# Linear Model of PEM Fuel Cell Power System for Controller Design Purposes

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Abstract—This paper describes a procedure of obtaining linear model with changeable parameters of the fuel cell (FC) power system. The start point was the nonlinear model of the FC power system based on Polymer Electrolyte Membrane (PEM) FC with Nafion 117 membrane. A basic stack voltage model, membrane hydration and anode flow model are included in nonlinear model. Transfer functions of stack voltage and power in relation to stack current and anode inlet flow are determined for different operating points. The influence of membrane humidity on the stack voltage and power is also considered. Parameters of transfer functions are determined by optimization using Matlab. The obtained results of parameters optimization show the significant variations in parameters, depending on the operating point, thus leading the authors to develop some advanced control strategies for the fuel cell power system.

# I. INTRODUCTION

Fuel cells have been studied intensively in the past decade. They are attractive because of the potential replacement of fossil fuels in the future, clean environment, mobile and transportation technologies and possibility of connecting with other renewable energy sources (wind, sun, etc.) into the existing energetic network.

The mathematical models are essential for investigation of the fuel cell power system behavior. There are many papers published concerning fuel cell modeling and simulation [3]–[10]. However, most of them are not oriented to the control of fuel cell power system.

This paper deals with the modeling of the fuel cell system with a Polymer Electrolyte Membrane (PEM), also known as Proton Exchange Membrane, with membrane of type Nafion 117. The accent of this paper is on linear model with changeable parameters, which is suitable for controller design.

Basics of the fuel cell are described in Section II. Section III describes a nonlinear model of the PEM FC system, which is taken from the literature [1], [2]. Section IV describes a linear models with changeable parameters, depending on the operating point. Final thoughts are given in the Conclusion.

#### II. BASICS OF THE FUEL CELL OPERATION

Fuel cells are electrochemical devices that convert the chemical energy of a fuel (hydrogen) and an oxydizer (oxygen) directly into electricity. The fuel cell consists of an electrolyte between two electrodes. The electrolyte allows the positive ions (protons) to pass through while blocking the electrons. Hydrogen gas passes over one electrode



Fig. 1. Basic principle of PEMFC operation.

(anode), and with the help of a catalyst, separates into electrons and hydrogen protons (Fig. 1, [13]):

$$2H_2 \to 4H^+ + 4e^-.$$
 (1)

The protons flow to the other electrode (cathode) through the electrolyte while the electrons flow through an external circuit thus creating electricity. The hydrogen protons and electrons combine with oxygen flow through the cathode, and produce water:

$$O_2 + 4H^+ + 4e^- \to 2H_2O.$$
 (2)

Therefore, the overall reaction of the fuel cell is:

$$2H_2 + O_2 \rightarrow 2H_2O. \tag{3}$$

# III. NONLINEAR MODEL OF A PEMFC SYSTEM

Nonlinear model of the PEM fuel cell according to [1], [2] is taken into consideration. A stack voltage model, anode flow model and membrane hydration model are considered in this paper because the FC phenomena described by these models are essential from the control point of view.

#### A. Stack Voltage Model

According to [1], the single cell voltage can be defined by following equation:

$$V_{FC} = E_{Nernst} - V_{act} - V_{ohmic} - V_{con}.$$
 (4)

The cell reversible voltage  $E_{Nernst}$  or an open circuit voltage is given by:

$$E_{Nernst} = 1.229 - 0.85 \cdot 10^{-3} \cdot (T - 298.15) + +4.31 \cdot 10^{-5} \cdot T \cdot \left[ \ln (p_{H_2}) + \frac{1}{2} \ln (p_{O_2}) \right],$$
<sup>(5)</sup>

where T denotes cell operation temperature (K),  $p_{H_2}$  and  $p_{O_2}$  are the partial pressures of hydrogen and oxygen (atm), respectively.

