State Feedback Optimal Controller Design for the Rotational Electromechanical System

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Abstract – In order to minimize operating time in the crane handling operation, fast load positioning with minimum load swinging has to be acquired. This conflicting control demand requires proper control action. In this paper optimal control design for the rotational crane control system is applied in order to achieve trade off between fast load transfer and minimum load swinging. In this SIMO (Single Input Multiple Output) system, state feedback controller based on Linear Quadratic (LQ) optimisation technique is realized. The controller is designed, simulated and experimentally verified on the planar rotary gantry crane laboratory test bench using Matlab/Simulink and Real Time Works. The controller performance is compared in simulation and experimentally with the performance of the state feedback Pole Placement (PP) based controller.

I. INTRODUCTION

As well as translational gantry cranes, rotational gantry cranes are widely used for the heavy loads transfer in the modern industrial systems. The problem faced in load transfer is similar as in translational crane type, considering negative influence of the crane acceleration required for the motion. Any fast ramp in reference position causes an undesirable load swing, having negative consequences on the system control and safety performances.

For the load transfer time minimization, in order to achieve more control efficiency, besides a fast load positioning the arm position and the swing of the suspended load should be controlled as well. This conflicting control demand can be solved with the state feedback linear controller designed according to the linear quadratic optimum criteria. This design technique is imposed as a logical solution and it is used by several authors for the similar control tasks.

For an associated control problem solving, most solutions are based on the linearized mathematical model. Typical control approach is adaptive (gain-scheduling logic with optimal controllers used by Corriga, Giua and Usai in [1]), optimal (Wang and Surgenor in [2]) or robust (G. Bartolini et al. in [3]), applied on the similar type of the electromechanical system.

Because of the crane system complexity and the fact that linearized mathematical model mostly doesn't represent real system good enough, some authors used fuzzy controller, [4]-[6]. Controller based on fuzzy logic can solve an undesirable effects caused by the system nonlinearities (e.g. static friction), [4]. From the real world applications point of view, the drawbacks can be in the large calculation time of the controller output caused by the number of controlled variables. This problem is reduced by applying Sugeno type fuzzy controller, but it is not completely solved. However, it is shown in [4] that for the four state variables necessarily to use (in this case those variables are arm and swing angle and theirs derivatives), four inputs and one output scaling gains have to be defined. In order to solve this problem, it is proposed to use LQ control gains as base for the fuzzy scaling coefficients determination. That is the reason to apply LQ optimisation technique based on the linearized mathematical model using well defined tools under Matlab/Simulink environment. This is first step which can be used after for the complex nonlinear controller design capable to handle with the systems nonlinearities in the real world application.

In this paper state feedback controller based on the LQ optimisation technique is realized. Using linearized mathematical model of the rotary electromechanical system, LQ controller is designed, simulated and experimentally verified on the laboratory test bench. The results are compared with the state feedback pole placement controller, in simulation and experimentally.

II. MATHEMATICAL MODEL OF ROTARY PENDULUM GANTRY SYSTEM

The rotary pendulum gantry model is presented in the Fig. 1. In comparison with the real industrial rotary crane (e.g. tower crane or boom crane), there are the following simplifications:

- Model doesn't include load hoisting drive, which means that rope length change will not be considered
- Load is represented with a lumped pendulum mass
- One degree of freedom is blocked compared to the real industrial crane; only planar gantry rotary motion is possible and load is moving in curved (round) plane (on cylinder sheat, defined by the length of the arm and pendulum length)

According to that, representing the real load, the tip of the pendulum is moving in steady state, without pendulum swinging, on the circle with radius defined with the length of the arm, Fig.1. In transient condition (acceleration, deceleration, disturbance,..) pendulum will swing in the plane π, which represents one instant of the movement where swinging is happening. This swinging plane is tangential to the plane assigned by "pendulum motion without swinging" plane (it is cylinder sheat with the base defined by the arm length).

