), \tag{4}$$

where K is the data record length and the steering vector may have the general form (3). As in MUSIC the noise power in each of the N receivers is assumed identical and uncorrelated among the receivers with the sample covariance $\hat{\sigma}_0^2\mathbf{I}_{\mathbf{NN}}$. Instead of identifying the noise eigenvectors directly from an eigendecomposition of $\hat{\mathbf{R}}_{\mathbf{zz}}$ as is done in MUSIC they are determined via the following alternative procedure. The noise covariance matrix is estimated first and subtracted from $\hat{\mathbf{R}}_{\mathbf{zz}}$. On the basis of the array signal model (1) this yields

$$\hat{\mathbf{W}} = \hat{\mathbf{R}}_{\mathbf{zz}} - \hat{\sigma}_0^2\mathbf{I} = \mathbf{A}\hat{\mathbf{R}}_{ss}\mathbf{A}^H, \tag{5}$$

where $\hat{\mathbf{R}}_{ss}$ is the signal sample covariance matrix $\hat{\mathbf{R}}_{\mathbf{ss}} = (1/K)\sum_{\ell=1}^{K}\mathbf{s}\,(t_\ell)\mathbf{s}^H\,(t_\ell)$.

[6] The (pseudo) rank of the matrix $\hat{\mathbf{W}}$ is L and $N - L$ dimensional (pseudo) null-space is spanned by the noise eigenvectors. When the sample size (K) is large $\hat{\mathbf{W}}$ converges in probability toward its asymptotic theoretical value (ensemble average) $\mathbf{W}$ in which case we can replace the estimators in (5) with true ensemble averages

$$\mathbf{W} = \mathbf{R}_{\mathbf{zz}} - \sigma_0^2\mathbf{I} = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H \tag{6}$$

so that the true null-space of $\mathbf{W}$ is now L. Vectors in this null-space can be determined with the aid of the following column partitioning of $\mathbf{W}$:

$$\mathbf{W} = \left[\mathbf{W}_{1,k-1}\middle|\mathbf{w}_k\middle|\mathbf{W}_{k+1,k+L}\middle|\mathbf{W}_{k+L+1,N}\right], \tag{7}$$

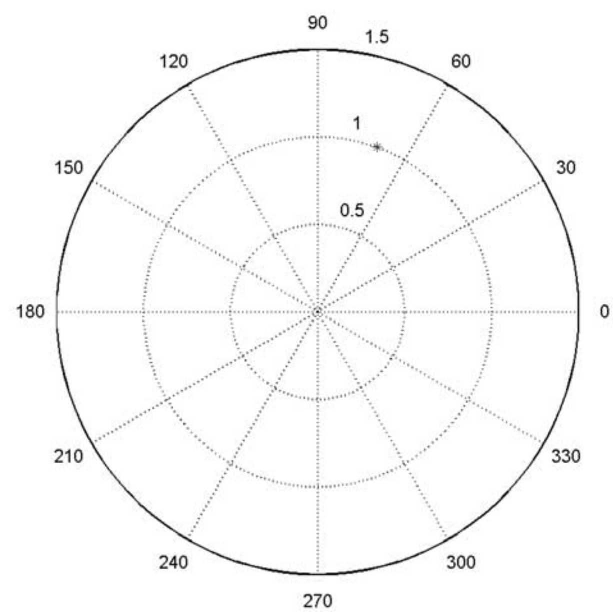

Figure 3. Root locus diagram of the MRP algorithm for one source and 30-element ULA.