Activation overpotential  $V_{act}$  can be calculated by:

$$V_{act} = -[\xi_1 + \xi_2 \cdot T + \xi_3 \cdot T \cdot \ln(C_{O_2}) + \\ + \xi_4 \cdot T \cdot \ln(i_{FC})],$$
(6)

where  $i_{FC}$  is the cell operating current (A), and the  $\xi$ 's represent parametric coefficients for each cell model (Table I).  $C_{O_2}$  is the concentration of oxygen in the catalytic interface of the cathode (mol/cm<sup>3</sup>) given by:

$$C_{O_2} = \frac{p_{O_2}}{5.08 \cdot 10^6 \cdot \exp\left(-498/T\right)}.$$
 (7)

Parametric coefficient  $\xi_2$  can be determined as:

$$\xi_2 = 0.00286 + 0.0002 \ln (A) + 4.3 \cdot 10^{-5} \cdot \ln (C_{H_2}), \quad (8)$$

where  $C_{H_2}$  is determined similarly as (7).

The ohmic voltage drop can be determined by:

$$V_{ohmic} = i_{FC} \cdot \left( R_M + R_C \right), \tag{9}$$

where  $R_C$  is the resistance to the transfer of protons through the membrane, usually considered constant, and  $R_M$  is the equivalent resistance of the membrane:

$$R_M = \frac{\rho_M \cdot l}{A},\tag{10}$$

where  $\rho_M$  is the specific resistivity of the membrane ( $\Omega \cdot$  cm), A is the cell active area (cm<sup>2</sup>) and l is the membrane thickness (cm). For the membrane of Nafion 117 type ( $l = 178 \ \mu$ m), specific resistivity equals:

$$\rho_M = \frac{181.6 \cdot \left[1 + 0.03 \cdot \frac{i_{FC}}{A} + 0.062 \cdot \left(\frac{T}{303}\right)^2 \left(\frac{i_{FC}}{A}\right)^{2.5}\right]}{\left[\lambda_m - 0.634 - 3 \cdot \frac{i_{FC}}{A}\right] \cdot \exp\left[4.18 \cdot \left(\frac{T-303}{T}\right)\right]},\tag{11}$$

where  $\lambda_m$  is the average membrane water content. The mass concentration voltage drop is given by:

$$V_{con} = -B \cdot \ln\left(1 - \frac{J}{J_{max}}\right),\tag{12}$$

where B (V) is a parametric coefficient, and J represents the actual current density of the cell (A/cm<sup>2</sup>).

The dynamics of the cell can be described with firstorder delay, which exists in the activation and concentration voltages. The associated time constant equals:

$$\tau = C \cdot R_a,\tag{13}$$

where C represents the equivalent capacitance (F) of the system and  $R_a$  the equivalent resistance ( $\Omega$ ). The value of the capacitance is considered constant (some few farads), while the resistance can be determined from the cell output current and the activation and concentration voltages:

$$R_a = \frac{V_{act} + V_{con}}{i_{FC}}.$$
(14)

The instantaneous electrical power (W) supplied by the cell to the load is given by the following expression:

$$P_{FC} = V_{FC} \cdot i_{FC}. \tag{15}$$

## B. Anode Flow Model

The mass balance equation for a lumped anode flow model, with the hydrogen mass  $m_{H_2,an}$  and water mass  $m_{w,an}$  as system states, is is according to [2] given by:

$$\frac{dm_{H_2,an}}{dt} = W_{H_2,an,in} - W_{H_2,an,out} - W_{H_2,reacted},$$
(16)

$$\frac{dm_{w,an}}{dt} = W_{v,an,in} - W_{v,an,out} - W_{v,membr} - W_{l,an,out},$$
(17)

where  $W_{H_2,an,in}$  is the mass flow rate of hydrogen gas entering the anode,  $W_{H_2,an,out}$  is the mass flow rate of hydrogen gas leaving the anode,  $W_{H_2,reacted}$  is the rate of hydrogen reacted,  $W_{v,an,in}$  is the mass flow rate of vapor entering the anode,  $W_{v,an,out}$  is the mass flow rate of vapor leaving the anode,  $W_{v,membr}$  is the mass flow rate of water across the membrane and  $W_{l,an,out}$  is the rate of liquid water leaving the anode.

All flows denoted with W terms have units of kg/s. In this paper the rate of liquid water leaving the anode  $W_{l,an,out}$  is set to zero.