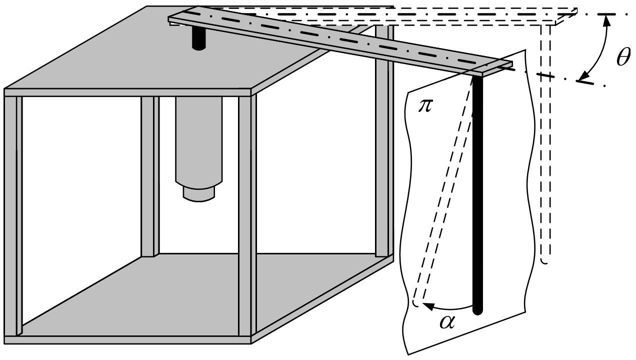


Fig. 1: Rotary Gantry Crane Model

The zero swing angle corresponds to a suspended pendulum in the vertical rest down position. Electromechanical system presented in the schematic in Fig 1. and in the real world (Fig 8), is a system with one input u (motor voltage), and two outputs: α (pendulum angle) and θ (arm position, angle). Mathematical equations of the motion can be defined via Lagrange equations using a total potential and kinetic energy,

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right)-\left(\frac{\partial L}{\partial \theta}\right)=T_m - B_{eq}\dot{\theta} \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\alpha}}\right)-\left(\frac{\partial L}{\partial \alpha}\right)=0 \quad . \tag{2}$$

Here L is Lagrangian defined as difference between total kinetic and potential energy ($L=E_k-E_p$), T_m is motor torque and B_{eq} is equivalent viscous damping coefficient according to Table 1. Solving equation (1) and (2) and linearizing around α=0 (cos α= 1 and sin α = α), yields

$$a\cdot\ddot{\theta}-b\cdot\ddot{\alpha}+G\cdot\dot{\theta}=F\cdot u \tag{3}$$

$$b\cdot\ddot{\theta}-c\cdot\ddot{\alpha}-d\cdot\alpha=0\,, \tag{4}$$

where unknown coefficients are defined in (5),

$$a=J_{eq}+mr^2\,,\ d=mgl_p\,,\ G=\frac{\eta_m\eta_g K_t K_m K_g^2+B_{eq}R_m}{R_m}\,,$$

$$c=\frac{4}{3}ml_p^2\,,\ F=\frac{\eta_m\eta_g K_t K_g}{R_m}\,,\ b=ml_p r\,,\ E=ac-b^2\,. \tag{5}$$

For obtained linear state space system in general form

$$\dot{x}=Ax+Bu \tag{6}$$

$$y=Cx \quad , \tag{7}$$

equation (6) can be transformed, using (3) and (4), in

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\dfrac{bd}{E} & -\dfrac{cG}{E} & 0\\ 0 & -\dfrac{ad}{E} & -\dfrac{bG}{E} & 0\end{bmatrix}\cdot\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \dfrac{cF}{E}\\ \dfrac{bF}{E}\end{bmatrix}\cdot u \tag{8}$$

and equation (6) in

$$\begin{bmatrix}\theta\\ \alpha\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix} \tag{9}$$

Substituting the parameters of the rotary electromechanical system from Table 1, and equation (5) to (8), follows the state space model of the rotary electromechanical system

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -40.486 & -30.061 & 0\\ 0 & -74.758 & -19.089 & 0\end{bmatrix}\cdot\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 52.916\\ 33.602\end{bmatrix}\cdot u \tag{10}$$

TABLE I

Parameters of the rotational crane model

Parameters	Description
l_p=0,3302 [m]	Length to pendulum's center of mass
m= 0,105 [kg]	Pendulum mass
r= 0,127 [m]	Rotating arm length
J_{eq}= 0.002 [kgm^2]	Equivalent moment inertia of the load
J_m= 3.87e-7 [kgm^2]	Motor moment of inertia
B_{eq}=0,004 [Nms/rad]	Equivalent viscous damping coefficient
K_t= 0.00767 [Nm/A]	Motor torque constant
K_m= 0.00767 [Vs/rad]	Back-emf constant
R_m= 2.6 [Ω]	Motor armature resistance
K_g= 70	Planetary gearbox gear ratio
η_m=0,69	Motor efficiency
η_g=0,9	Gearbox efficiency
g=9.81 [m/s^2]	Gravitational constant of earth

III. STATE FEEDBACK CONTROLLER DESIGN BASED ON LQR TECHNIQUE

State feedback controller is a simple controller suitable to handle high order systems, Fig.2. It allows the designer to locate the closed-loop poles of the system wherever they are needed. This method assumes that the state variables are measurable and are available for feedback.