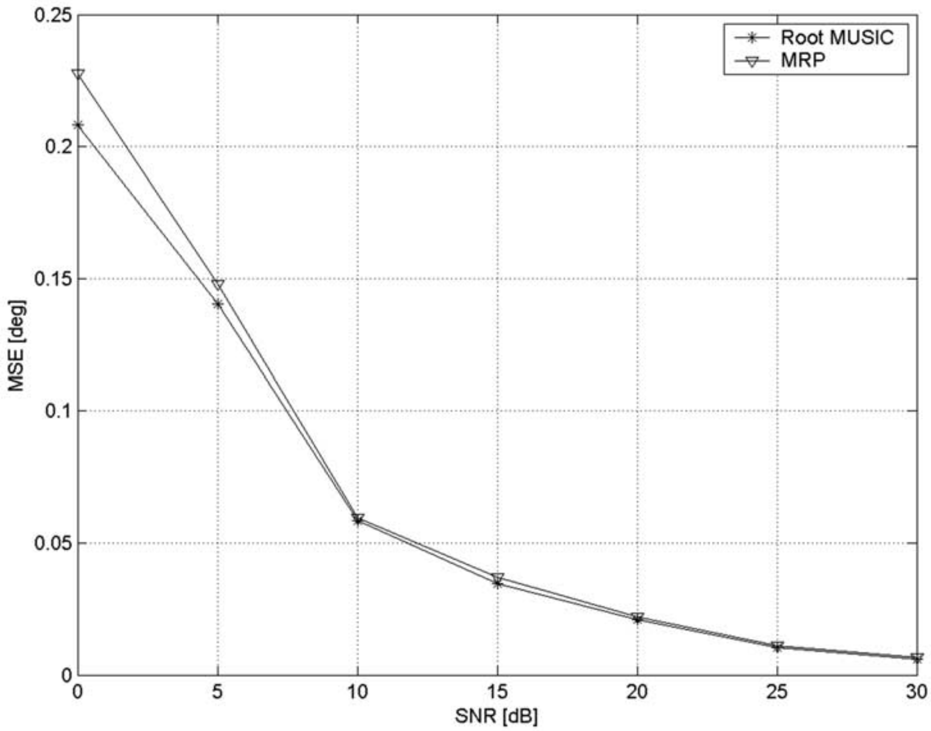

Figure 4. MSE in DOA estimation as a function of the SNR for one source and four-element ULA with sample size T = 1000 samples. Asterisks refer to RM, and inverted triangles refer to MRP.

where for $1 < k < N - L$ the $\mathbf{W}_{1,k-1}$ comprises columns 1 through $k - 1$, $\mathbf{w}_k$ is the kth column, $\mathbf{W}_{k+1,k+L}$ consists of columns $k + 1$ through $k + L$ and $\mathbf{W}_{k+L+1,N}$ comprises the remaining columns. For example for $k = 1$ and $k = N - L$ the partitions become

$$\begin{aligned}\mathbf{W} &= \left[\mathbf{w}_1 \middle| \mathbf{W}_{2,1+L} \middle| \mathbf{W}_{2+L,N}\right] \\ \mathbf{W} &= \left[\mathbf{W}_{1,N-L-1} \middle| \mathbf{w}_{N-L} \middle| \mathbf{W}_{N-L+1,N}\right].\end{aligned} \tag{8}$$

Because the number of linearly independent column vectors is L, $\mathbf{w}_k$ for $1 \leq k \leq N - L$ must be a linear combination of the column vectors comprising $\mathbf{W}_{k+1,k+L}$ so that

$$\mathbf{w}_k = \mathbf{W}_{k+1,k+L}\mathbf{c}_k, \tag{9}$$

where $\mathbf{c}_k$ are L-dimensional column vectors comprising coefficients expressing this linear dependence. We now define $L + 1$ dimensional vectors by

$$\mathbf{h}_k = \begin{bmatrix}-1 & \mathbf{c}_k^T\end{bmatrix}^T \tag{10}$$

and put (9) in the form

$$\begin{bmatrix}\mathbf{w}_k \; \mathbf{W}_{k+1,k+L}\end{bmatrix}\mathbf{h}_k = 0; \;\; 1 \leq k \leq N - L. \tag{11}$$

In view of (11) (5) and (7) the $N - L$ the N-dimensional vectors $\boldsymbol{\nu}_k$ spanning the null-space of $\mathbf{W}$ must have the structure

$$\nu_k = \begin{bmatrix}0 \; \ldots \; 0 \; \mathbf{h}_k^T \, 0 \; \ldots \; 0\end{bmatrix}^T \quad 1 \leq k \leq N - L, \tag{12}$$

where the $k - 1$ zero entries are followed by the $L + 1$ elements of the row vector $\mathbf{h}_k^T$ that are then followed by $N - L - k$ zero entries. Indeed, with the index change to $j = L + k$ we identify $\mathbf{e}_j$,

$$\nu_{j-L} = \mathbf{e}_j; \; j = L + 1, \ldots, N, \tag{13}$$

with the noise eigenvectors of the sample data covariance matrix $\hat{\mathbf{R}}_{\mathbf{zz}}$. However, here they are constructed from the solutions of $N - L$ equations (11). As in MUSIC

$$\mathbf{A}^H \nu_k = 0; \quad 1 \leq \; k \; \leq N - L, \tag{14}$$

but in view of (12) for each k only $L + 1$ elements of $\mathbf{a}^H$ the rows of $\mathbf{A}^H$ enter into (14). These can be identified by