The calculated masses are used to determine anode pressure  $p_{an}$ , hydrogen partial pressure  $p_{H_2}$ , and the relative humidity of the gas inside the anode  $\phi_{an}$ . The pressures are calculated using the ideal gas law:

Hydrogen partial pressure:

$$p_{H_2,an} = \frac{m_{H_2,an} R_{H_2} T_{st}}{V_{an}},$$
(18)

Vapor partial pressure:

$$p_{v,an} = \frac{m_{v,an} R_v T_{st}}{V_{an}},\tag{19}$$

Anode pressure:

$$p_{an} = p_{H_2,an} + p_{v,an}.$$
 (20)

In equations (18) – (20)  $R_{H_2}$  and  $R_v$  denote gas constants of hydrogen and vapor, respectively,  $T_{st}$  denotes the stack temperature and  $V_{an}$  denotes the anode volume.

The relative humidity of the gas inside the anode is:

$$\phi_{an} = \frac{p_{v,an}}{p_{sat}\left(T_{st}\right)},\tag{21}$$

where  $p_{sat}$  (kPa) is calculated by:

$$\log_{10} (p_{sat}) = -1.69 \cdot 10^{-10} \cdot T_{st}^4 + 3.85 \cdot 10^{-7} \cdot T_{st}^3 - 3.39 \cdot 10^{-4} \cdot T_{st}^2 + 0.143 \cdot T - 20.92.$$
(22)

The vapor pressure is calculated by:

$$p_{v,an,in} = \phi_{an,in} \cdot p_{sat} \left( T_{an,in} \right). \tag{23}$$

The hydrogen partial pressure of the inlet flow is then calculated as:

$$p_{H_2,an,in} = p_{an,in} - p_{v,an,in}.$$
 (24)

The anode humidity ratio is:

$$\omega_{an,in} = \frac{M_v}{M_{H_2}} \frac{p_{v,an,in}}{p_{an,in}},\tag{25}$$

where  $M_{H_2}$  and  $M_v$  are the molar masses of hydrogen and vapor, respectively.

The mass flow rates of hydrogen and vapor entering the anode are:

$$W_{H_2,an,in} = \frac{1}{1 + \omega_{an,in}} W_{an,in},$$
 (26)

$$W_{v,an,in} = W_{an,in} - W_{H_2,an,in}.$$
 (27)

The rate of hydrogen consumed in the reaction is a function of stack current:

$$W_{H_2,reacted} = M_{H_2} \cdot \frac{N \cdot i_{FC}}{2 \cdot F},\tag{28}$$

where F is a Faraday number and N is a number of stacks connected in series.

The anode exit flow rate  $W_{an,out}$  represents the purge of anode gas to remove both liquid water and other gases accumulated in the anode (if reformed hydrogen is used). In this paper that purge is assumed to be zero.

#### C. Membrane Hydration Model

The membrane hydration model calculates the water content in the membrane and the rate of mass flow of water across the membrane.

The water transport across the membrane is achieved through two distinct phenomena: an electro-osmotic drag and "back-diffusion" [2].

The net water flow  $N_{v,osmotic}$  (mol/(s·cm<sup>2</sup>)) from anode to cathode of one cell caused by electro-osmotic drag can be expressed as:

$$N_{v,osmotic} = n_d \frac{i_{FC}}{F},\tag{29}$$

where  $n_d$  is electro-osmotic drag coefficient.

The net water flow  $N_{v,osmotic}$  (mol/(s·cm<sup>2</sup>)) from cathode to anode of one cell caused by "back-diffusion" can be expressed as:

$$N_{v,diff} = D_w \frac{dc_v}{dy},\tag{30}$$

where  $c_v$  (mol/cm<sup>3</sup>) is the water concentration, y (cm) is the distance in the direction normal to the membrane,  $D_w$  (cm<sup>2</sup>/s) is the diffusion coefficient of water in the membrane.

Combining (29) and (30) and approximating the water concentration gradient in the membrane to be linear over the membrane thickness, the water flow across the membrane can be written as:

$$N_{v,membr} = n_d \frac{i_{FC}}{F} - D_w \frac{(c_{v,ca} - c_{v,an})}{l},$$
 (31)

where l (cm) is the thickness of the membrane.