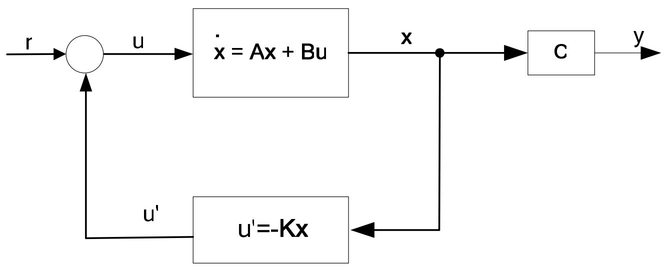


Fig. 2. State feedback controller based on the LQ optimisation technique

This controller is relatively easy to implement. Let (5) is expressed in discrete general form as

$$\mathbf{x}(k+1)=\mathbf{Ax}(k)+\mathbf{Bu}(k)\,, \tag{11}$$

where $\boldsymbol{x}(k)$ is n-state vector, $\boldsymbol{u}(k)$ is r-control vector, **A** is nxn non-singular matrix and **B** is nxr matrix. According to the Fig.2, the relationship of the feedback control $\boldsymbol{u}(k)$ to state $\boldsymbol{x}(k)$ for the pole-placement method applied to the linear system in discrete form is.

$$\mathbf{u}(k) = -\mathbf{K}\mathbf{x}(k) + \mathbf{r}(k) \,. \tag{12}$$

For the linear quadratic (LQ) case, control vector **K** in (12) is chosen as optimal vector. In the next few lines, just the main steps for optimal LQ gains determination will be presented. When the term *optimal* is involved in the control design it means that value of a function, chosen as the *performance index* or *cost function*, should be minimized (or maximized). In the rotary electromechanical system presented, the fixed finite final time quadratic performance index (13) is chosen to make the system behave in desired fashion

$$J = \frac{1}{2}\mathbf{x}^T(N)\mathbf{S}_N\mathbf{x}(N) + \\ + \frac{1}{2}\sum_{k=0}^{N-1}\left[\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k)\right] = Min \,. \tag{13}$$

Here **Q**, **S** and **R** are *nxn*, *rxr* and *nxn* positive definite or positive semidefinite symmetric matrices respectively. These matrices are selected to weight the relative importance of the performance measures caused by the state vector $\boldsymbol{x}(k)$, the final state $\boldsymbol{x}(N)$ and control vector $\boldsymbol{u}(k)$ for k=0, 1, 2,..N-1. N is the number of time steps.

Now, control vector **K** in (12) should be designed in the optimal way, minimizing performance index (13). This indicates on iterative process of changing vector **K** in order to minimize (13), before process begins. Formally, one can take **K** as **K**(k) *rxn* time-varying feedback matrix, but it is only in the process of performance index minimization, executing off line.

One of many different ways to solve (13) is concept of Lagrangian multipliers. On the base of detailed procedure described in [5], Riccati equation for solving **S**(k) is derived

$$\mathbf{S}_k = \mathbf{Q} + \mathbf{A}^T \cdot \mathbf{S}_{k+1} \cdot \left(\mathbf{I} + \mathbf{B}\cdot\mathbf{R}^{-1}\cdot\mathbf{B}^T\cdot\mathbf{S}_{k+1}\right)^{-1}\cdot\mathbf{A} \,, \tag{14}$$

and should be solved backward from k=N to k=0. Also, optimal control vector can be derived as

$$\mathbf{u}_k = -\left[\left(\mathbf{B}^T\cdot\mathbf{S}_{k+1}\cdot\mathbf{B} + \mathbf{R}\right)^{-1}\cdot\mathbf{B}^T\cdot\mathbf{S}_{k+1}\cdot\mathbf{A}\right]\cdot\mathbf{x}_k \,. \tag{15}$$

Let for the simplicity set input **r** =0, which means that system starts in t=0 from some state $\mathbf{x}(0)\neq 0$ reaching the final position $\mathbf{x}(0)=0$ in N steps, Fig.2. Then from (12) and (15) controller gains matrix can be calculated as

$$\mathbf{K}_k = \left(\mathbf{B}^T\cdot\mathbf{S}_{k+1}\cdot\mathbf{B} + \mathbf{R}\right)^{-1}\cdot\mathbf{B}^T\cdot\mathbf{S}_{k+1}\cdot\mathbf{A} \,. \tag{16}$$