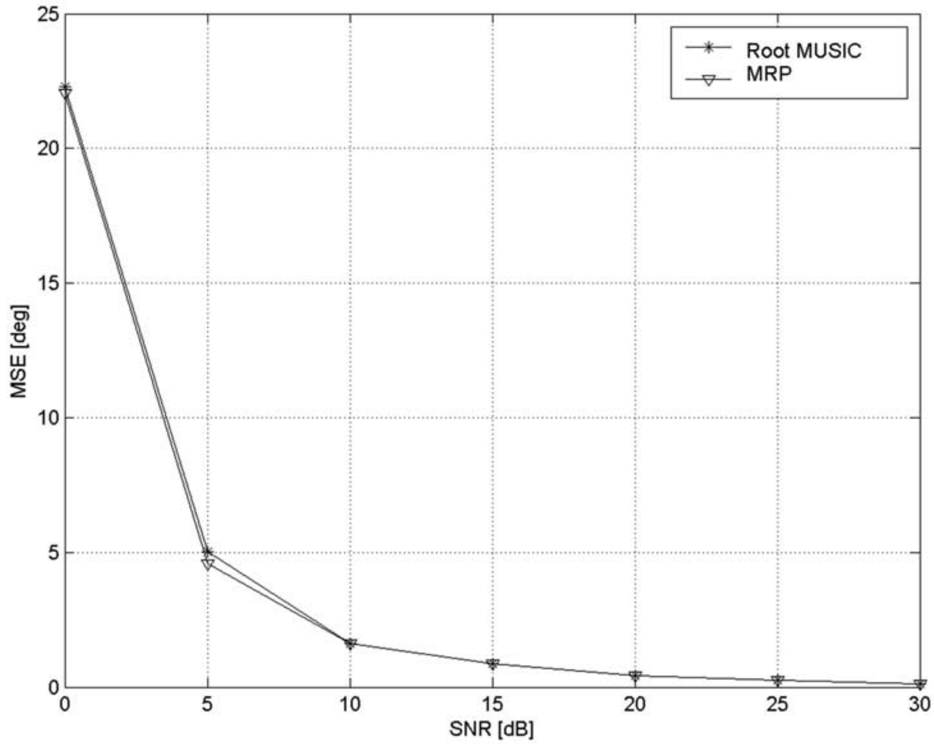

Figure 5. MSE in DOA estimation as a function of the SNR for three sources and four-element ULA with sample size T = 1000 samples. Asterisks refer to RM, and inverted triangles refer to MRP.

partitioning the steering vector (3) in a manner similar to **W**, i.e.,

$$\mathbf{a}(\Omega_l) = \left[\mathbf{a}^T_{1,k-1}(\Omega_l)|a_k(\Omega_l)|\mathbf{a}^T_{k+1,k+L}(\Omega_l)\middle|\mathbf{a}^T_{k+L+1,N}(\Omega_l)\right]^T, \quad (15)$$

where Ω_l stands for the solid angle $\Omega_l = (\theta_l, \varphi_l)$. Evidently (14) can then be replaced by

$$\mathbf{a}^H_k(\Omega_l)\mathbf{h}_k = 0; \quad 1 \le k \le N - L, \ l = 1, \ldots, L, \quad (16)$$

where

$$\mathbf{a}_k(\Omega_l) = \left[a_k(\Omega_l)\, \mathbf{a}^T_{k+1,k+L}(\Omega_l)\right]^T. \quad (17)$$

[7] By analogy with MUSIC one can now define the modified pseudospectrum (MPS)

$$\Lambda(\Omega) = \sum_{k=1}^{N-L} \mathbf{a}^H_k(\Omega)\mathbf{h}_k\mathbf{h}^H_k\mathbf{a}_k(\Omega), \quad (18)$$

which has zeros at the L DOA. The preceding relationships are exact since they presume perfect estimation, which in practice is unattainable. Instead, we have to deal with the set of equations that result from the partitioning of $\hat{\mathbf{W}}$ in (5) rather than **W** in (6) which will lead to a set of overdetermined but generally inconsistent systems. As a result we can sensibly seek a set of $\hat{\mathbf{c}}_k \approx \mathbf{c}_k$ that provide only an approximation to (9). We may phrase this symbolically as

$$\hat{\mathbf{w}}_k \sim \hat{\mathbf{W}}_{k+1,k+L}\hat{\mathbf{c}}_k. \quad (19)$$