The total stack mass flow rate across the membrane  $W_{v,membr}$  can be calculated from:

$$W_{v,membr} = N_{v,membr} \cdot M_v \cdot A \cdot N. \tag{32}$$

The membrane water content, and thus the electroosmotic and diffusion coefficients, can be calculated using the activities of the gas in the anode and the cathode:

$$a_i = \frac{y_{v,i}p_i}{p_{sat,i}} = \frac{p_{v,i}}{p_{sat,i}},\tag{33}$$

which, in the case of gas, is equivalent to relative humidity  $\phi_i$ . The index *i* is either anode (an) or cathode (ca),  $y_{v,i}$  is the mole fraction of vapor,  $p_i$  is the total flow pressure,

TABLE I

PARAMETRIC COEFFICIENTS OF THE FUEL CELL SYSTEM AND PHYSICAL CONSTANTS USED IN THE NONLINEAR MODEL.

Param.	Value	Param.	Value
T	343 K	A	50.6 cm <sup>2</sup>
l	178 μm	$p_{O_2}$	0.2 atm
B	0.016 V	$\bar{R_C}$	0.0003 Ω
$\xi_1$	-0.948	$\xi_3$	$7.6 \cdot 10^{-5}$
$\xi_4$	$-1.93 \cdot 10^{-5}$	$J_{max}$	1.5 A/cm <sup>2</sup>
$R_v$	461.5 J/(kg·K)	$R_{H_2}$	4124.3 J/(kg·K)
$M_v$	$18.02 \cdot 10^{-3}$ kg/mol	$M_{H_2}$	$2.016 \cdot 10^{-3}$ kg/mol
N	100	$ ilde{F}$	96485 Coulombs

 $p_{sat,i}$  is the vapor saturation pressure, and  $p_{v,i}$  is the vapor partial pressure.

For the Nafion 117 membrane, the water content in the membrane  $\lambda_i$  can be calculated as:

$$\lambda_i = \begin{cases} 0.043 + 17.81a_i - 39.85a_i^2 + 36a_i^3 & , \ 0 < a_i \le 1\\ 14 + 1.4(a_i - 1) & , \ 1 < a_i \le 3\\ (34) \end{cases}$$

The average water activity  $a_m$  can be calculated as:

$$a_m = \frac{a_{an} + a_{ca}}{2}.\tag{35}$$

The membrane average water content  $\lambda_m$  is calculated by equation (34) using the average water activity  $a_m$  between the anode and cathode water activities. The electro-osmotic drag coefficient  $n_d$  and the water diffusion coefficient  $D_w$ are then calculated from the membrane water content  $\lambda_m$ :

$$n_d = 0.0029\lambda_m^2 + 0.05\lambda_m - 3.4 \cdot 10^{-19}$$
(36)

and

$$D_w = D_\lambda \exp\left(2416\left(\frac{1}{303} - \frac{1}{T_{FC}}\right)\right),\qquad(37)$$

where

$$D_{\lambda} = \begin{cases} 10^{-6} & , \lambda_m < 2\\ 10^{-6} \left(1 + 2 \left(\lambda_m - 2\right)\right) & , 2 \le \lambda_m \le 3\\ 10^{-6} \left(3 - 1.67 \left(\lambda_m - 3\right)\right) & , 3 < \lambda_m < 4.5\\ 1.25 \cdot 10^{-6} & , \lambda_m > 4.5 \end{cases}$$
(38)

The water concentration at the membrane surfaces on anode and cathode sides, used in (31), is given by:

$$c_{v,an} = \frac{\rho_{m,dry}}{M_{m,dry}} \lambda_{an},\tag{39}$$

$$c_{v,ca} = \frac{\rho_{m,dry}}{M_{m,dry}} \lambda_{ca},\tag{40}$$

where  $\rho_{m,dry}$  (kg/cm<sup>3</sup>) is the membrane dry density and  $M_{m,dry}$  (kg/mol) is the membrane dry equivalent weight.

To form a fuel cell stack model, the membrane hydration model is integrated with the stack voltage and anode flow models, previously described.

The parametric coefficients of the fuel cell system and physical constants are given in Table I [1], [2].



Fig. 2. Optimization scheme for determination of linear model parameters.