Once the state matrix **A**, control matrix **B**, and weighting matrices **Q**, **R** and **S** are known, the time-varying controller gains matrix **K**(k) may be predetermined off-line. This is important from the real world application point of view. For a given set of **Q**, **R** and **S** weighting matrices, Linear Quadratic Regulator (LQR) control is optimal and unique. However, more than one starting set of matrices will result specifications to be satisfied, and any of such set of matrices will be acceptable as optimal problem solution. Therefore, from this point of view, the solution of this problem is not unique. Formally, it means that any initial choice of weighting matrices used in (13) as symmetric and positive definite or positive semidefinite, is acceptable.

However, a set of specifications in the form of maximum magnitudes for each state variable (arm and pendulum angle), control variable (motor voltage) and state variables at the final time (reference arm position) must be imposed in the system. In each off-line LQ controller gains session (**K**(k) calculation), states and control physical limitations have to be checked. The calculation must be repeated with different **Q**, **R** and **S** matrices as long as all specifications are satisfied or becomes evident that can not all be satisfied at the same time. Using some techniques like Bryson's method, will often speed up the design process by requiring a fewer iterations to reach the final design, [5]. With Bryson's method, the diagonal elements of the weighting matrices are the reciprocals of the squares of the maximum allowed magnitudes for the states, final state and control variables. Other elements are zero.

$$\underline{Q}_k = \begin{bmatrix} q_{11} & 0 & . & . & . \\ 0 & q_{22} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & 0 \\ . & . & . & 0 & q_{nn} \end{bmatrix} \quad q_{ii} = \left(\frac{1}{|x_{ki}|^2_{\max}}\right)_{\forall k} \quad i = 1, 2, ., \mathrm{n} \tag{17}$$

$$\underline{R} = \begin{bmatrix} r_{11} & 0 & . & . & . \\ 0 & r_{22} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & 0 \\ . & . & . & 0 & r_{nn} \end{bmatrix} \quad r_{ii} = \left(\frac{1}{|u_{ki}|^2_{\max}}\right)_{\forall k} \quad i = 1, 2, ., \mathrm{n} \tag{18}$$

$$\underline{S}_N = \begin{bmatrix} s_{11} & 0 & . & . & . \\ 0 & s_{22} & . & . & . \\ . & . & . & . & . \\ . & . & . & . & 0 \\ . & . & . & 0 & s_{nn} \end{bmatrix} \quad S_{Nii} = \left(\frac{1}{|x_{ki}|^2_{\max}}\right)_{k=N} \quad i = 1, 2, ., \mathrm{n} \tag{19}$$

This method for initializing weighting matrices shows faster convergence to an acceptable design than some other methods (e.g. identity matrices). The corrections of the coefficients of matrix **Q** in iterative procedure is done by following expression

$$(q_{ii})_{NEW} = (q_{ii})_{OLD} \cdot \left(\frac{|x_i|_{\max\_ACTUAL}}{|x_i|_{\max\_SPECIFIED}}\right)^2 \,. \tag{20}$$

In the similar way, (20) can be applied for coefficients correction of **R** and **S** matrices too.

IV. SIMULATION RESULTS

System performances with the LQ controller, applied on the rotary single pendulum gantry electromechanical system, are tested in the simulation, Fig.3.

Model is realized in Matlab/Simulink environment, and prepared also for the real-time running, according to the simulation schematic shown in the Fig.3. Because the system is SIMO type, control variable is scalar and then according to (13) matrix **R** is scalar too. The matrix **Q** is dimension of 4x4, which means that first diagonal element **Q**(1,1) corresponds to the first state variable (arm angle) in state vector $\mathbf{x}=[\theta\ \alpha\ d\theta/dt\ d\alpha/dt]^T=[x_1\ x_2\ x_3\ x_4]^T$, second element **Q**(2,2) corresponds to the pendulum angle, etc. There is no rule to set coefficients of **Q** matrix;

it is a matter of designer's requirements. The large value of **Q** relative

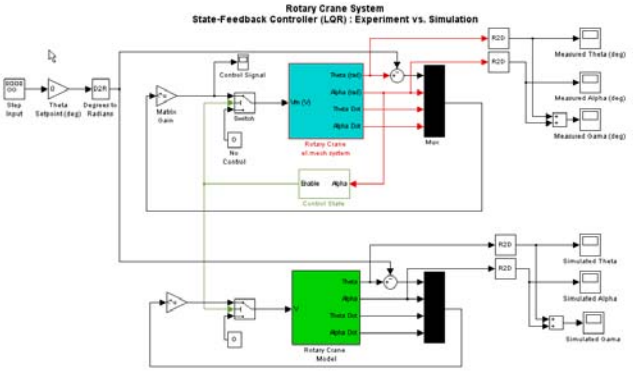


Fig.3. Experimental (upper) and simulation (lower) model.