[8] For example, we can seek $\hat{\mathbf{c}}_k$ that minimize $\left\|\hat{\mathbf{w}}_k - \hat{\mathbf{W}}_{k+1,k+L}\hat{\mathbf{c}}_k\right\|^2$ (LS approximation). In that case we get

$$\hat{\mathbf{c}}_k = \hat{\mathbf{W}}^{\dagger}_{k+1,k+L}\hat{\mathbf{w}}_k, \quad (20)$$

where $\hat{\mathbf{W}}^{\dagger}_{k+1,k+L}$ is the Moore-Penrose pseudoinverse. The vectors (10) must now be replaced by their approximate forms

$$\hat{\mathbf{h}}_k = \left[-1 \ \ \hat{\mathbf{c}}^T_k\right]^T \quad (21)$$

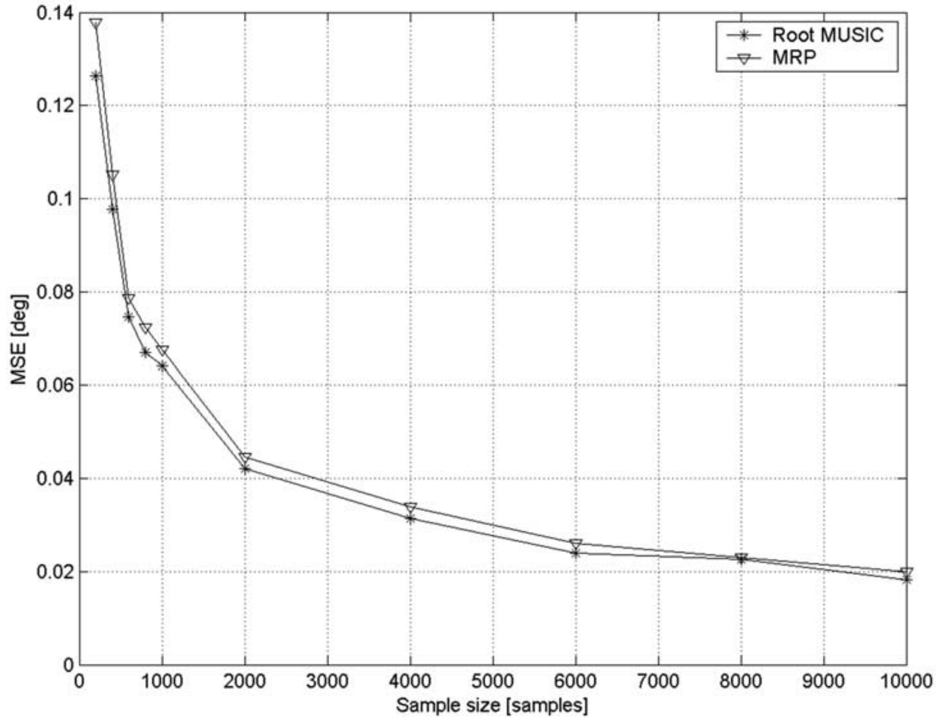

Figure 6. MSE in DOA estimation as a function of the sample size for one source and four-element ULA with SNR = 10 dB. Asterisks refer to RM, and inverted triangles refer to MRP.

and (18) by

$$\hat{\Lambda}(\Omega) = \sum_{k=1}^{N-L} \mathbf{a}_k^H(\Omega)\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\mathbf{a}_k(\Omega), \tag{22}$$

which no longer has zeros at the DOA but merely minima. Just like in MUSIC for purposes of calculation it is more appropriate to use the reciprocal of (22) and search for maxima rather than minima. Accordingly we define the reciprocal of the modified pseudospectrum (RMPS) spectrum by

$$S(\Omega) = \frac{\mathbf{a}_k^H(\Omega)\mathbf{a}_k(\Omega)}{\sum_{k=1}^{N-L} \mathbf{a}_k^H(\Omega)\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\mathbf{a}_k(\Omega)}. \tag{23}$$

[9] What is the connection between the RMPS and the MUSIC spectra? When the preceding steps are compared with any of the derivations of the MUSIC algorithm [*Roy and Kailath*, 1989] that the essential difference between the two approaches lies in the manner in which the noise eigenvectors are being solved for. Evidently the correct asymptotic form (corresponding to perfect estimation, i.e., an infinite sample size) of the noise eigenvectors is (12) which has only $L + 1$ entries not identically zero. The MRPS forces this constraint on each noise eigenvector, MUSIC does not. When the steering vector of the linear uniformly spaced array can be represented by (2) (omitting the constant multiplier with the element pattern)

$$\begin{aligned}\mathbf{a}_k(\Omega_l) \equiv \mathbf{a}_k\left(e^{j\psi}\right)\left[e^{jk\psi}\ e^{j(k+1)\psi}\ e^{j(k+2)\psi}\ \ldots\ e^{j(k+L)\psi}\right]^T \\ = e^{jk\psi}\left[1\ \ e^{j\psi}\ e^{j2\psi}\ \ \ldots\ \ e^{jL\psi}\right]^T,\end{aligned} \tag{24}$$