## IV. LINEAR MODEL OF A PEMFC SYSTEM

Based on the nonlinear model, the structure of a linear model of fuel cell system can be determined exactly. However, parameters are not so easy to determine analytically because of the complexity of nonlinear model. Therefore, the parameters of linear model are determined using MATLAB – SIMULINK [11] and OPTIMIZATION TOOLBOX [12]. The general optimization scheme for obtaining the parameters of the linear model is shown on Fig. 2. Input in the linear model is the change of input value in the nonlinear model, which determines the operating point, and the output of the linear model is the change of output value of the nonlinear model around the operating point. The integral square error criterion was used for optimization:

$$I_{crit} = \int_{0}^{\infty} e^2(t) dt.$$
(41)

All changes of input values  $\Delta u$  (Fig. 2) are made as step changes with value of 10% of operating point:

$$\Delta u = 0.1 \cdot u_0 \cdot \mathbf{S} \left( t \right). \tag{42}$$

Linear model of the fuel cell system is determined for six operating points, which are given in Table II.

The linear models of the stack voltage and power, with current density as its input, can be described with transfer functions:

$$G_{11}(s) = \frac{\Delta V_{FC}(s)}{\Delta J_{FC}(s)} = K_V \cdot \frac{1 + T_V s}{1 + T_1 s}$$
(43)

#### TABLE II

STATIONERY STATE VALUES FOR OPERATING POINTS DETERMINED BY CURRENT DENSITY VALUES.

$\begin{array}{c} J_{FC} \\ ({\rm A/cm^2}) \end{array}$	$\frac{W_{an,in}/(N\cdot 10^6)}{(\text{kg/s})}$	${m_{w,an,0}/10^4} \over { m (kg)}$	$\lambda_m$
0.2	0.3678	7.5864	11.2
0.4	0.7356	6.6836	9.4
0.6	1.1034	5.9451	8.2
0.8	1.4711	5.3211	7.3
1	1.8389	4.7836	6.6
1.2	2.2067	4.3148	6.1

and

$$G_{12}(s) = \frac{\Delta P_{FC}(s)}{\Delta J_{FC}(s)} = K_P \cdot \frac{1 + T_P s}{1 + T_1 s}.$$
 (44)

The parameters of transfer functions (43) and (44) determined for uncontrolled membrane water content  $\lambda_m$  are given in Table III. It is assumed that the anode inlet flow is controlled ideally to insure the exact amount of hydrogen to be reacted on the membrane, depending on the current (load) demands.

As seen from Tables II and III, the membrane water content  $\lambda_m$  drops down very quickly with current increase, dropping down the voltage and power as well as slowing down the cell dynamics. It is seen that already at 1 A/cm<sup>2</sup> of current density, the stack voltage drops down to zero. This is why the membrane water content (humidity) has to be controlled. Therefore, all other transfer functions determined in this paper will assume that the membrane water content is controlled at value  $\lambda_m = 14$ .

In that case, the parameters of transfer functions (43) and (44) are given in Table IV.

From the Table IV it is seen that the dynamics of the stack is faster with increase of the current density (time constant  $T_1$  is smaller). It is also seen that efficiency of the cell falls with higher current density (gain coefficient  $K_P$  is smaller or even negative).

The maximum error values  $e_m$  do not exceed 1 % in any case, so the linear model is very accurate for given operating point. The voltage, power and error responses of nonlinear and linear model for operating point  $J_{FC} = 0.4 \text{ A/cm}^2$ , with current density as the model input are given on Fig. 3.

The transfer functions of the voltage and power in relation to anode mass flow rate can be described by integral behavior:

$$G_{21}(s) = \frac{\Delta V_{FC}(s)}{\Delta W_{an,in}(s)} = \frac{K_{iV}}{s}$$
(45)

## TABLE III

Parameters of transfer functions (43) and (44) determined for uncontrolled membrane water content  $\lambda_m$ .

$J_{FC}$ (A/cm <sup>2</sup> )	$K_V$	$T_V$ (ms)	<i>T</i> <sub>1</sub> (ms)	$K_P$	$T_P$ (ms)
0.2	-60.58	78.06	187.4	2187	221.1
0.4	-62.53	283.9	414.8	975.1	601.8
0.6	-88.3	472.4	672.2	-1236	195.3
0.8	-148	525.9	778.6	-5891	496
1	0	0	0	0	0
1.2	0	0	0	0	0

#### TABLE IV

Parameters of transfer functions (43) and (44) determined for controlled membrane water content at value  $\lambda_m = 14$ .