to **R** forces state variables $\boldsymbol{x}$ to rapidly reduce from its initial states. Also, the final state of $\boldsymbol{x}$ will be very low for a large $\mathbf{S}_N$ weighting matrix (it is set to 100). This leads to the high control gain and quickly reaching of the steady state values. When the values of **Q** and **R** interchanged, the performance index (13) is now placing more emphasise on keeping the magnitude of the control signal $\boldsymbol{u}$ small, than on making the magnitude of the state variable small. Now, the value of control gain **K** becomes smaller and consequently control signal $\boldsymbol{u}$ and state variables will not be driven towards the origin as rapidly.
The constraints (CNS) on the state variables and the control signal are set as

CNS=[15*π/180, 5*π/18,0 4*π, 4*π, 4.9], (21)

which correspond to θ, α, $d\theta/dt$, $d\alpha/dt$ and u respectively. According to the Bryson's method described, **Q** and **R** matrices are determined from (17)-(20) and LQ gains **K** from (16) as

$$Q=\begin{bmatrix}14.5903 & 0 & 0 & 0\\ 0 & 44.9860 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix};\ R=0.0416 \qquad (22)$$

$$K=\begin{bmatrix}18.7166 & 21.4887 & 2.8692 & -3.4549\end{bmatrix}. \qquad (23)$$

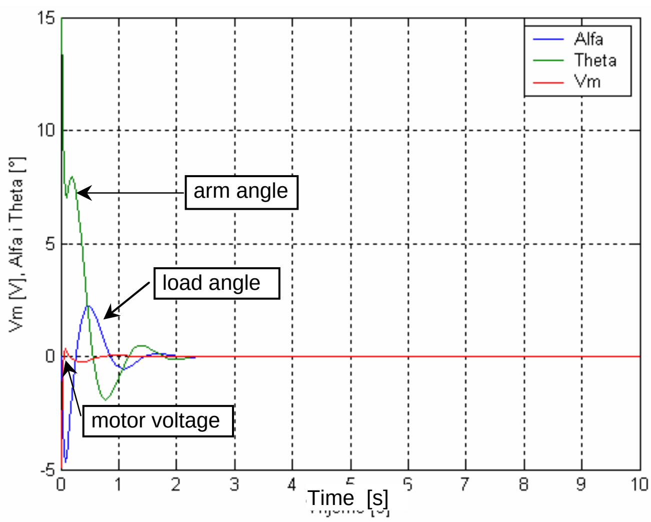


Fig.4. Arm angle, load (pendulum) angle and motor voltage for matrices **Q** and **R** coefficients (22) and controller gains (23) with initial arm angle disturbance of 15°.

According to (22) and (23), system response on disturbance θdist=15° is shown in Fig.4.

The response is very fast inside the constraints set (21). The response on the arm angle step reference of θref=15° is presented in Fig.5.

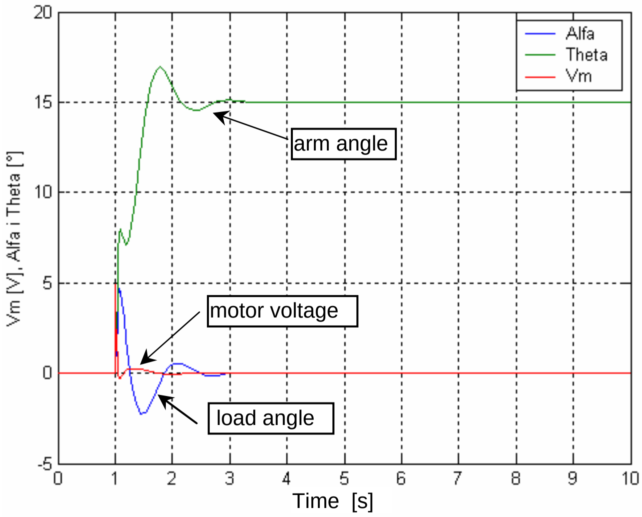


Fig.5. Arm angle, load (pendulum) angle and motor voltage for matrices coefficients (22) and controller gains (23) with arm angle step reference of 15°.