(where $\psi = k_0 d\xi$ and ξ the direction cosine between the direction of incidence and the array axis), the location of the zeros in (18) and the maxima in (23) can be accomplished by finding the roots of a polynomial. This leads to the modified root polynomial (MRP) algorithm as described in the sequel. Substituting (24) into (18) we get

$$\Lambda\left(e^{j\psi}\right) = \sum_{n=1}^{L+1}\sum_{m=1}^{L+1} e^{j(m-n)\psi}\mathbf{H}_{nm}, \tag{25}$$

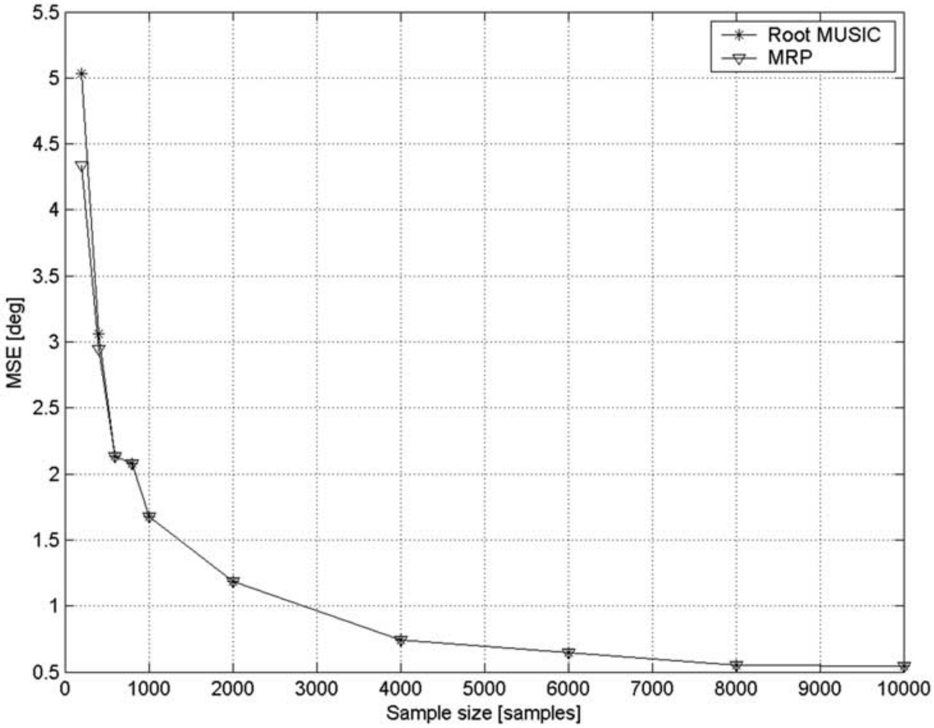

Figure 7. MSE in DOA estimation as a function of the sample size for three sources and four-element ULA with SNR = 10 dB. Asterisks refer to RM, and inverted triangles refer to MRP.

where $\mathbf{H}_{nm}$ are elements of the $L + 1 \times L + 1$ matrix

$$\mathbf{H} = \sum_{k=1}^{N-L} \mathbf{h}_k \mathbf{h}_k^H. \tag{26}$$

[10] With the substitution $z = e^{j\psi}$ (25) becomes

$$\Lambda(z) = z^{-L} P_{2L}(z), \tag{27}$$

where $P_{2L}(z)$ is a $2L$ degree polynomial. Changing summation index from m to $k = m - n + L$, $P_{2L}(z)$ can be written as

$$P_{2L}(z) = \sum_{n=1}^{L+1} \sum_{k=L+1-n}^{2L+1-n} \mathbf{H}_{n;k+n-L} z^k. \tag{28}$$

[11] Relationship between the coefficients of the polynomial $P_{2L}(z)$ and elements of the matrix $\mathbf{H}$ can be established by writing out the partial sums in (28):

$$\begin{aligned} P_{2L}(z) = &\sum_{k=L}^{2L} \mathbf{H}_{1;k+1-L} z^k + \sum_{k=L-1}^{2L-1} \mathbf{H}_{2;k+2-L} z^k \\ &+ \sum_{k=L-2}^{2L-2} \mathbf{H}_{3;k+3-L} z^k + \sum_{k=L-3}^{2L-3} \mathbf{H}_{4;k+4-L} z^k \\ &+ \ldots + \sum_{k=0}^{L} \mathbf{H}_{L+1;k+1} z^k. \end{aligned} \tag{29}$$