$\begin{array}{c} J_{FC} \\ (\mathrm{A/cm^2}) \end{array}$	$K_V$	$T_V$ (ms)	<i>T</i> <sub>1</sub> (ms)	$K_P$	$T_P$ (ms)
0.2	-50.94	50.23	140.6	2338	162.5
0.4	-37.84	41.11	75.48	1727	92.25
0.6	-36.31	34.67	52.51	1045	73.21
0.8	-38.93	29.92	40.68	193.4	137.1
1	-44.89	26.14	33.5	-946.9	14.08
1.2	-55.78	22.61	28.83	-2623	19.99



Fig. 3. The voltage, power and error responses of nonlinear and linear model for operating point  $J_{FC} = 0.4$  A/cm<sup>2</sup>, with current density as the model input.

 TABLE V

 The parameters of transfer functions (45) and (46)

 determined for operating points given in Table II.

$J_{FC}$ (A/cm <sup>2</sup> )	$K_{iV}/10^4$	$K_{iP}/10^5$	$e_{Vm}$ (%)	$e_{Pm}$ (%)
0.2	6.991	7.075	1.84	1.84
0.4	3.555	7.196	2.49	2.49
0.6	2.384	7.239	2.71	2.71
0.8	1.794	7.261	2.83	2.83
1	1.438	7.274	2.90	2.90
1.2	1.199	7.283	2.94	2.94

and

$$G_{22}(s) = \frac{\Delta P_{FC}(s)}{\Delta W_{an,in}(s)} = \frac{K_{iP}}{s}.$$
(46)

The parameters of transfer functions (45) and (46) determined by optimization, for operating points given in Table II, are given in Table V.

It is seen from Table V that speed of integration is different for each operating point (gain coefficients  $K_{iV}$  and  $K_{iP}$ ). The errors of the parameters determination are larger than in the case of transfer functions (43) and (43) (Table V), but still satisfying for control purposes. The voltage, power and error responses of nonlinear and linear model for operating point  $J_{FC} = 0.6$  A/cm<sup>2</sup>, with anode mass flow rate as the model input are given on Fig. 4.

To illustrate the fuel cell power system nonlinearities, the obtained parameters of transfer functions from Tables IV and V are shown graphically on Fig. 5. It can be concluded that every parameter from stated tables changes significantly with the current density.



Fig. 4. The voltage, power and error responses of nonlinear and linear model for operating point  $J_{FC} = 0.6$  A/cm<sup>2</sup>, with anode mass flow rate as the model input.



Fig. 5. Graphical dependance of linear model parameters from Tables IV and V on the change of current density.

# V. CONCLUSION

The contribution of this paper lies in determination of linear models of the cell voltage and power in relation to the stack current density and anode inlet flow. The linear model with variable parameters is important for control purposes. The variation in model structure and parameters is closely related to controller type selection for particular control loop.

The parameters of the linear model (transfer functions) are determined by optimization using program package MATLAB, for different operating points determined by stack current density. The optimization goal was minimizing the difference between the nonlinear model taken from the literature and linear model, so that linear model follows the nonlinear model behavior in the given operating point. The obtained errors of linear models have had satisfying values.

The optimization results show that parameters of transfer functions vary significantly with the load demand (change of current density). For better illustration, these variations in parameters are graphically presented.

The change of the membrane humidity, caused by load demand, drastically effects the cell voltage and power drops. Therefore, the humidity (membrane water content) has to be controlled at constant level, ideally  $\lambda_m = 14$ . The humidity is in this paper supposed as a constant value. The FC systems have humidity control loop which would hold the humidity at desired level. The humidity model and control loop is subject of further investigation.

The nonlinear model used in this paper does not consider the possibility of increasing the cell voltage and power by increasing the oxygen pressure by using a compressor. To make the stack power more usable, a DC/AC converter should be added to the nonlinear and linear model. This is what the authors are planning to do next.

The final plans are to design the controllers of cell voltage and power based on obtained linear models. Because of the significant variations in the parameters, it is likely that these controllers will have advanced structure, but the basic PID controllers will also be considered.

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