The simulation results in Fig.5. show that constraint of the arm angle in (21) is accessed. System is "too fast", which doesn't mean that is not good! One way to fix that is to placing more emphasise on the first state variable (θ) in state vector, changing coefficient $\mathbf{Q}(1,1)=q_{11}$ or changing other diagonal coefficients in matrix **Q**. Just for the test, it is taken identity matrix **Q** (all diagonal coefficients of matrix as 1). In that case, calculation for the optimal controller gains **K** gives

$$K=\begin{bmatrix}4.9000 & 33.3842 & 4.8032 & 0.8581\end{bmatrix}, \qquad (24)$$

with matrix **R** unchanged, as in (22).

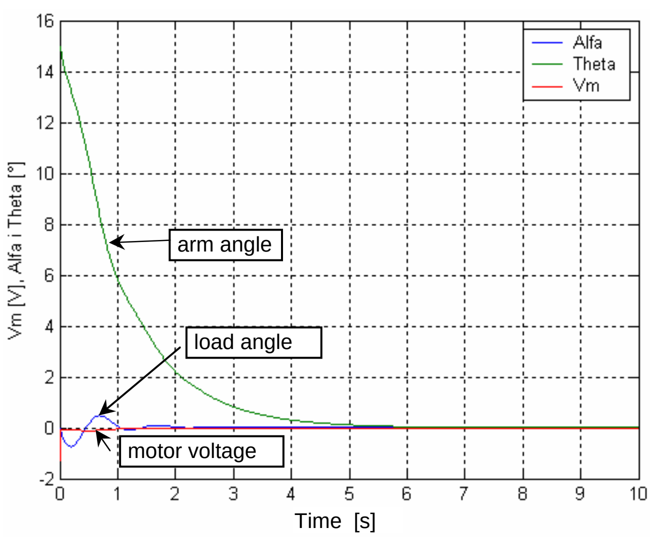


Fig.6. Arm angle, load (pendulum) angle and motor voltage for **Q** identity matrix, **R** matrix (22) and controller gains (24) with initial arm angle disturbance of 15°.

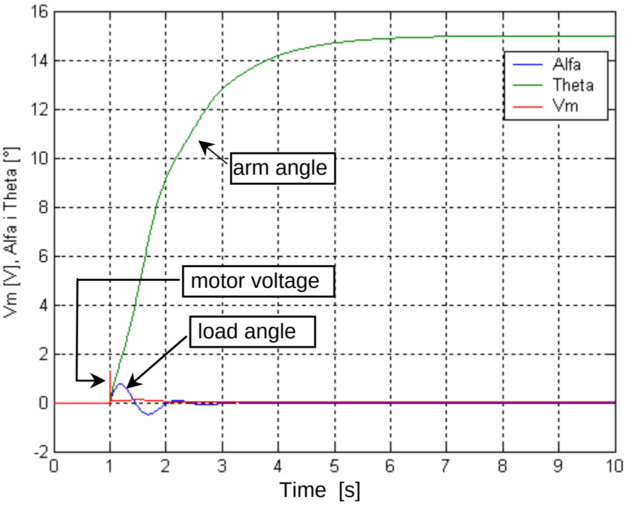


Fig.7. Arm angle, load (pendulum) angle and motor voltage for **Q** identity matrix, **R** matrix (22) and controller gains (24) with arm angle step reference of 15°.

From these two simulation tests, one can conclude that the first simulation (Fig.4. and 5.) gives the better control performance even though the arm angle constraint is reached. Second simulation session (Fig.6. and 7.) gives slow arm angle response with the smaller load (pendulum) swinging. From the application point of view, one has to think here also in "optimal" way. That means, if there are enough manoeuvring space around the crane and fast load positioning is requested, the first solution would be acceptable even the swing is higher than in the second case (but still inside the constraints (21)). Of course, there can be solutions between presented two. It is necessary to change matrices **R** and **Q** and in iterative procedures, analysed in this section, find acceptable solution.