[12] Rewriting (29) with the powers of z in descending order we get

$$\begin{aligned} P_{2L}(z) = &\mathbf{H}_{1;L+1} z^{2L} + \left(\mathbf{H}_{1;L} + \mathbf{H}_{2;L+1}\right) z^{2L-1} \\ &+ \left(\mathbf{H}_{1;L-1} + \mathbf{H}_{2;L} + \mathbf{H}_{3;L+1}\right) z^{2L-2} \\ &+ \left(\mathbf{H}_{1;L-2} + \mathbf{H}_{2;L-1} + \mathbf{H}_{3;L} + \mathbf{H}_{4;L+1}\right) z^{2L-3} \\ &+ \ldots + (\mathbf{H}_{1;1} + \mathbf{H}_{2;2} + \mathbf{H}_{3;3} + \ldots \\ &+ \mathbf{H}_{L+1;L+1}) z^L + (\mathbf{H}_{2;1} + \mathbf{H}_{3;2} + \mathbf{H}_{4;3} + \ldots \\ &+ \mathbf{H}_{L+1;L}) z^{L-1} + (\mathbf{H}_{3;1} + \mathbf{H}_{4;2} + \mathbf{H}_{5;3} + \ldots \\ &+ \mathbf{H}_{L+1;L-1}) z^{N-3} + \ldots + \mathbf{H}_{L+1;1}. \end{aligned} \tag{30}$$

Denoting its coefficient of $P_{2L}(z)$ by β_j we can write (30) as

$$P_{2L}(z) = \sum_{l=-L}^{L} \beta_l z^{l+L}. \tag{31}$$

[13] Because the matrix $\mathbf{H}$ is hermitian the $2L$ zeros of P_{2L} (z) are symmetrically disposed with respect to the unit circle. Unlike the Root MUSIC algorithm [*Barabell*, 1983; *Rao and Hari*, 1989] which always yields $2N - 2$

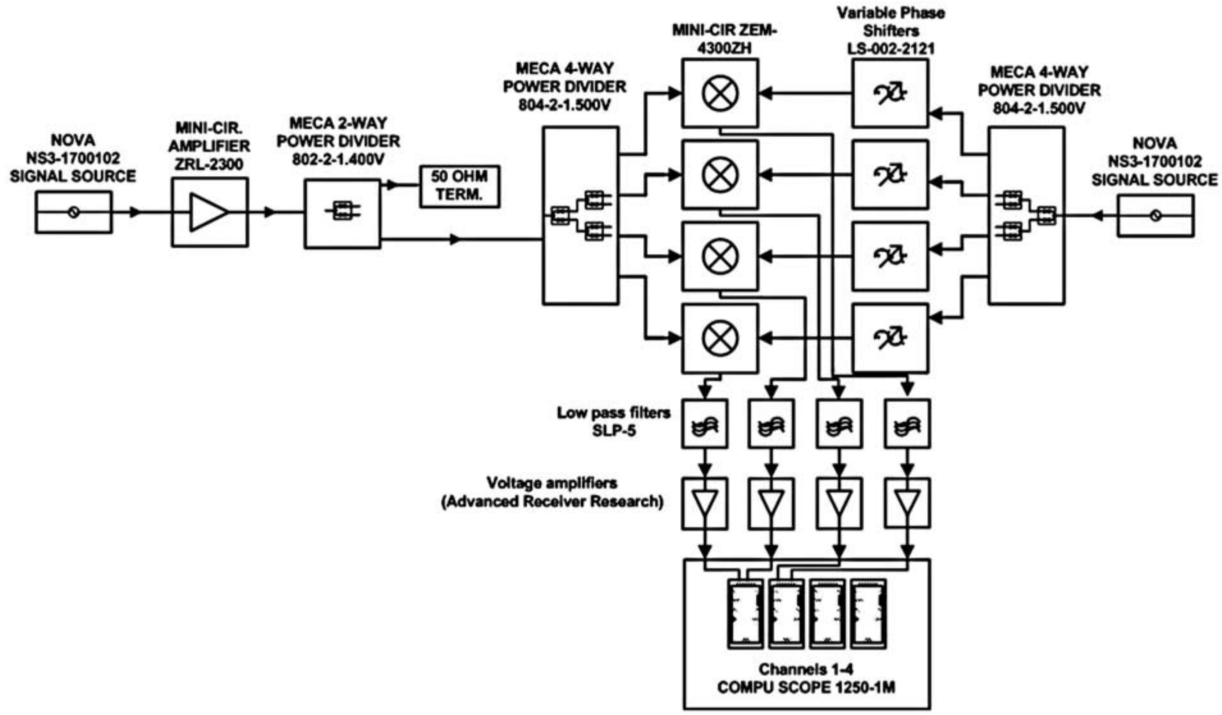

Figure 8. Schematic diagram of the hardware setup for performance evaluation of the root-based DF algorithms.

roots from which the roots that correspond to the actual DOA must be downselected, there are no extraneous roots produced by the MRP method.

[14] The determination of the roots of (31) can be readily mechanized in MATLAB. Thus with β a $2L + 1$ element vector and H the matrix (26), the root finding algorithm can be phrased succinctly in MATLAB notation as follows:

for $\quad j = 1 : 2^{*}L + 1$

$\qquad \beta(j) = \mathrm{sum}(\mathrm{diag}(H, L + 1 - j))$

end

$z = \mathrm{roots}(\beta)$.

[15] The DOA α_l are then computed from

$$\cos\alpha_l = \frac{1}{k_0 d}\mathrm{angle}(z_l) \;\; l = 1, .., L. \qquad (32)$$

4. Numerical and Experimental Results

[16] We evaluate here performance of both RM and MRP algorithms and show that MRP algorithm is as accurate as RM algorithm having at the same time significantly simpler algebraic structure. In this regard we show in Figures 2 and 3 root locus diagrams produced by RM and MRP algorithms respectively, for a case when one QPSK emitter was impinging on the 30-element uniform linear arrays (ULA) with $\lambda/2$ interelement spacing. Root locus diagram produced by RM algorithm is evidently very crowded containing 28 pairs of roots despite the fact that only one emitter was impinging on the ULA. Figures 4 and 5 show mean square error (MSE) in DOA estimation as a function of the SNR value. The MSE was estimated when one (Figure 4) or three (Figure 5) emitters were impinging on the four-element ULA. The sample size was kept constant at T = 1000 samples. Emitters were located at 70°, 50° and 30°. Evidently there is no significant difference in accuracy between MRP and RM algorithms. The same conclusion can be drawn from Figures 6 and 7 where MSE in DOA estimation is shown as a function of the sample size value for one (Figure 6) and three (Figure 7) QPSK sources. SNR was kept constant at 10dB value. Results presented in Figures 4–7 were obtained as an average of 100 runs for each value of the SNR or sample size.

[17] In addition, all results presented in Figures 4–7 were obtained assuming no mutual coupling between the array elements. The DOA estimation performance achievable with root algorithms in an ideal decoupled array environment may therefore be regarded as a

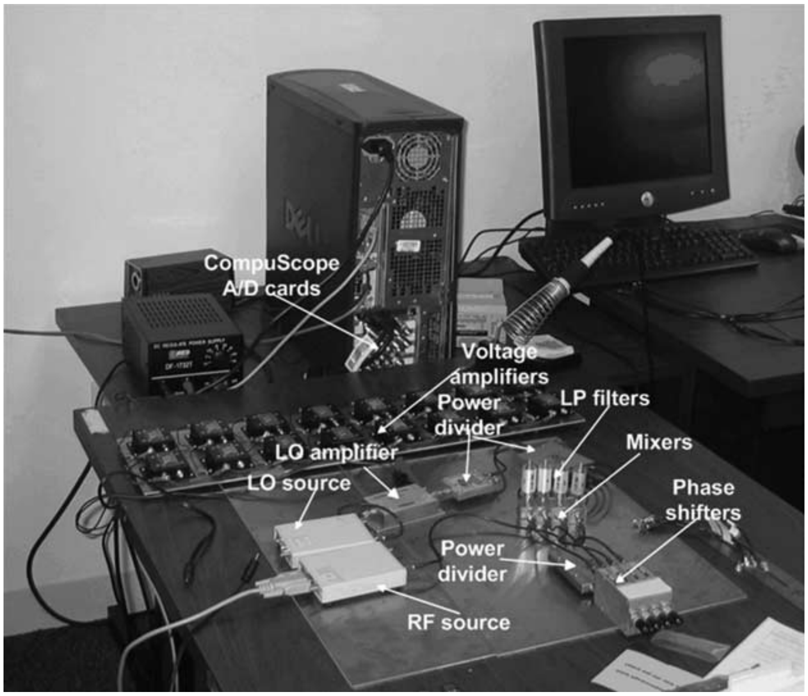

Figure 9. Photograph of the hardware setup for performance evaluation of the root-based DF algorithms.