V. ROTARY PENDULUM GANTRY EXPERIMENTAL SETUP

Experimental tests have been made on electromechanical model of a rotary pendulum gantry, which has three basic parts: rotary base with the DC motor driven arm with the pendulum, AD interface and digital control system implemented on the personal computer, Fig.8.Laboratory system use microprocessor of personal computer (PC) for the simulation and software development as well as for the real-time control. Control algorithm, designed and simulated in MATLAB/Simulink environment, uses graphically oriented software interface for the real-time code generation. This application oriented code is running in the real time under the same PC as for simulation. Details about experimental test bench can be found in [8].

In Fig.9.a) and 9.b) simulation and experimental results are compared for arm angle step reference of 25°. The matrices **Q** and **R** are identical for the simulation and the real time experiment. One can see a few differences. Firstly, arm position response has overshot in simulation; it seems faster then in real time experiment and, as a consequence, settling time in simulation is considerably smaller. Secondly, steady state error in simulation is zero, but in the experiment this error is cca 8%! In the first case these differences are taken from the simplified (linearized) model used in simulation. Due the linearization, model is acceptable only for small load angle (small swinging). For the second case, critical phase is arm positioning because of unmodeled static friction which unavoidably results in the steady state positioning error.

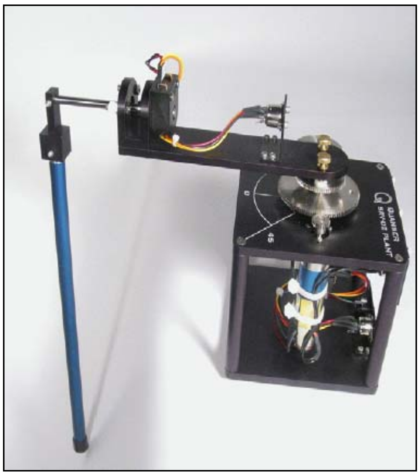
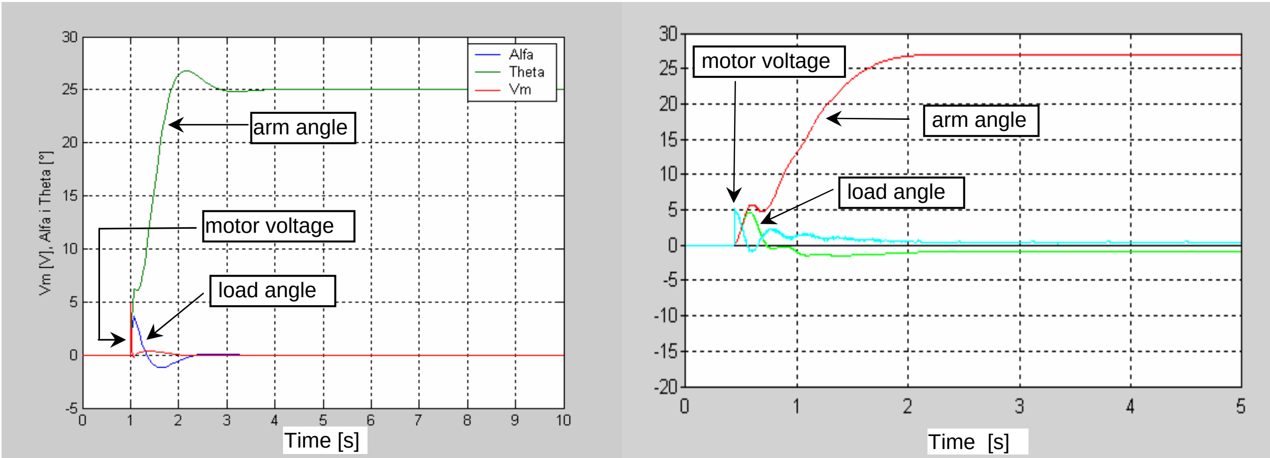
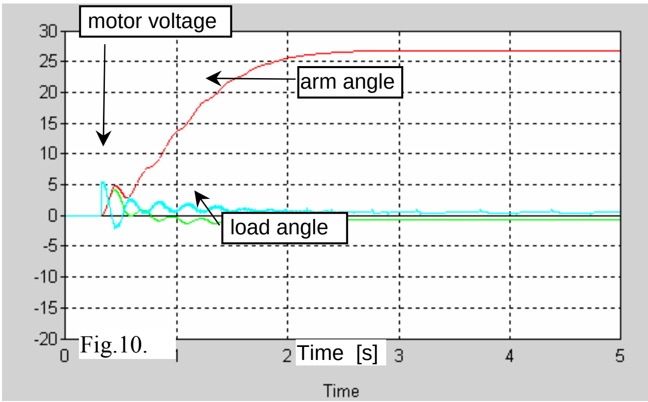


Fig. 8. Rotary Crane laboratory model



a) b)

Fig.9. **LQR** based controller: Simulation a) and experimental responses (arm angle, motor voltage, load angle), b) for **Q**(1,1)= 5.2525, **Q**(2,2)= 131.3123, **Q**(3,3)= 0.0063, **Q**(4,4)= 0.0063, **R**= 0.0416, under arm angle step reference of 25°



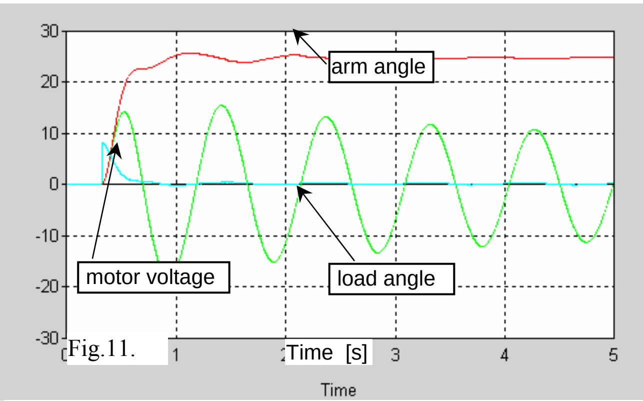


Fig.10. **PP** based controller: Experimental responses (arm angle, motor voltage, load angle) for the same parameter as in Fig.9.b). The Comparison LQR-PP controller. Step arm angle reference of 25°.

Fig.11. **LQR** based controller: Experimental responses (arm angle, motor voltage, load angle) for matrices coefficients (22) and controller gains (23), same parameter as in Fig.4. and 5. Step arm angle reference of 25°. NO load angle control.

This analyze confirm Fig.10, where pole placement (PP) based state feedback controller is designed only for comparison with LQ controller. The latter is designed with control requirements on transient response settling time of 1,5 s and overshoot 2%. These requirements were fulfilled only in the simulation, but in experiment on the rotary electromechanical gantry drive, settling time about 2,5s without overshoot and steady state error about 8% were accomplished. More or less, experimental results for the PP and LQ based state feedback controller are practically identical, see Fig. 9.b).and Fig.10. In both cases steady state error is to high and this problem can be solved using either integrator (not enough good solution), or some of control technique such as tensor product transformation based modelling, proposed in [6,7] Fig.11. undoubtedly presents that without load (swing) angle control, crane control task can not be performed effectively; load swinging is too high for the accurate load positioning.

VII. CONCLUSION

For the real time application, state feedback LQ optimal controller is designed, simulated in Matlab/Simulink and experimentally verified on the laboratory test bench. This model is scaled and simplified model of a real industrial rotary crane.

LQ optimal design for the state feedback controller request iterative 'off line' procedure for controller gains calculation, minimizing selected performance index, taking into account all states and control systems constraints, as well as weighting matrices **Q**, **R** and **S**. One set of these matrices guarantees a unique solution for the optimal controller gains, but some other set can also result predefined specifications to be satisfied. The comparison of simulated and experimental results in Fig.9.a) and 9.b), show different transient response; smaller overshoot, long settling time and large steady state error of experimental response vs. simulation. These unsatisfactory experimental results are practically same for the PP controller too, Fig.10. The logical explanation is that the linear controllers cannot perform crane control task effectively as nonlinear controllers can do.

Although the optimality between crane arm positioning and load swinging is just what the cranes "desperately" need, LQ optimisation technique cannot handle problems like the unavailability of accurate linear model, static friction compensation, and other system's nonlinearities.

The nonlinear industrial crane system require definitely nonlinear control and in this respect LQ controller can serve as a base controller for supporting main nonlinear controller in order to fulfil control task in satisfactory manner. The example of such LQR application is presented in [4].

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