benchmark that can be approach through the application of mutual coupling compensation techniques of increasing sophistication (Wasylkiwskyj, submitted manuscript, 2004). Since a physical antenna array does not provide an ideal decoupled environment in the experimental approach adopted herein we wanted to emulate the ideal array steering vector using variable RF phase shifters thus experimentally creating a decoupled array environment and to compare results with results obtained by numerical evaluation presented on Figures 3–7. The array outputs were down converted using standard analogue circuits and fed to a four-channel receiver providing four separate digitized data channels. The performance of the MRP algorithm was evaluated experimentally using the hardware setup shown on Figures 8 and 9. Single channel downconversion with four Mini Circuits ZEM-4300ZH mixers was implemented (thus saving one fast A/D converter per array element) and the complex baseband signal format required by the DF algorithm was synthesized in software with the aid of the Hilbert transform. The image contribution from the adjacent band was canceled employing a novel recursive sample data covariance matrix subtraction scheme [*Wasylkiwskyj et al.*, 2007]. The LO was generated by the NOVA NS3-17001002 RF signal source and amplified by the Mini Circuits RL-2300 RF amplifier and fed through the four-way MECA 804-2-1.500V power splitter delivering + 10.3 dBm to each of the four mixers. Measurements were carried out in the 1910–1915 MHz band using a –15 dBm CW signal generated by another NOVA NS3-17001002 RF signal source. The signal was fed to the second four-way MECA 804-2-1.500V power splitter followed by four Spectrum LS-002-2121 continuously adjustable RF phase shifters emulating the outputs of a linear array of decoupled elements. Power delivered to the mixer RF port was −21 dBm. Mixer insertion losses were approximately 6.6dB. IF low-pass filters (Mini Circuits SLP-5) provided 40 dB attenuation for frequencies higher than 6 MHz acting effectively as antialiasing filters. Voltage amplifiers (Advanced Receiver Research model P.O.1-30/20VD) that provided 20 dB amplification were used to amplify the signals fed to the A/D converters resulting in approximately −8 dBm signal power. The signals were sampled at 25 MHz using two Compuscope 1250-1M two-channel 12-bit A/D cards with 10 effective bits thus setting the upper limit on the SNR at almost 60dB. The phase difference between the channels was set to 25°. Assuming that this corresponds to an interelement spacing of $\lambda/2$ the equivalent DOA computed from (32) is 82.0164°. The DOA estimated by the MRP algorithm was 82.0083° giving an error of 0.008°. The record length was 2.6 ms or 65000 samples and estimated SNR was 26 dB. Figure 10 shows the

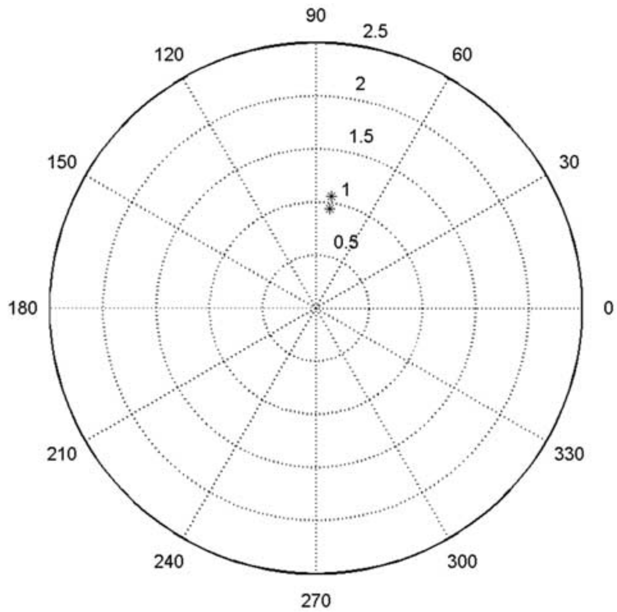

Figure 10. DOA estimation using MRP algorithm. True DOA was 82.0164°, while estimated DOA was 82.0083°, giving estimation error 0.008°.

polar diagram of the two reciprocal roots provided by the MRP algorithm. If we compare these result with result presented in Figure 4 for the same SNR value it can be seen that both numerical and experimental approaches gave the same result verifying that with an equivalent physical array aperture of only 1.5 wavelength the DOA in a decoupled array environment can be estimated with errors smaller than 0.01°.

5. Conclusion

[18] The new root-based DF algorithm was derived. The algorithm coined modified root polynomial estimates the DOAs from the polynomial of degree $2L$ where L represents number of emitters. The well known Root-MUSIC algorithms estimates DOAs from the polynomial of degree $2N - 2$ where N represents number of antennas by selecting L roots closest to the unit circle in order to estimate DOA. We have shown by extensive numerical performance evaluation that both algorithms have the same accuracy while MRP algorithm has at the same time simpler algebraic structure. Our analysis, based on both numerical and experimental data, shows that with an equivalent physical array aperture of only 1.5 wavelength the DOA in a decoupled array environment can be estimated with errors smaller than 0.01°.

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